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BULLETIN NO. 51

STREET LIGHTING

BY

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AND

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UNIVERSITY OF ILLINOIS
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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 51 DECEMBER 1911

STREET LIGHTING

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STREET LIGHTING

I. Introduction

1. *Preliminary.*—It is the purpose of this bulletin to make available information concerning street illumination. The suggestion which led to this compilation came from the many inquiries received by the Electrical Engineering Department each year from those interested in framing ordinances permitting corporations or individuals to operate street lighting systems. An attempt has been made to present this information in such a form as to be readily understood by the general public, without requiring any special technical knowledge. The data have been compiled from reliable sources, and checked in many instances by tests conducted by the writers.

This bulletin is designed to be of assistance to central station superintendents, and to the general public in selecting the proper lamp and fixing the charge for the same. It is also designed to be of value to the illuminating engineer and to the manufacturer, and in clearing up, or perhaps in preventing misunderstandings, which so frequently arise between municipalities and power companies. These misunderstandings, being usually the outcome of imperfect knowledge of the effectiveness and limitations of ordinary street lighting units, may readily be prevented if specifications and contracts be made definite and clear.

Many contracts are based upon the rates charged in cities of about the same size for the same class of service, regardless of the comparative operating cost of the installation in each case. Many companies do not compute the cost of any one particular branch of their service, but are satisfied if their total yearly balance shows a profit. Other companies, while knowing the cost of operation of the street lights, are still governed by the prevailing rate in similar cities. Thus it is that some companies may furnish the light to the city streets at an actual loss, but are compensated for this loss by special privileges in other branches of the service. It is believed by the writers that fewer misunderstandings would arise between the consumer and the company if each branch of service received its proper compensation.

This bulletin is also designed to show that the same type of light will not serve equally well for the illumination of all classes of city streets; and that certain accepted standard spacings of lights can never produce adequate and economical street illumination.

II. Production of Light

2. *General Theory.*—Probably the most familiar form of illuminant is the flame. At least, this is the earliest historical form. In the flame, the light is produced by small particles of carbon being brought to a temperature at which they will unite with the oxygen of the air. The heat which gives to these particles their incandescence comes from the union of the various compounds in the combustible with the oxygen. If the flame is cooled in any manner, the carbon will deposit in the form of soot before it reaches a sufficient temperature to burn. If enough air be mixed with the gas formed by the heated illuminant to consume the compounds, an almost non-luminous flame results. Thus the alcohol flame is nearly non-luminous; as is also that of the oxyhydrogen flame and the Bunsen burner. It is well-known that non-luminous flames may be made luminous by the introduction of some foreign substance which need not necessarily be consumed; of such character is the Welsbach burner.

In the Welsbach burner, another interesting phenomenon takes place. If the substance being heated is of a certain character, which is represented by carbon, a combination of colors is obtained for a given temperature. This is known as white light. If, however, the substance be some one of the metals, certain of the colors will be more pronounced; i. e., not all substances give off the same color when heated to the same temperature in a flame. This phenomenon is known as selective radiation. Use is made of this in the newer forms of illuminants in producing more light for a given amount of power or of illuminant. In the Welsbach burner, thoria and seria are spread upon a thin web of heat resisting material. These compounds, when heated to the high temperature of the gas flame, give off a much greater amount of light than would be given by carbon at the same temperature. Many other metals display this property to a greater or less extent. In this field, lie the recent advances in the development of arc lights.

3. *The Incandescent Lamp.*—In the incandescent lamp, light is produced by a fine filament or wire of some material, heated to

incandescence by a current of electricity. Formerly, all such lamps were made with filaments of carbon. These were followed by metallized filaments, and later by tantalum and tungsten, the latter being now used in the form of a drawn wire. All of these lights have their filaments in a vacuum to prevent rapid oxidation of the material, and also to prevent the heat from being conducted away from the filament as rapidly as it would be in air or any dense gas. Such lamps are made for house lighting and street lighting.*

4. *The Arc Lamp.*—(*a*) If two pieces of conducting material be connected in an electrical circuit with their ends touching, and these ends be separated slowly, a spark will be seen between them. If the voltage of the circuit be sufficient, portions of the terminals will be burned away, forming a vapor between them. This vapor is in itself a conductor, and current will continue to flow, even though the terminals are rather widely separated. Light will also be given off from the electrodes and the arc stream, thus giving the arc light.

In the earlier forms of arc lamp, plain carbon rods were commonly used for the terminals, or electrodes, as they are called. The voltage to be used across a given lamp, and the current necessary, are so determined for each type as to produce the proper characteristics. The old direct current open arc lamp required about 50 volts and took 9.6 amperes. This lamp was known as the "full arc" or 2000 "nominal candle-power" lamp. "A half arc" lamp using 6.6 amperes, known as the 1200 "nominal candle-power" lamp, was also used to a considerable extent. In any given type of lamp, the amount of light and the consumption of the carbons depend upon the current used and the size of the carbons. The light given off by an arc light proceeds partly from the electrodes and partly from the vapor or arc stream.

(*b*) In the direct current open arc, most of the light comes from a crater formed in the positive electrode. The upper electrode is made positive to throw most of the light in a downward direction, i. e., in the lower hemisphere. Only about five per cent of the light comes from the arc stream. The voltage across such a lamp must be increased when the arc is lengthened, but not directly in proportion to the distance between the electrodes. This is due to the fact that nearly a constant value of electromotive-force is consumed in vaporizing the electrode material.

*Tests on multiple burning lamps of different classes have been made by the Electrical Engineering Department, and reported in Engineering Experiment Station Bulletins No. 19 and 33.

Since any energy used in the arc stream produces but little light and lowers the efficiency, it is desirable to use as short an arc as possible without cutting off too much of the light. A long arc is also more difficult to maintain steady on account of the influence of air currents. The electrodes in such an arc are consumed rapidly; the positive being consumed about twice as fast as the negative. From 1.5 to 2.0 in. of positive electrode are consumed each hour. Such a lamp must be trimmed each day, i. e., the electrodes must be replaced, entailing expense in labor and material.

(*c*) The rapidity of the consumption of the electrodes in an open arc is due to their oxidation in the air. In order to lengthen the life, or number of hours of burning, of the electrodes, and decrease the expense due to trimming, the enclosed arc lamp was invented. In such a lamp, the arc is produced as in the open arc between carbon electrodes. It is, however, enclosed in a nearly air-tight chamber of glassware known as the inner globe, to distinguish it from the much larger outer globe used on nearly all arc lamps. The air being excluded the electrodes are consumed slowly, their life averaging about 100 hrs., or about 10 times that of the open arc. However, more expensive electrodes must be used, since they must fit quite closely into a gas cap at the top of the inner globe, and yet be fed by the mechanism with perfect freedom. Forced carbons are used in enclosed arcs and moulded carbons in open arcs. Enough air must be introduced into the inner globe of such a lamp to consume all the carbon volatilized by the arc, in order that it shall not deposit on the interior of the inner globe and shut in the light, lowering the efficiency of the lamp.

(*d*) Later improvements in the production of light from an arc lamp consist in securing more light from the arc stream. This has been accomplished by impregnating the electrodes with some metallic salt. The light from an arc stream of pure carbon is purplish in color, but since it comprises such a small proportion of the total light from the lamp, its effect is scarcely noticeable except when colors are being matched. Salts of certain metals, such as calcium, strontium, barium, titanium, sodium, etc., give their own characteristic color to the light, raising its efficiency by luminescence. These salts tend to make the arc unsteady or flickering, necessitating the introduction of other compounds to steady the light. Arc lamps using impregnated electrodes are known as flame arcs. When magnetite is used for

one of the electrodes, the lamp is called a magnetite or luminous arc, the latter being a trade name. Magnetite, containing iron, furnishes the conducting material in the arc stream. Most of the light is produced by a compound containing titanium, which is mixed with the magnetite. The electrode consists of a thin iron tube into which ground magnetite, titanium oxide, and chromite are packed. The magnetite melts, volatilizes, and carries with it the titanium. The chromite is used to absorb the fluid magnetite and steady the arc.

The electrodes in a flame arc lamp are consumed very rapidly. Even with long electrodes (14 to 18 in.), they do not last more than one night, or about 10 to 17 hours. The magnetite electrode, on the other hand, lasts from 85 to 200 hours, depending upon the current used. Since the electrodes contain considerable quantities of foreign impregnating materials which are not consumed in the arc, it is impossible to apply an ordinary inner globe to such a lamp. In fact, a draft tube or chimney is usually employed to carry away the products so that they are not deposited on the outer globe. It is obvious that such an opaque deposit would shut in most of the light after the lamp had operated a short time.

In the long burning flame arc, an attempt has been made to partially enclose the arc. Such a lamp has a fairly close-fitting inner globe and a condensing chamber. The gases pass first from the arc to the cool condensing chamber, where they deposit the foreign material. They return along the sides of the inner globe by natural draft. Otherwise the operation is similar to that of the enclosed arc lamps. The electrodes have a life of about 70 hours with ordinary current densities.

III. Systems of Distribution

5. *Series.*—There are two systems of electrical distribution, series and multiple. The series system is almost universal for street lighting. Either direct or alternating current may be used on either of these systems. Certain types of arc lamps will operate on either direct or alternating current, others on direct current only. In the direct current system, the current always flows in the same direction, i. e., from the positive terminal of the generator through the external circuit, and back to the negative terminal of the generator. In the alternating current system, the direction of flow of current changes very rapidly, and the electrodes of a lamp must change in polarity at the same time as

that of the circuit. In a 60-cycle alternating current circuit, the polarity changes 120 times per second. It is obvious that the current must be zero at some instant during the change. Hence the conducting path must have a low enough resistance to allow the current to pass again easily. In order to produce sufficient conducting material for an alternating current arc, one or both of the electrodes is made hollow, and filled with a softer and more volatile substance in the form of a continuous core. These electrodes are slightly more expansive than those used in the direct current lamps.

For street lighting, lamps are usually operated in series, i. e., the same current passes from the station through all the lamps in one group or circuit and back to the station. Apparatus is provided at the station to keep the current constant for each circuit of lamps. The mechanism of the lamp regulates the arc length and holds the voltage across the arc constant. This mechanism must also be able to start the lamp at any time, or to cut it out of service without breaking the circuit, if, for any reason, the lamp is inoperative, as any break in the circuit will cut out all the lamps of that circuit.

6. *Multiple.*—The first machines for supplying electrical energy from central stations were for the production of direct current. These machines were nearly always made to supply power to incandescent lamps at a low potential, about 100 volts. They were unsuitable for operating more than two arc lamps in series. Accordingly, direct current arc light generators were devised. Many of the early types are still in use with very little change from the first design. When only direct current is available from a station, it is necessary to provide a separate machine for the arc lights, since there is no means of changing low voltage direct current to a higher voltage. These machines can not be used to assist the low voltage machines in times of emergency. They thus form an additional investment and source of expense to the station for repairs.

Upon the introduction of alternating current generators and systems of distribution, direct current was driven out of the field where power must be transmitted for any considerable distance, on account of the fact that alternating currents may be transformed from one voltage to another readily and with very little loss. With low voltage distribution, a large investment, and consequent high fixed charge is required, due to the heavy copper conductors necessary for efficient transmission. Since the

greater part of the load on most central stations is due to incandescent lamps operated on multiple circuits, it was found desirable to install apparatus of constant voltage for their supply. Alternating current arc lamps were readily devised, and the direct current arc light generators were replaced by regulators or by constant current transformers, thus reducing the cost of station equipment.

Alternating current arc lamps are not as efficient light producers as direct current arc lamps, but the combination of alternating current arc lamps and regulators is more desirable than separate direct current arc light machines. Recently, rectifiers have been invented to convert constant alternating current into constant direct current, so that nearly any type of lamp may be operated with fair efficiency and continuity of service from an alternating current constant potential generator. This has led to increased activity in the perfection of direct current arc lamps for street lighting. Thus, to operate constant alternating current arc lamps from a central station supplying constant potential alternating current for multiple incandescent lighting, there will be required additional equipment of arc light transformers or regulators. To operate direct current arc lamps from such a station requires the same equipment as for alternating current lamps and the addition of a rectifier outfit for each group of lamps; or separate direct current arc light generators must be installed. Series incandescent lamps may be operated on either of these circuits.

A standard voltage of 110 volts has been adopted for use on multiple lighting circuits. Since arc lamps by their characteristics require considerably less voltage, some device must be used in series with the arc on multiple arc lamps to limit the current to the proper value. Any such device entails a considerable loss. In D. C. lamps, this loss is proportional to the differences between the line and arc voltage, multiplied by the current flowing. When the line voltage is a little greater than twice the arc voltage two arcs may be operated in series. This method is not satisfactory, since if one arc goes out, it extinguishes the other in series with it, or some complicated mechanism must introduce a resistance capable of absorbing the same voltage as one arc lamp.

In an A. C. system, a device known as a reactance coil may be used to reduce the voltage, at the same time absorbing only a nominal amount of power, not over 10% of the lamp watts. Such a device, however, introduces other undesirable factors into the circuit.

The reason for using series instead of multiple lights, is partly due to the less efficient lamps, and partly to the larger and much more expensive conductors required for the multiple system. The current for each group of lamps may be carried over a rather small size of conductor, without regard to the number of lamps in circuit, if they are all connected in series. A No. 8 B & S gauge wire would be of sufficient current carrying capacity. Some companies, however, use No. 6 or No. 4 on account of the greater mechanical strength when open wiring is used. If the same number of lamps were fed over a multiple circuit, several times the amount of money would have to be invested in copper, making a more expensive and less efficient installation.

IV. Photometry and Illumination

7. *Photometry.*—The accurate photometric measurement of the intensity of light from an arc lamp is very difficult to make even with the facilities of the best laboratories. This is due to the unsteadiness of the light. Devices have been made for determining the mean spherical intensity of such lights at one setting. These devices are, however, very expensive, and are beyond the means of most central station laboratories. The distribution curves, as published by many different writers, are usually quite reliable. They are sufficiently accurate for calculating street illumination. Such curves are illustrated in the following pages.

It must be remembered in these comparisons that most curves are taken in a laboratory where the lamp is kept in proper adjustment throughout the test, and the glassware carefully cleaned. The actual illumination received from such lights, when in continuous service, and with ordinary care in trimming, does not average over 80 per cent of the laboratory values. This loss of light is due to dirt on the glassware, deposit on the inner globe of enclosed lamps, and lack of adjustment of the mechanism due to wear, corrosion, etc. On the whole, illumination readings of actual installations are much more reliable, if made with sufficient care.

8. *The X Value.*—There is great need of a new standardization of the method of specifying street illumination in this country. In 1907, the National Electric Lamp Association adopted a standard specification for street lights. Comparative tests of the illumination thrown upon the street, at distances of between 200 and 300 ft. from the arc, were made upon the various street lights

then in use. From these values, a factor known as the "X" value of the lamp was computed. The X value shows the relation between the illumination from the lamp in question, and that given by a standard 16 candle power incandescent lamp at 1/X of the distance. Thus, for example, an arc lamp, having an X value of 4, gives the same illumination as a 16 candle power incandescent lamp at $\frac{1}{4}$ the distance, and a lamp having an X value of 5 gives the same illumination as the 16 candle power incandescent lamp at $\frac{1}{5}$ the distance. Thus a lamp which has an X value of 5, at a distance of 250 ft., gives the same illumination as a 16 candle power incandescent lamp, at a distance of 50 ft. With the advent of the demand for brighter illumination in this country, necessitating closer spacing of the lamps, this specification is no longer a measure of the value of the lamp. Other methods of specification used in different countries, or by different engineers, include horizontal illumination on a plane about four feet above the street surface, illumination on a vertical plane at a given point, and illumination on a plane normal to the rays of light from the lamp and at various distances.

As will be shown later, the illumination on the street surface depends not only upon the light at a certain angle, but upon the distribution of light about the arc at all angles, and upon the height and spacing of the lamps. The specification of the illumination should also include the distribution of light about the lamp for a given condition. The amount and character of the illumination required depends upon the character of the street surface and the use made of the street. Thus a lamp with a good X value may be suitable for lighting one street, but wholly unsuitable for another.

9. *Measurement of Illumination.*—Since the light from any one arc lamp varies continuously on account of the arc shifting about on the ends of the electrodes, it is difficult to arrive at accurate results in the measurement of street illumination. In thoroughfares where several arcs per pole or incandescent lamps are used, the illumination is much more steady. The illumination may be measured by some form of illuminometer. Where accurate results are not necessary, some form of reading photometer may be used. Such an instrument is shown in section in Fig. 1. The observer stands with his back toward the light, and looks into the photometer through the aperture A, his eyes being shielded from other lights. The illumination to be measured is that upon the surface C, which is a card containing lines of letters

or characters of varying boldness. Such a card is illustrated in Fig. 2. The light which falls on the surface C comes into the

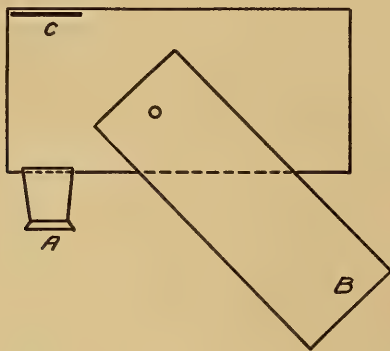

FIG. 1. SECTION OF A READING PHOTOMETER

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FIG. 2. SAMPLE CARD FOR READING PHOTOMETER

photometer through the tube B. The line of type on the card is so selected as to show the small characters dimly or blurred, while the large characters and capitals remain clear-cut. The distance from the observer to the light is also measured.

By previously calibrating the instrument by comparison with a standard lamp, values of illumination may be determined with an accuracy of from 5 to 10 per cent, depending upon the intensity of the illumination to be measured. Calibration is performed by finding the distance from the standard light at which the different lines of characters may be read. The illumination in foot candles may be computed from the candle power of the lamp and from the distance in feet as follows. Suppose the lamp giving 16 candle power to be in a horizontal position, and suppose the distance from the lamp to be 4 ft. Then since the illumination, from a small object as a source, varies inversely as the square of the distance, the illumination would be $\frac{16}{4^2} = 1$, or one foot candle. This instrument measures the illumination on a normal plane. To reduce the illumination to that on a horizontal plane, these values must be multiplied by the cosine of the angle between this plane and the street surface at that point. This form of illuminometer is inexpensive and may be constructed in any carpenter shop. Other more expensive illuminometers are on the market which are more accurate, but are also more difficult to operate. Unless there is considerable reflection from buildings, the street illumination may be calculated quite approximately from the light distribution curve of the lamp, its height and the distance from its base to the point considered, as will be explained later.

10. *Distribution Curves.*—On the following pages will be found the distribution curves of the more common lamps used for street lighting. Data for the distribution curves are obtained by measuring the candle power at different angles in a vertical plane about the lamp by means of a photometer. In such a device, the lamp to be measured is compared with a standard lamp. This comparison is difficult to make for an arc lamp on account of the high intensity of the light in some directions, and the continual shifting about of the arc on the electrodes, with the consequent change in the distribution. The color of the light, in some instances differing from that of the comparison lamp, makes the determination more difficult. Special and expensive apparatus must also be used for obtaining the intensity at different angles. The results of such determinations are shown in Fig. 3, 4, 5 and 6. A complete circle is divided into 360°, and radii are drawn at convenient angles. On these radii, the intensity in candle power is laid off at convenient scale. The points thus determined are connected by a smooth curve known as the "distribution curve". The candle power in any direction in a vertical circle is the length of the

radius, at that angle, intercepted by the curve, multiplied by the candle power scale. The distribution in a horizontal plane through the source is usually a circle; so that the distribution in one vertical plane represents the lamp distribution with sufficient accuracy.

Since, in street lighting, only the light thrown downward, i. e., in the lower hemisphere, is useful for illumination, only that portion of the curve is shown. All light in the upper hemisphere is wasted unless a reflector is used to divert these beams into useful directions. In order to save space and allow a larger scale, only one-half the lower hemisphere is shown in this bulletin, the other being exactly like it.

In Fig. 3, A is the distribution curve from a 9.6 amperes, direct current open arc lamp. It will be seen that the maximum

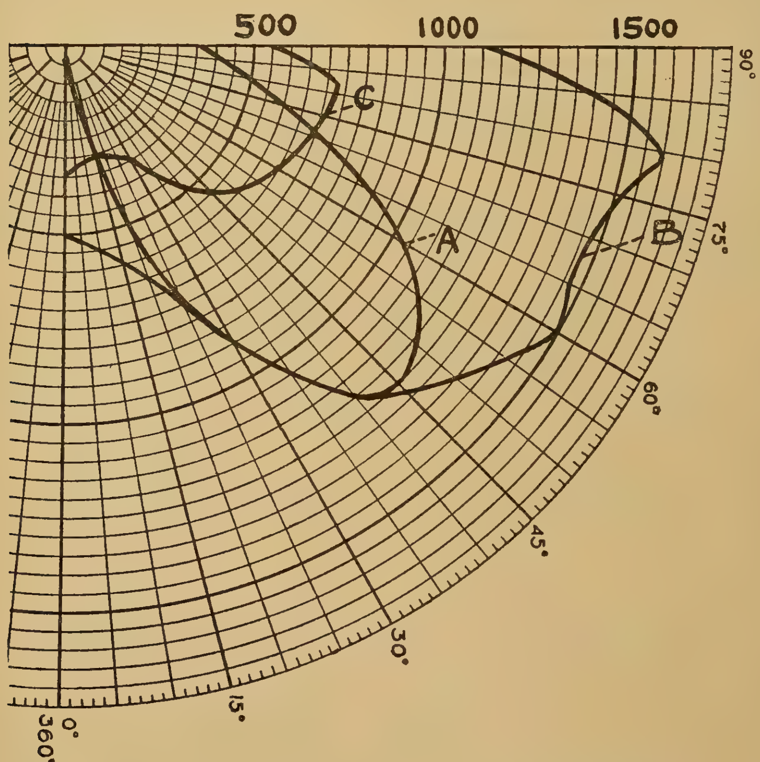

FIG. 3. DISTRIBUTION CURVES OF ARC LAMPS WITH CLEAR OUTER GLOBES

A. 9.6 Amperes, D. C. Open Arc.
B. 6.6 Amperes, D. C. Magnetite Arc.
C. 4 Amperes, D. C. Magnetite Arc.

intensity is at 45° from the vertical axis. The ray in this direction will strike the ground at a distance equal to the height of the lamp above the street surface. The light from this lamp comes mostly from the crater in the upper or positive electrode. The arc being short, the shadow cast by the lower electrode will cover a considerable angle, as seen in Fig. 7. The source of light being a small area, its intrinsic brilliancy, or candle power per sq. in., is of necessity very high. Shadows from the side rods and lower carbon of the lamp are also more or less annoying, because they are so much magnified on the surface of the ground.

In the enclosed direct current arc lamp, the curve for which is shown in Fig. 4 as C, the same shadow from the lower electrode appears, but it is not so intense on account of the fact that the curve is modified by the presence of the inner globe; the arc also

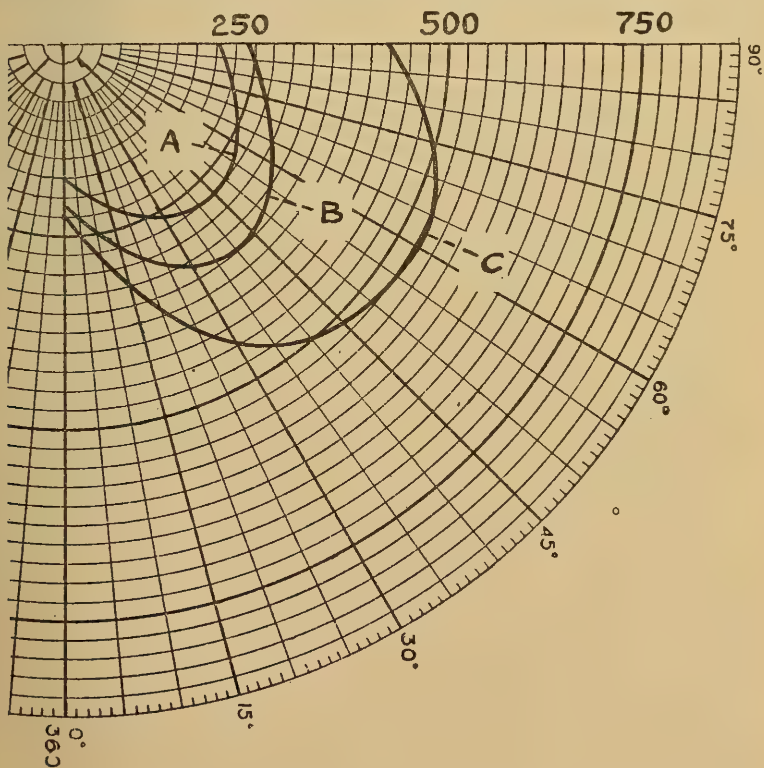

Fig. 4. Distribution Curves of Arc Lamps

A. 6.6 Amperes A C. Enclosed, Opal Inner. Clear Outer Globe.
B. 7.5 Amperes A. C. Enclosed, Opal Inner, Clear Outer Globe.
C. 6.6 Amperes D. C. Enclosed. Opal Inner, Clear Outer Globe.

is longer. The globe absorbs some of the light, and also becomes a secondary source of light, making the intensity less in the maximum direction and greater in other directions and at wider angles.

In the alternating current enclosed arc lamp, as much light is thrown in the upper hemisphere as in the lower, since craters are formed alternately in the upper and the lower electrodes. A reflector or shade should be used always with such a lamp, to return as much of the light as possible to the lower hemisphere. Two curves for such lamps are shown as A and B in Fig. 4, A being for a 6.6 amperes lamp, and B for a 7.5 amperes lamp.

In the flame arc lamp, the distribution curve depends upon whether the arc is formed between vertical electrodes, as shown in Fig. 8, or between inclined electrodes as shown in Fig. 9. In the latter case, the light will all be in a downward direction and strongest immediately below the light, as in B, Fig. 5. When

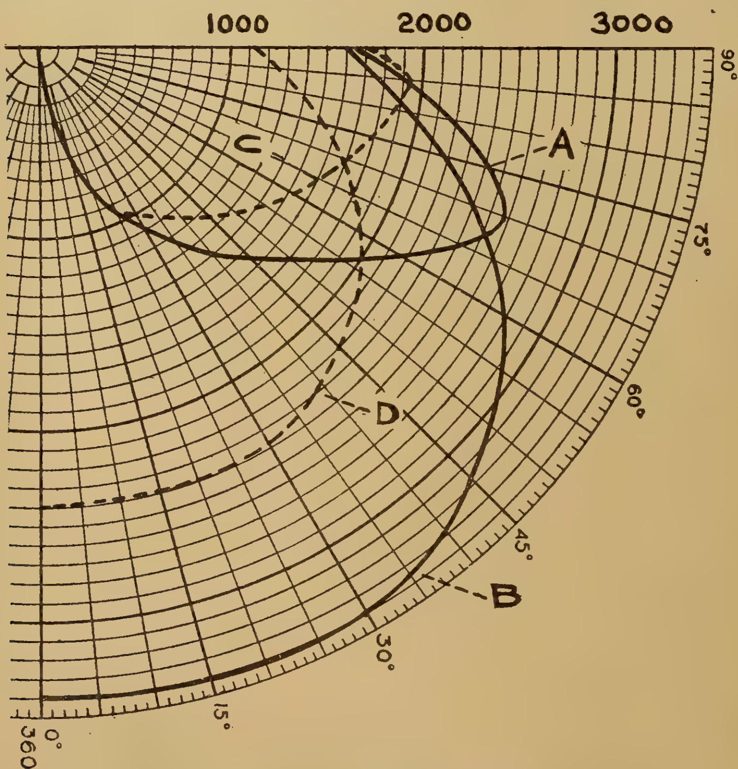

FIG. 5. DISTRIBUTION CURVES OF ARC LAMPS

A. Long Burning Flame, 5.5 Amperes, D. C., Clear Inner and Clear Outer Globe.
B. Inclined Electrode Short Burning Flame, 10 Amperes D. C., Clear Outer Globe.
C. Long Burning Flame, 5.5 Amperes, D. C., Clear Inner, Opal Outer Globe.

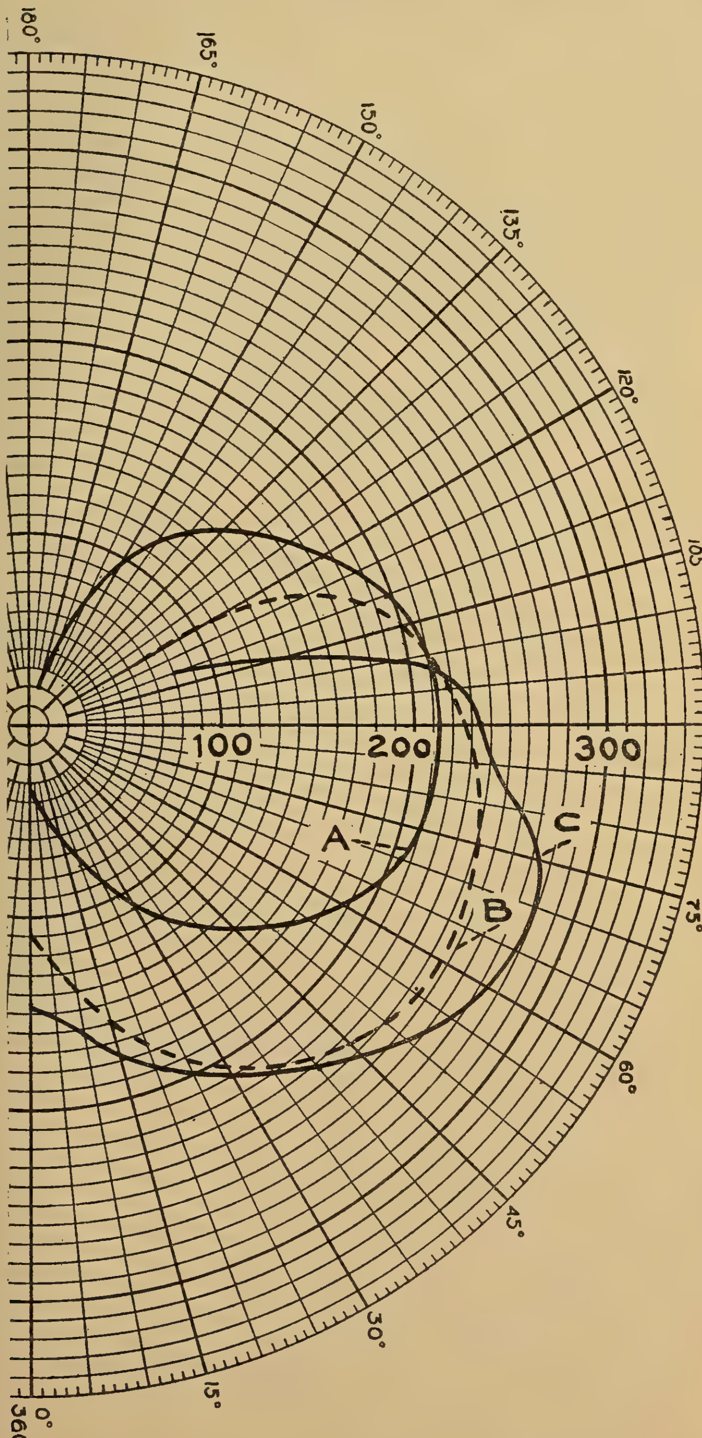

FIG. 6. DISTRIBUTION CURVE FOR 200–CANDLE POWER SERIES TUNGSTEN LAMP

A. Bare Lamp.
B. Lamp with 22 inch Enamelled Reflector.
C. Same with Radial Wave Reflector.

vertical electrodes are used, the light will be about equal in each hemisphere, but may be modified by use of the shade.

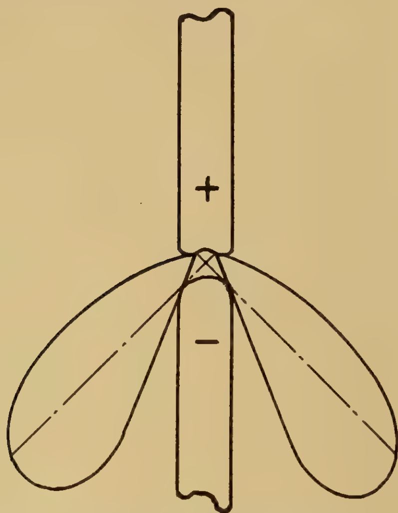

FIG. 7. TYPICAL DISTRIBUTION CURVE OF D. C. OPEN ARC LAMP

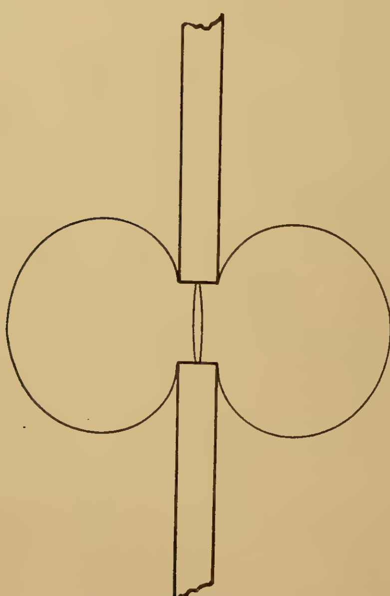

FIG. 8. TYPICAL DISTRIBUTION CURVE OF VERTICAL ELECTRODE, FLAME ARC LAMP

The distribution curve for a tungsten series incandescent light is similar in shape to that shown in Fig. 8, or in A, Fig. 6, which is the distribution curve for a rated 200 candle-power lamp. Various shades and reflectors have been developed for use with these lamps. They vary from a 22-in. enamel reflector, giving a distribution as shown in B, Fig. 6, or the radial reflector with its curve C, Fig. 6, and various unsymmetrical reflectors throwing most of the light into the middle of the street and very little on the sidewalk. For this latter shade, the light must be supported at the side of the street.

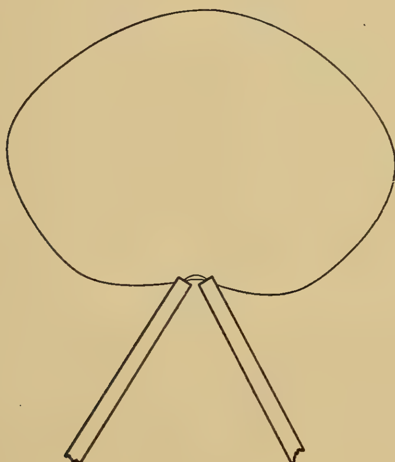

FIG. 9. TYPICAL DISTRIBUTION CURVE, INCLINED ELECTRODE, FLAME ARC LAMP

11. *Effect of Shades and Reflectors.*—That the distribution curves of lamps may be greatly altered by the use of reflectors and globes has already been mentioned. To illustrate the effect of a reflector, two curves of the same lamp are shown in Fig. 10. Curve A is a distribution curve for a long burning flame arc lamp with clear inner globe and thin opal outer globe but with no reflector. Curve B is for the same lamp, with the same globes and with a 24-in. white porcelain enameled reflector added. The shape of the distribution curve has thus been altered to suit a certain condition. The mean lower hemispherical candle-power has been increased by about 5 per cent, but the mean spherical candle-power has been decreased by 24 per cent. In arc lamps using stationary lower electrodes, the shape of the distribution

curve, with reflector, changes during the life of the electrodes, due to the change of position of the source of light. Fig. 11 illustrates the effect of globes on the distribution curve of the

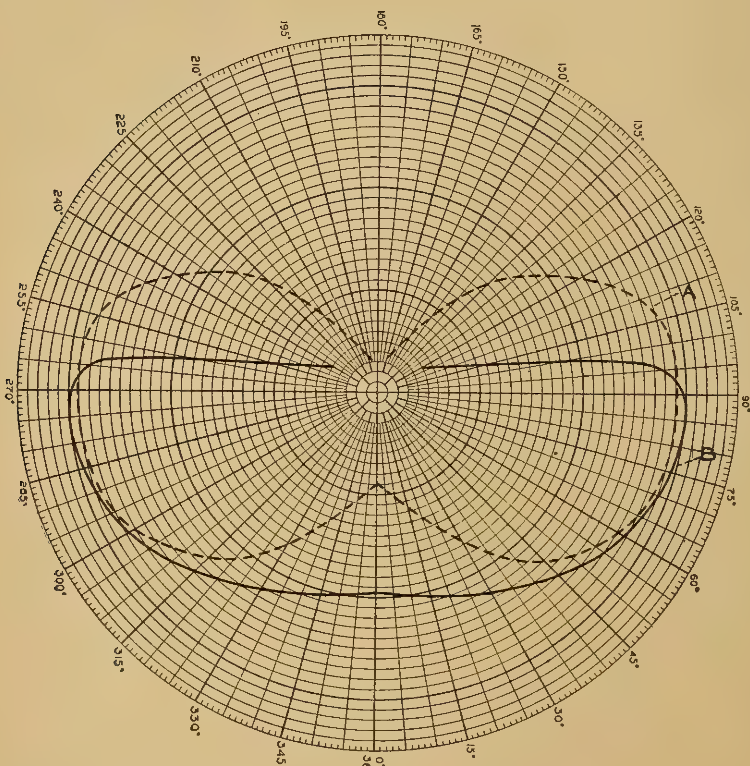

FIG. 10

A. Distribution Curve of the Long Burning Flame Arc Lamp with Opal Globe, No Reflector.
B. Same Lamp with Reflector.

lamp. Curve A is for a vertical carbon flame arc lamp with 26 in. reflector and with clear outer globe. Curves B and C are for the same lamp, with the same reflector, but with opal globes of different densities. Nearly the same shape of distribution curve has been obtained by both of the globes. However, 10 per cent of the light has been absorbed by one of the globes and 25 per cent by the other. Thus it is seen that an opal globe tends to make the distribution curve more uniform, but in so doing, it

absorbs a considerable portion of the light. This type of globe also lowers the intrinsic brilliancy, since it becomes, in effect, the source of light.

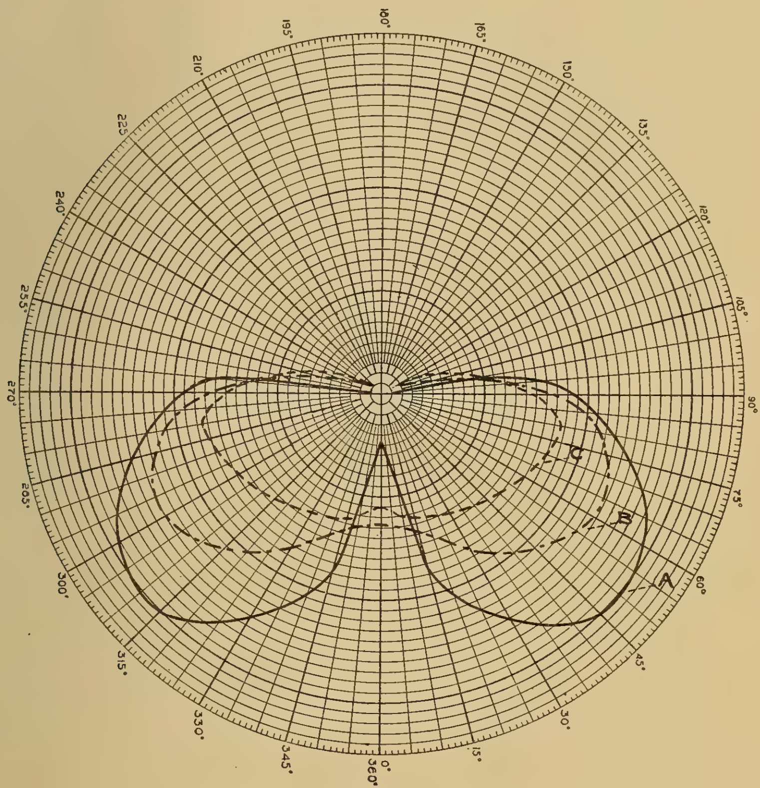

FIG. 11. EFFECT OF GLOBE

A. Vertical Electrode, Flame Arc Lamp with Reflector and Clear Outer Globe.
B. Same Lamp with Alba Globe.
C. Same Lamp with Dense Opal Globe.

V. CALCULATION OF ILLUMINATION

12 *General Theory.*—In order to obtain the best results from general illumination, it should be as nearly uniform as possible over the whole surface. The object of street illumination is to make the street safer for travel after nightfall. To accomplish this result, the eye must be able to distinguish objects with sufficient clearness and at a fair distance, so that they may be avoided if necessary. With the advent of the automobile and other swiftly moving vehicles, the distinctness with which objects may be seen must be improved, so that danger may easily be avoided.

The illumination produced by full moon has a maximum of about .02 foot candle. The eye has become accustomed to distinguish objects fairly well by this illumination, even at a considerable distance. Moreover, this illumination is uniform except where some object, such as a tree or a building, intervenes to shut off the light. This illumination is not sufficient for reading ordinary type without fatigue. For reading purposes from 1 to 3 foot candles are required. The eye is accustomed to adjust itself to quite a wide range of intensity, if the changes in intensity do not follow each other too rapidly. A street lighted by 6.6 amperes alternating current enclosed arc lights placed at the street crossings 25 ft. above the street surface where blocks are 400 ft. long, has a variation in its horizontal illumination of from .28 foot candle, near the lamp, to .0014 foot candle midway between the lamps. With the lamps placed at alternate street intersections, as is the custom in smaller cities, the minimum illumination, i. e. the illumination near the other street intersections, will be less than .0001 foot candle, an almost negligible quantity. An automobile traveling at a speed of 25 miles per hour along a street lighted every 400 ft. will pass from the point of minimum to maximum illumination in about 5.5 seconds. During this time, the eye must accustom itself to a range in illumination of 200 to 1, producing fatigue in the eye and uncertainty in discerning objects. When the illumination is not uniform, but varies over a wide range from lamp to lamp, the eye is not able to distinguish objects nearly so well, at the points of minimum illumination, as it is when the illumination is more nearly uniform but low.

The intrinsic brilliancy of the light source in the line of vision is another consideration in selecting the proper lamps, as well as their height and spacing. The pupil of the eye adjusts itself so as to admit a certain quantity of light. A source of light of high intrinsic brilliancy, in the line of vision, closes the pupil of the eye so that insufficient light is admitted to distinguish dimly lighted objects clearly. This effect is made apparent, in ordinary street lighting, by the fact that a person facing an arc light is unable to see objects beyond the light until he is so near the light that it is no longer in his line of vision. The effect is not so apparent in brightly lighted streets where the illumination is also more uniform.

13. *Tables and Curves.*—(*a*) In order to calculate the illumination on the street surface, the following equations have been

derived. Fig. 12 represents a light, L, suspended at a height, h, above the center of the street, BC. The intensity of the light in the direction, LB, at an angle ϕ with the vertical may be

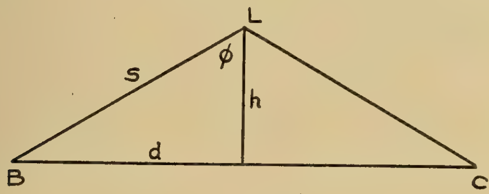

FIG. 12

obtained from the distribution curve of the lamp. Let this intensity of the light source be I. Then the intensity of the illumination at B, on a plane perpendicular to the ray of light, would be

$$i = \frac{I}{s^2} \qquad (1)$$

The illumination on the horizontal surface of the street would be

$$i_h = \frac{I \cos \phi}{s^2} \qquad (2)$$

since the ray is at the angle ϕ with the vertical. Taking the point B, at a horizontal distance d, from the lamp

$$\tan \phi = d/h \qquad (3)$$

and

$$s = h/\cos \phi \qquad (4)$$

Hence the normal illumination, or the illumination on a plane perpendicular to the ray of light would be

$$i = \frac{I \cos^2 \phi}{h^2} \qquad (5)$$

and on the street surface

$$i = \frac{I \cos^3 \phi}{h^2} \qquad (6)$$

In order to have uniform normal illumination, the intensity in any direction about the lamp must vary according to the equation

$$I = \frac{i\, h^2}{\cos^2 \phi} \qquad (7)$$

derived from equation (5) or

$$I = \frac{i\, h^2}{\cos^3 \phi} \qquad (8)$$

from equation (6), for uniform horizontal distribution. In these two equations,

if

h = height of lamp in feet above the street;

and

I = intensity of the light in candle power at the angle ϕ from the vertical;

then

i = intensity of illumination in foot candles.

Table 1 has been calculated from equation (7) for different values of h, ranging from 15 to 50 ft. In the same manner, Table 2 has been calculated from equation (8). In each table, a uniform intensity of illumination of .01 foot candle has been assumed. For any other intensity, these values should be

TABLE 1

CANDLE POWER AT LAMP FOR .01 FOOT CANDLE NORMAL ILLUMINATION ON STREET.

ϕ	$\cos\phi$	$\frac{1}{\cos^2\phi}$	Height of lamp—feet					
			15	20	25	30	40	50
0	1.000	1	2.25	4.00	6.25	9.00	16.00	25.00
5	.996	1.01	2.27	4.04	6.31	9.09	16.16	25.25
10	.985	1.03	2.32	4.12	6.44	9.27	16.48	25.75
15	.966	1.07	2.41	4.28	6.69	9.63	17.12	26.75
20	.940	1.13	2.57	4.56	7.13	10.17	18.26	28.50
25	.906	1.22	2.75	4.88	7.63	10.98	19.52	30.50
30	.866	1.33	2.99	5.32	8.31	11.97	21.30	33.30
35	.819	1.49	3.35	5.96	9.32	13.41	23.80	37.3
40	.766	1.70	3.83	6.80	10.60	15.3	27.2	42.5
45	.707	2.00	4.50	8.00	12.50	18.0	32.0	50.0
50	.643	2.43	5.47	9.72	15.16	21.8	38.9	60.8
55	.547	3.03	6.33	12.12	18.93	27.3	48.5	76.0
60	.500	4.00	9.00	16.0	25.0	36.0	64.0	100.0
65	.423	5.59	12.68	22.4	35.0	50.3	89.7	140.0
67.5	.383	6.82	15.35	27.3	42.6	61.4	109	170.5
70	.342	8.35	18.8	33.4	52.5	75.1	133.5	209
72.5	.301	11.05	24.9	44.2	69.1	99.5	177	276
75	.259	14.90	34.0	60.4	94.4	136	242	378
76	.242	17.10	38.5	68.4	107.	154	274	428
77	.225	19.75	44.5	79.0	123	178	316	494
78	.208	23.10	52.0	92.4	145	209	370	577
79	.191	27.4	61.7	109.6	171	246	438	687
80	.1736	33.2	74.8	133.	208	299	532	832
81	.1564	40.8	92.2	163	255	367	653	1020
82	.1392	51.7	116.	207	323	465	830	1292
83	.1219	67.3	151	269	421	606	1077	1682
84	.1045	91.7	206	367	573	826	1467	2290
85	.0872	131.7	296	527	824	1180	2110	3290
85.5	.0785	162.5	366	650	1010	1460	2600	4060
86.	.0698	205	461	820	1300	1850	3280	5120
86.5	.0610	269	605	1070	1680	2420	4300	6730
87.	.0523	366	824	1460	2290	3300	5850	9150
87.25	.0480	434	980	1740	2710	3900	6960	10850
87.5	.0436	527	1185	2110	3300	4740	8450	13170
87.75	.03926	650	1460	2600	4060	5850	10400	16250
88.	.03490	822	1850	3290	5140	7400	13150	20550
88.25	.03054	1072	2300	4290	6700	9540	17150	26800
88.5	.02618	1460	3290	5840	9120	13100	23400	36500
89.	.01745	3290	7420	13100	20500	29600	52700	82400

multiplied by the ratio between .01 and the intensity chosen. Thus, for .10 foot candle, the values should be multiplied by 10.

TABLE 2

CANDLE POWER AT LAMP FOR .01 FOOT CANDLE HORIZONTAL ILLUMINATION AT STREET

ϕ	$\cos\phi$	$\frac{1}{\cos^3\phi}$	Height of Lamp					
			15′	20′	25′	30′	40′	50′
0	1.000	1.000	2.25	4.00	6.25	9.00	16.00	25.0
5	.996	1.015	2.28	4.06	6.35	9.14	16.25	25.4
10	.985	1.045	2.35	4.18	6.43	9.41	16.72	26.15
15	.966	1.11	2.50	4.44	6.94	9.99	17.76	27.75
20	.940	1.20	2.70	4.80	7.50	10.80	19.20	30.0
25	.906	1.35	3.04	5.00	8.44	12.15	21.60	33.7
30	.866	1.54	3.47	6.16	9.53	13.86	24.65	38.5
35	.819	1.82	4.09	7.28	11.35	16.4	29.1	45.5
40	.766	2.22	5.00	8.88	13.85	20.0	35.5	55.2
45	.707	2.83	6.37	11.30	17.68	25.4	45.3	70.8
50	.643	3.73	8.42	14.9	23.3	33.6	59.7	93.5
55	.547	5.27	11.85	21.1	32.9	47.4	84.5	131.7
60	.500	8.00	18.00	32.0	50.0	72.0	128.0	200
65	.423	13.20	29.7	53.8	82.5	119	211	330
67.5	.383	17.8	40.1	71.2	111	160	285	445
70	.342	24.4	54.9	97.6	152	220	390	610
72.5	.301	36.7	82.6	146	229	330	587	918
75	.259	58 3	131.2	233	364	525	935	1460
76	.242	70.5	159	282	440	635	1130	1760
77	.225	87.8	198	351	549	790	1410	2200
78	.208	111.2	250	445	695	1000	1780	2780
79	.191	143.3	323	574	896	1290	2290	3590
80	.1736	191.0	430	764	1190	1720	3060	4780
81	.1564	261.0	574	1040	1630	2350	4180	6530
82	.1392	371.0	835	1480	2320	3340	5940	9280
83	.1219	552.0	1242	2210	3450	4970	8850	13800
84	.1045	876.0	1970	3500	5480	7880	14000	21900
85	.0872	1510	3400	6040	9440	13600	24200	37800
85.5	.0785	2070	4660	8280	12900	18600	33100	51800
86	.0698	2930	6600	11720	18320	26380	46800	73400
86.5	.0610	4410	9950	17650	27600	39700	70600	110000
87	.0523	6970	15700	27900	43500	62700	111500	174200
87.25	.0480	9050	20400	36200	56600	81500	145000	226000
87.5	.0436	12100	27200	48400	75600	118000	194000	303000
87.75	.03926	16500	37100	66000	103000	148000	264000	413000
88	.03490	23500	52900	94000	146500	211000	376000	588000
88.25	.03054							
88.5	.02618							
89	.01745							

From equation (3), the value of ϕ has been calculated for different values of d and h over as wide a range as necessary for ordinary exterior illumination. These angles are given in Table 3. Factors have also been included in Table 3 to aid in the calculation of illumination. For each height and distance, two factors are given by which the intensities in thousands of candle power must be multiplied to give the illumination at the chosen point. The first line is for normal and the second for horizontal illumination, respectively. By referring to Tables 1

and 2, the difficulty will be seen of securing uniform low illumination over large areas with economical height and spacing of lamps. Higher values of intensity may be secured by placing the lamps sufficiently close so that the illumination from one assists that from the others, increasing the minimum illumination.

TABLE 3

TABLE OF ANGLES AND FACTORS FOR VARIOUS HEIGHTS AND DISTANCES

$$a = \frac{\cos^2 \phi}{h^2} \times 1000\text{—Normal illumination}$$

$$b = \frac{\cos^3 \phi}{h^2} \times 1000\text{—Horizontal illumination}$$

Distance feet	Height—feet					
	15	20	25	30	40	50
25	59°2′	51°20′	45°-0′	39°50′	31°-0′	26°-3″
a	1.18	.977	.800	.656	.458	.321
b	.607	.610	.566	.504	.393	.287
50	73°18′	68°12′	63°27′	59°2′	51°20′	45°0′
a	.366	.345	.323	.295	.244	.200
b	.105	.128	.143	.150	.152	.141
75	78°41′	75°4′	71°34′	68°12′	62°0′	56°20′
a	.171	.168	.161	.153	.138	.123
b	.0335	.043	.051	.057	.065	.0682
100	81°28′	78°41′	75°57′	73°20′	68°12′	63°26′
a	.100	.0987	.0915	.0956	.0863	.0804
b	.0148	.0193	.022	.027	.032	.036
125	83°10′	80°55′	78°41′	76°30′	72°20′	68°10′
a	.0662	.0625	.0615	.0607	.0577	.0553
b	.0079	.010	.012	.0141	.0125	.0205
150	84°17′	82°24′	80°32′	78°41′	75°4′	71°34′
a	.0452	.0436	.0433	.0427	.0416	.0400
b	.0045	.0058	.0071	.0084	.0107	.0126
200	85°42′	84°17′	82°52′	81°28′	78°41′	75°57′
a	.0254	.0252	.0247	.0245	.0234	.0233
b	.0019	.0025	.0031	.0036	.0046	.0056
250	86°34′	85°25′	84°17′	83°10′	80°55′	78°41′
a	.016	.016	.016	.0158	.0156	.0154
b	.00106	.00128	.0016	.0019	.00247	.0030
300	87°8′	86°11′	85°14′	84°17′	82°24′	80°32′
a	.0111	.0111	.0111	.011	.0109	.0108
b	.00055	.00173	.00091	.0011	.00144	.0018
350	87°33′	86°44′	85°50′	85°5′	83°29′	81°57′
a	.00812	.00812	.00812	.00812	.00807	.0077
b	.00035	.00046	.00058	.00070	.00091	.0011
400	87°51′	87°8′	86°25′	85°42′	84°17′	82°52′
a	.00625	.00625	.00625	.00625	.0061	.0060
b	.00023	.00031	.00039	.00047	.0006	.00074
450	88°5′	87°27′	86°49′	86°11′	84°55′	83°40′
a	.0050	.00494	.00492	.00491	.00490	.00487
b	.00017	.00022	.00027	.00032	.00043	.000536
500	88°17′	87°42′	87°8′	86°34′	85°25′	84°17′
a	.004	.004	.004	.004	.004	.004
b	.00012	.00016	.00020	.00024	.00032	.0004

This method is used in interior illumination, and in the brilliant illumination of streets. From Table 3 and the distribution curves of the several lamps, Tables 4 to 13 have been computed. These tables give the foot candles illumination on the normal and horizontal planes for different heights and spacing of the lamps.

TABLE 4

Normal and Horizontal Distribution at Street Surface for 9.6 Amperes D. C. Open Arc Lamp

Height feet	Distance—feet											
	0	25	50	75	100	125	150	200	250	300	350	400
15	.222	1.185	.242	.089	.0476	.030	.019	.0104	.0065	.0044	.0032	.0025
	.222	.610	.0693	.0174	.0070	.00355	.0019	.00078	.00043	.00028	.00014	.00009
20	.125	1.18	.228	.101	.050	.031	.021	.011	.0067	.0044	.0031	.0024
	.125	.735	.0845	.0258	.0098	.0049	.0028	.0011	.00054	.00029	.00017	.00012
25	.080	1.00	.294	.120	.057	.031	.022	.011	.0064	.0046	.0038	.0025
	.080	.700	.133	.0384	.0130	.0061	.0035	.0014	.00068	.00038	.00024	.00016
30	.0555	.788	.303	.116	.063	.036	.022	.012	.0068	.0046	.0034	.0026
	.0555	.605	.154	.0435	.018	.0083	.0043	.0018	.00082	.00046	.0003	.00019
40	.0312	.417	.296	.138	.069	.038	.025	.012	.0076	.0050	.0034	.0026
	.0312	.356	.185	.065	.026	.012	.0064	.0023	.0012	.00066	.00038	0 0025
50	.020	.284	.250	.140	.0735	.044	.028	.0137	.0079	.0052	.0037	.0026
	.020	.253	.176	.0775	.033	.0165	.0089	.0033	.0015	.00089	.00052	.00032

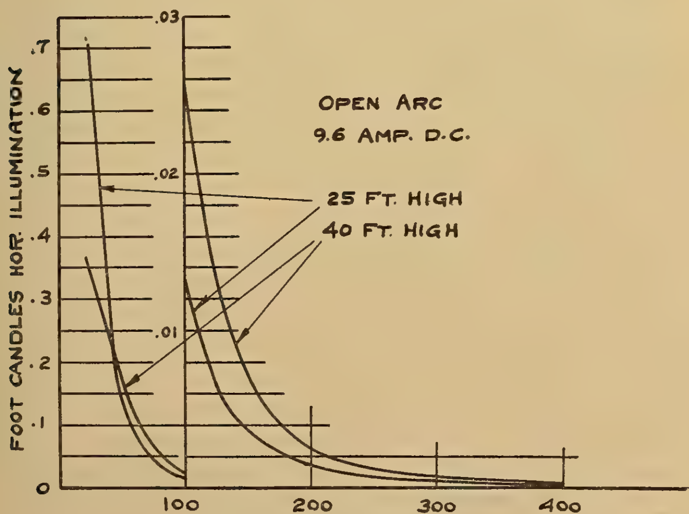

Fig. 13. Horizontal Distribution at Street Surface for 9.6 Amp. D. C. Open Arc Lamp

TABLE 5

NORMAL AND HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR 6.6 AMPERES D. C. MAGNETITE ARC LAMP

Height feet	Distance—feet											
	0	25	50	75	100	125	150	200	250	300	350	400
15	2.22	1.76	.550	.269	.155	.100	.0660	.0320	.0208	.0142	.0101	.0075
	2.22	.91	.169	.053	.023	.0118	.0066	.0024	.00138	.00070	.00044	.00028
20	1.25	1.30	.506	.456	.134	.0950	.0658	.0357	.0223	.0146	.0104	.0080
	1.25	.815	.188	.0656	.0267	.0153	.0088	.0035	.00178	.00096	.00059	.0004
25	.60	.985	.480	.242	.142	.0970	.0675	.0366	.0236	.0154	.0109	.0083
	.60	.696	.225	.0765	.0342	.0189	.0111	.0046	.0024	.0013	.00078	.00051
30	.556	.590	.437	.225	.143	.0937	.0635	.0387	.0235	.0160	.0114	.0085
	.556	.454	.223	.0840	.0405	.0218	.0125	.0056	.0028	.0016	.0010	.00064
40	.313	.435	.330	.203	.126	.0872	.0635	.0370	.0246	.0168	.0119	.0086
	.313	.373	.205	.0955	.0470	.0265	.0163	.00727	.0039	.0022	.00134	.00084
50	.200	.257	.246	.178	.117	.0813	.0595	.0356	.0243	.0171	.0116	.0090
	.200	.230	.173	.099	.052	.0301	.0188	.00856	.0047	.00286	.0017	.00111

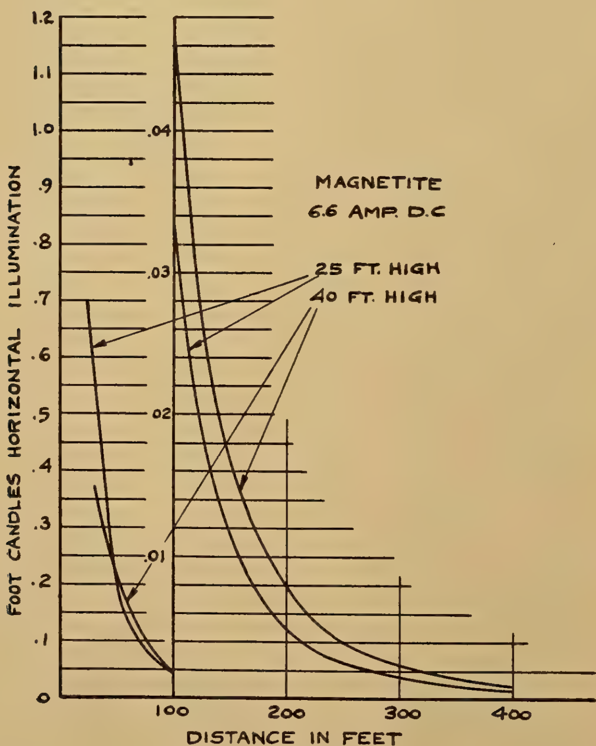

FIG. 14. HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR A 6.6 AMP. MAGNETITE ARC LAMP

TABLE 6

Normal and Horizontal Distribution at Street Surface for 4 Amperes D. C. Magnetite Arc Lamp

Height feet	Distance—feet											
	0	25	50	75	100	125	150	200	250	300	350	400
15	1.55	.768	.259	.125	.0742	.0459	.0303	.0155	.01015	.0069	.0050	.00375
	1.55	.395	.0743	.0244	.0110	.00547	.00302	.00116	.00067	.00034	.00021	.00014
20	.875	.596	.237	.1210	.0713	.0454	.0305	.0162	.0107	.0071	.0050	.0039
	.875	.372	.0872	.0310	.0140	.00726	.00406	.00161	.00086	.00047	.00028	.00019
25	.560	.440	.213	.113	.066	.0448	.0311	.0163	.0110	.00743	.00540	.0040
	.560	.311	.095	.0357	.0159	.00875	.0051	.00217	.0011	.00061	.00037	.00025
30	.389	.328	.192	.098	.0677	.0438	.0311	.0167	.0110	.00765	.00543	.00445
	.389	.252	.0975	.0366	.0191	.0102	.00613	.0026	.0013	.00076	.00047	.0003
40	.219	.167	.148	.091	.0588	.0400	.0300	.0160	.0112	.0076	.0057	.0040
	.219	.143	.092	.0428	.0218	.0123	.0077	.00334	.00177	.0010	.00063	.0004
50	.140	.108	.110	.0718	.0537	.0380	.0278	.0158	.0112	.0079	.0055	.0042
	.140	.0965	.0777	.0398	.0241	.0141	.00875	.0040	.0022	.0013	.00078	.0005

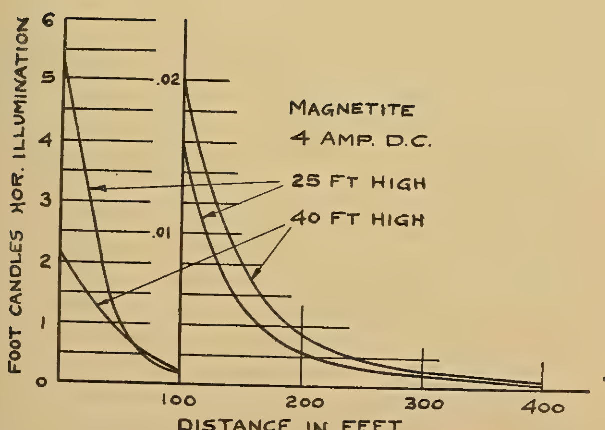

Fig. 15. Horizontal Distribution at Street Surface for a 4.0 Amp. Magnetite Arc Lamp

TABLE 7

NORMAL AND HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR 6.6 AMPERES D. C. ENCLOSED ARC LAMP

Height feet	Distance—feet											
	0	25	50	75	100	125	150	200	250	300	350	400
15	1.00	.62	.187	.083	.0475	.0310	.0183	.0113	.0075	.00485	.00355	.0027
	1.00	.32	.054	.012	.007	.0036	.0018	.00045	.00036	.00024	.00015	.0001
20	.563	.503	.180	.084	.05	.0296	.0202	.0115	.0072	.0049	.0035	.0027
	.563	.314	.067	.022	.0097	.0048	.0027	.0011	.0006	.0003	.0002	.0001
25	.36	.400	,167	.083	.046	.0296	.0206	.0116	.0073	.0050	.0036	.0028
	.36	.283	.075	.026	.011	.0058	.0034	.0014	.00072	.00041	.00026	.00017
30	.25	.311	.155	.0803	.0490	.0304	.0207	.0116	.0074	.0050	00366	.0028
	.25	.240	.089	.0298	.0138	.0071	.0041	.0017	.00089	.00050	.00031	.00021
40	.14	.197	.127	.0725	.0453	.0297	.0208	.0112	.0074	.00518	.0037	.0027
	.14	.167	.079	.0341	.0168	.0090	.0054	.0022	.0012	.00067	.00042	.00027
50	.091	.128	.100	.0645	.0420	.0290	.0207	.0117	.0075	.00512	.00364	.0028
	.091	.115	.072	.0356	.0189	.0108	.0065	.0050	.00146	.00085	.00052	.00034

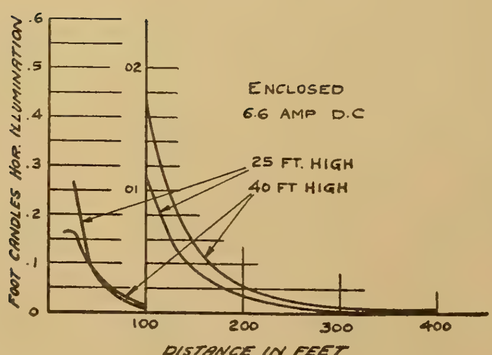

FIG. 16. HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR A 6.6 AMP. D. C. ENCLOSED ARC LAMP

TABLE 8

NORMAL AND HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR 6.6 AMPERES A. C. ENCLOSED ARC LAMP

Height feet	Distance—feet											
	0	25	50	75	100	125	150	200	250	300	350	400
15	.78	.313	.086	.036	.021	.014	.0097	.0053	.0033	.0022	.0016	.0012
	.78	.161	.0247	.0075	.0032	.0017	.00096	.0004	.0002			
20	.44	.259	.077	.038	.022	.014	.0094	.0053	.0034	.0023	.0017	.0013
	.44	.168	.0285	.0097	.0043	.0022	.0012	.0005	.00027			
25	.28	.22	.080	.038	.022	.014	.0095	.0054	.0034	.0024	.0017	.0013
	.28	.156	.032	.012	.0044	.0027	.00156	,0007	.00034			
30	.19	.18	.071	038	.022	.0137	.0094	.0054	.0034	.0023	.0017	.0013
	.19	.139	.0362	.014	.0063	.032	.0018	.0008	.0004	.0002		
40	.11	.124	.067	.036	.022	.0135	.0095	.0054	.0034	.0024	.0017	.0013
	.11	.116	.042	.017	.008	.0041	.0024	.0011	.0005	.0003		
50	.07	.083	.055	.033	.020	.0135	.010	.005	.0034	.0024	.0016	.0013
	.07	.074	.039	.018	.0092	.005	.003	.0022	.0006	.0004	.0002	.0001

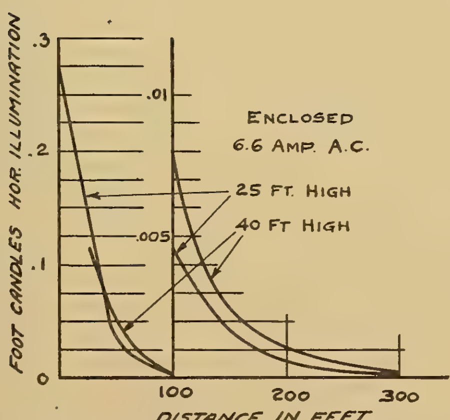

FIG. 17. HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR A 6.6 AMP. A. C. ENCLOSED ARC LAMP

TABLE 9

Normal and Horizontal Distribution at Street Surface for 7.5 Amperes A. C. Enclosed Arc Lamp

Height feet	Distance—feet											
	0	25	50	75	100	125	150	200	250	300	350	400
15	.94	.376	.103	.043	.025	.017	.011	.0063	.004	.0022	.0019	.0014
	.94	.193	.030	.009	.0038	.002	.001	.0005	.0002			
20	.53	.311	.092	.045	.026	.017	.011	.0064	.0041	.0027	.0020	.0015
	.53	.201	.e34	.0016	.0051	.0026	.0014	.0006	.0003			
25	.336	.26	.096	.045	.026	.017	.011	.0065	.0041	.0029	.0020	.0015
	.336	.187	.038	.014	.0053	.0032	.0019	.0008	.0004			
30	.228	.216	.085	.045	.026	.0165	.011	.0065	.004	.0028	.002	.0015
	.228	.167	.043	.017	.0076	.0038	.0022	.0011	.0005	.0002		
40	.132	.149	.080	.043	.026	.0162	.011	.0066	.0041	.0029	.0020	.0015
	.132	.139	.050	.020	.0096	.0049	.0029	.0013	.0006	.0004		
50	.084	.10	.066	.040	.024	.016	.012	.006	.0041	.0029	.0019	.0015
	.084	.09	.047	.022	.011	.006	.0036	.0026	.0007	.0005	.0002	.0001

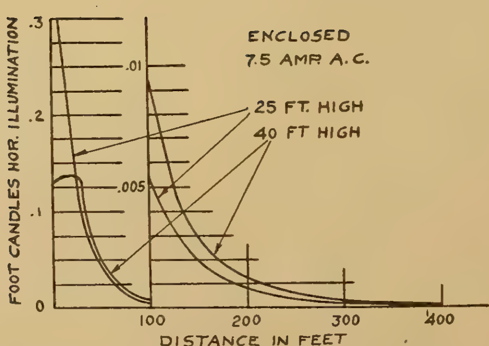

Fig. 18. Horizontal Distribution at Street Surface for a 7.5 Amp. A. C. Enclosed Arc Lamp

TABLE 10

Normal and Horizontal Distribution at Street Surface for 10 Amperes D. C. Flame Arc Lamp, Inclined Electrodes, Opal Globes

Height feet	Distance											
	0	25	50	75	100	125	150	200	250	300	350	400
15	10.6	2.33	.585	.240	.142	.085	.056	.031	.019	.013	.010	.007
	10.6	1.20	.168	.047	.021	.010	.0055	.0022	.0013	.0006	.0004	.0003
20	5.95	1.70	.595	.259	.136	.084	.057	.031	.020	.013	.010	.008
	5.95	1.28	.224	.066	.027	.0135	.0075	.0031	.0015	.0008	.0005	.0004
25	3.81	1.77	.602	.262	.140	.085	.058	.031	.020	.013	.010	.0075
	3.81	1.25	.265	.084	.034	.017	.010	.004	.002	.0011	.0007	.0005
30	2.65	1.51	.580	.266	.152	.091	.059	.032	.020	.014	.010	.0075
	2.65	1.15	.297	.100	.043	.021	.012	.0047	.0025	.0014	.0008	.0075
40	1.49	1.09	.52	.253	.150	.094	.064	.032	.021	.014	.010	.0077
	1.49	.94	.32	.123	.055	.029	.0165	.0064	.0032	.0018	.0011	.0008
50	.95	.76	.44	.25	.16	.096	.065	.035	.022	.015	.010	.0084
	.95	.68	.315	.138	.072	.036	.021	.0084	.0042	.0024	.0014	.001

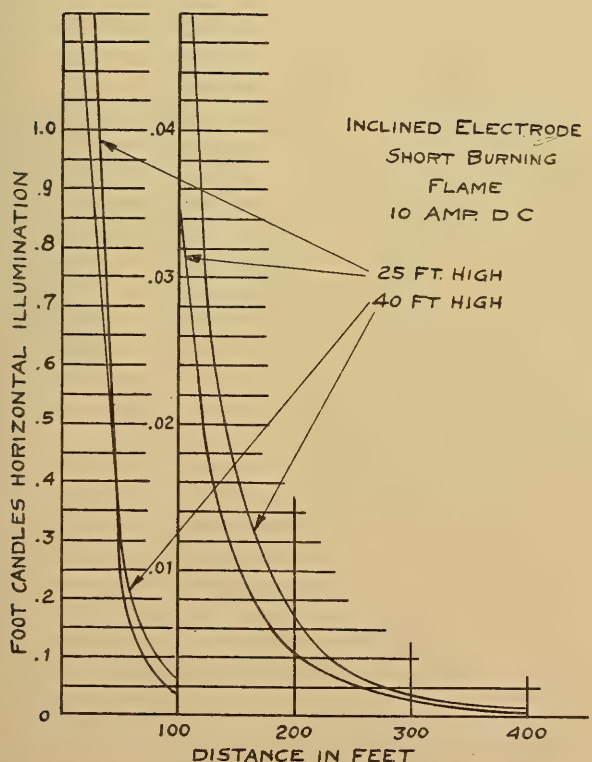

Fig. 19. Horizontal Distribution at Street Surface for a 10 Amp. D. C. Flame Arc Lamp, Inclined Electrodes

TABLE 11

Normal and Horizontal Distribution at Street Surface for 5.5 Amperes D. C. Long Burning Flame Arc Lamp Vertical Electrodes, Opal Globes

	Distance—feet											
	0	25	50	75	100	125	150	200	250	300	350	400
15	3.38	1.77	.65	.32	.19	.125	.087	.049	.0304	.021	.015	.011
	3.38	.91	.186	.063	.0281	.015	.0087	.0036	.0020	.001	.0006	.0004
20	1.90	1.49	.587	.302	.182	.119	.082	.0485	.031	.021	.0154	.0115
	1.90	.945	.218	.0775	.036	.019	.011	.0048	.0025	.0033	.0009	.0006
25	1.26	1.0	.550	.278	.166	.114	.081	.047	.031	.021	.0156	.012
	1.26	.707	.243	.088	.040	.022	.0132	.0059	.0031	.0018	.0011	.0007
30	.845	.760	.443	.260	.166	.110	.079	.046	.030	.021	.0156	.012
	.845	.657	.225	.095	.047	.0256	.0155	.0068	.0036	.0021	.0013	.0009
40	.475	.458	.398	.214	.146	.101	.075	.043	.03	.021	.015	.0116
	.475	.393	.204	.101	.054	.022	.0193	.0085	.0047	.0027	.0017	.0011
50	.304	.321	.25	.180	.128	.094	.069	.042	.029	.020	.015	.0114
	.304	.287	.176	.10	.057	.035	.0217	.010	.0056	.0034	.0021	.0014

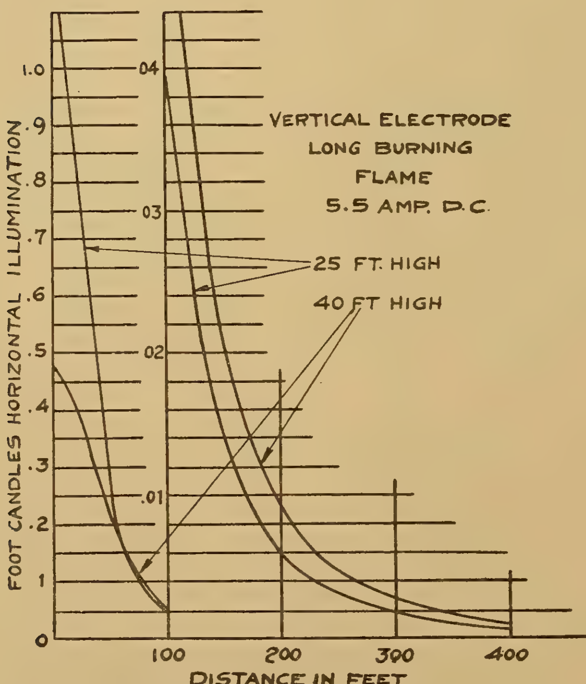

Fig. 20. Horizontal Distribution at Street Surface for a 5.5 Amp. D. C. Long Burning Flame Arc Lamp Vertical Electrodes

TABLE 12

NORMAL AND HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR 200 C. P. TUNGSTEN WITH RADIAL WAVE REFLECTOR

Height feet	Distance—feet									
	0	25	50	75	100	125	150	200		
15	.643	.316	.097	.043	.026	.015	.0102	.0057		
	.643	.163	.028	.0084	.0039	.0018	.0010	.0004		
20	.362	.243	.093	.044	.025	.0153	.0105	.0058		
	.362	.155	.035	.0112	.0049	.0024	.0014	.0006		
25	.232	.200	.086	.0427	.0238	.0154	.0107	.0059		
	.232	.141	.035	.0135	.0057	.0030	.0017	.0007		
30	.161	.154	.078	.0413	.0251	.0156	.0107	.0060		
	.161	.118	.040	.0154	.0071	.0036	.0021	.0009		
40	.091	.098	.063	.0376	.0233	.0153	.0108	.006		
	.091	.084	.039	.0175	.0086	.0046	.0028	.0011		
50	.058	.062	.049	.0326	.0217	.0151	.0106	.006		
	.058	.058	.035	.018	.0097	.0054	.0033	.0014		

TABLE 13

NORMAL AND HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR 350 C. P. TUNGSTEN WITH FLUTED ENAMELED REFLECTOR

Height feet	Distance—feet									
	0	25	50	75	100	125	150	200	250	300
15	1.03	.503	.165	.072	.041	.0265	.0179	.010	.0062	.0043
	1.03	.259	.0473	.014	.006	.0031	.0018	.00074	.00041	.00021
20	.575	.395	.156	.074	.042	.0256	.0178	.010	.0063	.0043
	.575	.247	.0575	.019	.0082	.0041	.0024	.001	.0005	.00024
25	.370	.324	.141	.0725	.040	.0252	.0180	.017	.0063	.00435
	.370	.230	.063	.023	.0095	.0045	.0030	.0012	.0006	.00036
30	.257	.256	.126	.0685	.0425	.0262	.018	.010	.0063	.0044
	.257	.197	.064	.026	.0120	.0061	.0035	.0015	.00076	.00044
40	.145	.168	.099	.060	.0392	.0259	.0181	.0098	.0064	.0044
	.145	.145	.0615	.0383	.0145	.0079	.0046	.0019	.0010	.00058
50	.093	.111	.081	.052	.0354	.025	.0181	.010	.0065	.0045
	.093	.099	.057	.0288	.0160	.0093	.0057	.0024	.0013	.00074

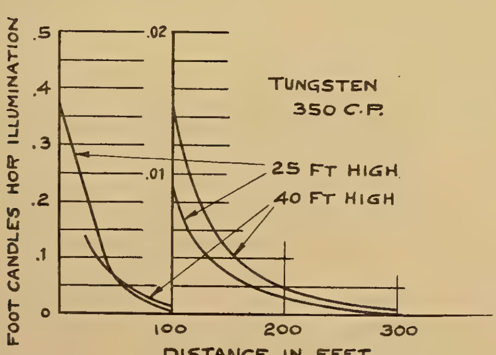

FIG. 21. HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR A 350 C. P. TUNGSTEN LAMP WITH ENAMELLED REFLECTOR

(*b*) *Surface Illumination Curves.*—From the data in Tables 4 to 13, curves have been plotted, showing the distribution of illumination over the street surface for each of the lamps. Two curves are plotted in each case, one for a height of 25 ft. and the

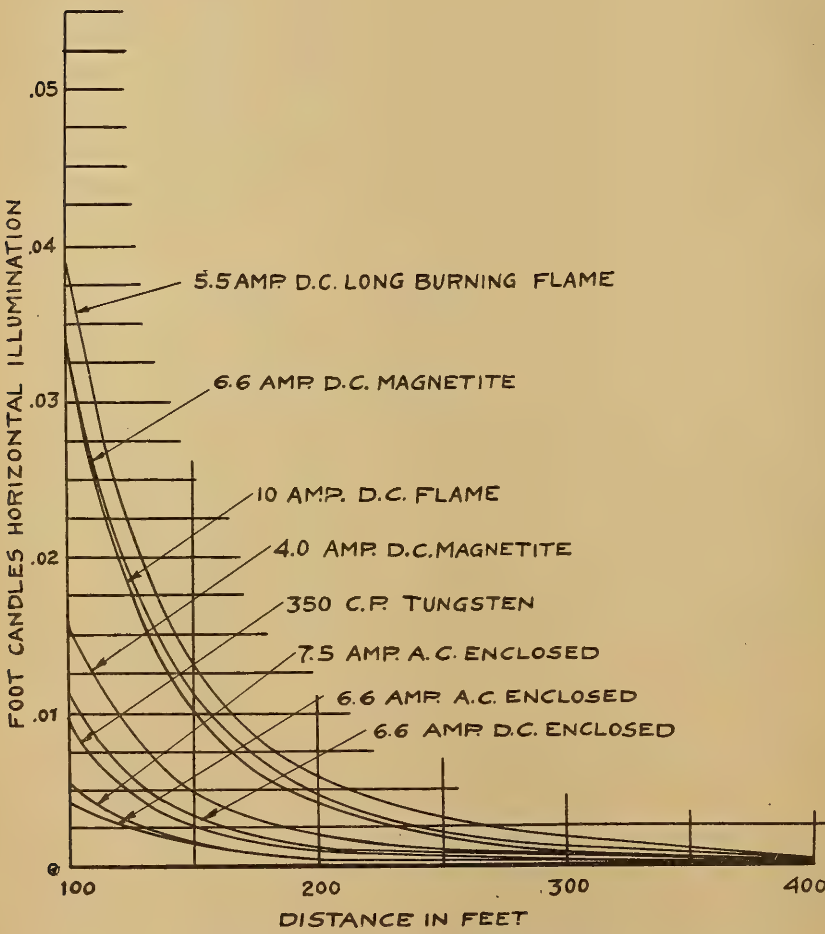

FIG. 22. COMPARISON OF HORIZONTAL DISTRIBUTION AT STREET SURFACE BY DIFFERENT LAMPS

other for 40 ft. At a distance of 100 ft. from the base of the lamp, the illumination has become so low that it is not easily comparable with that near the lamp. In order to read these values

more easily, the foot candle scale has been magnified between 100 and 400 ft. These curves are shown in Fig. 13 to 21. In Fig. 22, a comparison is made between the different lamps between 100 and 400 ft.

In order to indicate the method of computing the distribution of illumination on the street surface due to several lamps, and the effect of height and spacing, the following calculations have been made. Take, as an example, the 6.6 ampere alternating current enclosed arc, suspended 25 and 40 ft. above the center of the street surface, and spaced 800, 400 and 200 ft. apart, these distances being controlled by local conditions. In Fig. 23, OO′ represents the surface of the street and 1-2 lamps. It is obvious that the normal illumination can not be added for lamps on

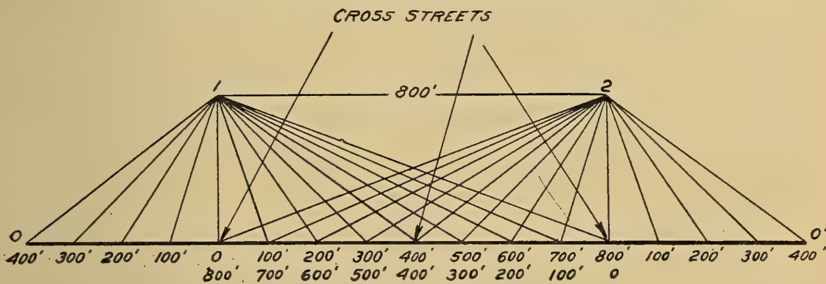

FIG. 23

opposite sides of an object, but that the horizontal is increased. Adding the illumination from the two lights at the various distances, line 1, Table 14 is obtained from Table 8. It is seen that the illumination beyond 400 ft. from the lights is so small that the sum need be considered only at the middle point.

For 400 ft. spacing, line 2 is obtained from the two lights 1-2 at street intersections (see Fig. 24). It will be seen that the

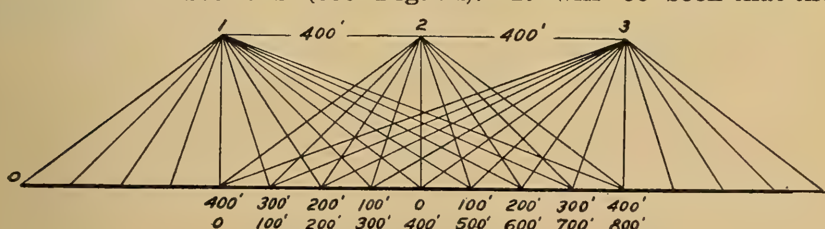

FIG. 24

illumination at any point is that produced by the sum of the illuminations from all lights at their respective distances from that point. Only two lights need be considered at 400 ft. At 200 ft. it will be found that four lights must be considered. In each

case, the maximum illumination is affected but little, but the general effect becomes better. For 200 ft. spacing, line 3, Table 14 is computed. In a similar manner, the illumination for other types of lights has been computed. These are also illustrated in Table 14. These values have not been plotted in the form of curves.

TABLE 14

HORIZONTAL DISTRIBUTION AT STREET SURFACE FOR DIFFERENT HEIGHTS AND SPACINGS

	Spacing feet	h feet	Distance—feet								
			0	50	100	150	200	250	300	350	400
6.6 Amp. D. C. Enclosed	800	25	.36	.075	.011	.0034	.0014	.0007	.0004	.0004	.0004
	400	25	.36	.075	.011	.04	.003	.004	.011	.075	.36
	200	25	.372	.082	.023	.082	.372	.082	.023	.082	.372
	800	40	.14	.079	.0168	.0054	.0022	.0012	.0007	.0006	.0006
	400	40	.14	.08	.017	.0066	.0044	.0066	.017	.08	.14
	200	40	.142	.085	.034	.085	.142	.085	.034	.085	.142
7.5 Amp. A. C. Enclosed	800	25	.336	.038	.005	.0019	.0008	.0004	.0003	.0002	.0001
	400	25	.336	.038	.0053	.0023	.0016	.0023	.0053	.038	.336
	200	25	.34	.05	.012	.05	.34	.05	.012	.05	.34
	800	40	.132	.05	.0096	.003	.0013	.0006	.0004	.0003	.0002
	400	40	.132	.05	.011	.0035	.0026	.0035	.011	.05	.132
	200	40	.135	.10	.022	.10	.135	.10	.022	.10	.135
6.6 Amp. D. C. Magnetite	800	25	.80	.225	.034	.0111	.0046	.0024	.0013	.001	.001
	400	35	.80	.226	.035	.0135	.0092	.0135	.035	.226	.80
	200	25	.85	.238	.069	.238	.85	.238	.069	.235	.85
	800	40	.315	.205	.047	.0163	.0073	.0039	.0022	.002	.0016
	400	40	.316	.206	.049	.020	.014	.020	.049	.206	.316
	200	40	.329	.225	.096	.225	.329	.225	.096	.225	.329
4.0 Amp. D. C. Magnetite	800	25	.56	.095	.016	.005	.002	.001	.0016	.0006	.0005
	400	25	.56	.095	.016	.006	.004	.006	.016	.095	.56
	200	25	.564	.101	.033	.101	.564	.101	.033	.101	.564
	800	40	.219	.092	.022	.0077	.0033	.0018	.001	.0009	.0008
	400	40	.219	.092	.044	.0095	.0066	.0095	.044	.092	.219
	200	40	.222	.094	.044	.094	.222	.094	.044	.094	.222
9.6 Amp. D. C. Open	800	25	.08	.133	.013	.0035	.0014	.0007	.0004	.0003	.0003
	400	25	.08	.133	.013	.042	.0028	.042	.013	.133	.08
	200	25	.081	.136	.026	.136	.081	.136	.026	.136	.081
	800	40	.031	.185	.026	.006	.002	.001	.0007	.0006	.0005
	400	40	.031	.185	.026	.007	.004	.007	.026	.185	.031
	200	40	.035	.192	.053	.192	.035	.192	.053	.192	.035

TABLE 14 (*Continued*)

	Spacing feet	h feet	Distance—feet 0	50	100	150	200	250	300	350	400
Long Burning Flame Vertical	800	25	1.26	.243	.040	.0132	.0059	.0031	.002	.0016	.0014
	400	25	1.26	.244	.042	.0163	.012	.0163	.042	.244	1.26
	200	25	1.27	.261	.084	.261	1.27	.261	.084	.261	1.27
	800	40	.475	.204	.054	.0193	.0085	.0047	.0029	.0023	.0022
	400	40	.476	.206	.057	.024	.017	.024	.057	.206	.476
	200	40	.486	.230	.114	.230	.486	.230	.114	.230	.486
Flame Arc Inclined	800	25	3.81	.265	.034	.010	.004	.002	.001	.001	.001
	400	25	3.81	.266	.035	.012	.008	.012	.035	.266	3.81
	200	25	3.82	.276	.07	.276	3.82	.276	.07	.276	3.82
	800	40	1.49	.32	.055	.0165	.0064	.0032	.0021	.0017	.0016
	400	40	1.49	.32	.057	.0168	.0128	.0168	.057	.32	1.49
	200	40	1.50	.34	.112	.34	1.50	.34	.112	.34	1.50
6.6 Amp. A. C. Enclosed	800	25	.28	.032	.0044	.0016	.0007	.0003			
	400	25	.28	.032	.0044	.0016	.0014	.0016	.0044	.032	.28
	200	25	.28	.035	.009	.035	.28	.035	.009	.035	.28
	800	40	.11	.042	.008	.0024	.0011	.0005	.0003		
	400	40	.11	.042	.008	.003	.0022	.003	.008	.042	.11
	200	40	.11	.045	.016	.045	.11	.045	.016	.045	.11
350 C. P. Tungsten	800	25	.37	.063	.0095	.003	.0012	.0006	.0004	.0002	.0002
	400	25	.37	.032	.01	.004	.0024	.004	.01	.032	.37
	200	25	.37	.038	.025	.038	.37	.038	.025	.038	.37
200 C. P. Tungsten	400	20	.362	.035	.011	.005	.003	.005	.011	.035	.362
	200	20	.365	.04	.023	.04	.365	.04	.023	.04	.365

	d	h	0	25	50	75	100	125	150	175	200
100 C. P. Tungsten	200	20	.181	.078	.015	.0068	.005	.068	.018	.078	.181
	100	20	.183	.084	.034	.084	.183	.084	.034	.084	.183
60 C. P. Tungsten	200	20	.11	.046	.01	.004	.003	.004	.01	.046	.11
	100	20	.11	.052	.021	.052	.11	.052	.021	.052	.11
32 C. P. Tungsten	200	20	.06	.024	.005	.002	.002	.002	.005	.024	.06
	100	20	.09	.028	.010	.028	.06	.028	.010	.028	.06

VI. Street Lighting

Street lighting in different parts of a city must serve various purposes. For utility and safety, the business streets require a different treatment from the residence streets and suburban districts. Parks and open places require a still different arrangement of lights. A good division of the city is the following:

A Principal business streets;
B Important cross streets and boulevards;
C Residence streets;
D Outlying districts.

14. *Business Streets.*—In the principal business streets, the illumination should be more or less brilliant. It should be as uniform as possible, and of sufficient intensity to enable one to read ordinary sized print, i. e., at least .25 to 1 foot candle. In many cities in Europe, the inclined carbon flame arc is used for these streets. The lamps are suspended over the center of the street from tall ornamental poles on both sides, and at a height of from 40 to 50 ft. They are spaced from four to five times their height. These produce a very beautiful effect, and the street is brightly illuminated. With the reflection from buildings, the illumination averages about two foot candles. This lighting is also carried out for short distances into the cross streets. In this country, the flame arc has not met with favor on account of the high operating cost for the short burning arc. With the advent of the more modern long burning flame arc, this objection can not be made, since, in lighting from large units spaced comparatively near together, the number of the lights will be nearly inversely to the mean lower hemispherical candle power. This will be more apparent later. The standard for such lighting has not yet been set in this country. There is now a strong tendency towards clusters of tungsten lights, or arc lights on ornamental poles, placed low on both sides of the street. It is claimed justly that the best lighted streets attract the crowds and hence merchants are insisting upon brighter and brighter illumination on their streets, even assisting the city in meeting the additional cost of maintenance.

15. *Cross Streets and Boulevards.*—The boulevards and cross streets require a much less brilliant illumination. The principal cross streets do not usually have shade trees, and may be lighted either by high intensity lamps suspended high or low intensity lamps more closely spaced and supported low. The general characteristics of the streets will indicate the type of lighting to be used. In such streets, heavy slow moving traffic is usual. The

boulevards, on the other hand, are usually shaded and require the lights to be suspended low unless the streets are very wide. For shaded streets, lights upon ornamental poles at the sides of the streets are preferable to those suspended over the center, on account of the character of the street. By this is meant that since such a street is used mostly for light vehicles, moving more or less rapidly, lights should not be hung so as to blind the drivers. The appearance of such a street is improved by side lighting, or lights suspended fairly high over the center of wide streets. Probably from .10 to .5 foot candle is a good figure to use.

16. *Residence Streets.*—Residence streets should have one light at each street crossing, for the safety of vehicles and pedestrians approaching these crossings, as collisions are more liable to occur at such points. These streets are usually rather narrow and densely shaded, thus requiring the lights to be supported not over 15 to 25 ft. above the street surface. The maximum illumination should not be too high or the minimum too low. The general average of the illumination should, however, be low. The character and color of the surface has a marked effect upon the lighting of such streets. It is well known that a street with snow upon its surface appears much better lighted than the same one when the snow has disappeared. It is usual in dim illumination to see nearly all obstructions as dark objects against the lighter background of the lighted street. Such a contrast is more distinct if the background is lighter. A macadamized road or asphalt pavement needs much more illumination than one paved with light bricks. It will be seen from Tables 4 to 13 that lights will not give good illumination at the middle of blocks 400 ft. long, if the lights are 25 ft. or less above the street. The illumination in the immediate neighborhood of most arc lights is also higher than necessary for this class of street. In crooked streets or where the surface is uneven or hilly, lights must be spaced more closely than at every street crossing. Residence street lighting is the particular field for the incandescent light. In choosing a light for this class of service, the economic limit comes in the ordinary spacing of poles on the street, and the lowest value of illumination allowable immediately below the light. A street having many small lights, hung low and spaced rather closely together, has long been associated with the narrow street of the poorer quarter with its dingy appearance. The brilliant effect and white light from the tungsten light has done much to dispel this prejudice.

17. *Outlying Districts.*—In the outlying districts, either the tungsten light or an arc light having low operating cost may be used. It is seldom that a community can afford to have an arc light at each street intersection in such a district. Hence for protection and safety, the tungsten light may be used to good advantage. A difference here may also be made between streets connecting trade centers and cross streets. The former may be treated as boulevards or as residence streets, according to the traffic conditions.

It is thus seen that no specific rules can be laid down in street lighting. Each case to be handled is more or less special. However in choosing a general lighting scheme for any city, the central station furnishing the power must also be considered. It would of course, not be feasible to specify a variety of lights, requiring varied generating apparatus. In any system, the station apparatus should be interchangeable from one circuit to another for continuity of service and low first cost and maintenance. Thus, if D. C. lamps are chosen, they should all have the same current consumption and this should also fit well with incandescent lamps on the market. On the other hand small compensators may be supplied with each A. C. lamp to adapt it to any A. C. circuit. Wires in series circuits have a voltage upon them at the station, dangerous to personal safety. The circuit may become accidentally grounded at some point, either by contact with a telephone wire or the limb of a tree when wet. In such case, the full voltage may exist between the other wire and ground. Hence, for safety to persons, these wires should not be allowed too near the ground. They are usually required to be strung on the upper cross arm of the poles. If they are brought down to incandescent lamps, suspended too low or strung on low poles, serious accidents are liable to occur. To avoid this, especially low poles should not be used for series lighting. For such service, tungsten multiple 110 volt lamps may be used from special feeder circuits or series lights on ornamental posts fed from armored cables buried in the ground. This latter applies especially to ornamental park lighting.

VII. Cost of Operation

The choice of a lamp for a given class of street lighting depends upon the illumination which may be obtained from the lamp and upon its cost of operation. A flat rate is usually made for street lighting; i. e., a charge of so many dollars per lamp per

year. The items which enter into a consideration of these charges are:

A Fixed charges;
B Maintenance charges;
C Energy charge.

These items, together with the number of hours in operation, or schedule, determine the rate to be charged for the service. These will be considered in the order named.

18. *Fixed Charges.*—In this item is considered a charge of 6 per cent, covering interest on the capital invested, taxes, insurance, etc., and a depreciation charge of 10 per cent. This latter charge, if deposited at 5 per cent in a sinking fund, would replace the equipment in from eight to nine years. With the present rapid advance in the development of lamps, this figure seems conservative. It means that replacement is due to advance in the art, rather than to the complete deterioration of the apparatus. On the other hand, the depreciation charge depends also upon the life of the city ordinance. If the ordinance is for five years, the depreciation should be 18 per cent instead of 10 per cent, since the city may require an entire change of its system at the end of the contract or award the contract to some other competing company.

19. *Maintenance Charges.*—In the following table, this charge is itemized and computed on the basis of 1000 hours' operation. It covers the cost of renewals, due to the consumption of electrodes, breakage of glassware, repairs to the mechanism of the lamp, and a charge for labor in trimming, cleaning of glassware and reflectors, inspection, store room charges, etc. These figures represent an average installation, say of 400 arc lamps or their equivalent in incandescent lamps. The trimming cost for tungsten lamps is low on account of the long life of these units. The arc lamps are supposed to be cleaned and inspected by the trimmer. In the case of the tungsten lamps, it is assumed that the reflectors will be thoroughly inspected and cleaned each time the lamps are renewed and also at regular intervals, say four times per year, independent of the schedule. Hence this cleaning charge is placed as an annual charge instead of on the basis of 1000 hours.

20. *Energy Charge.*—Energy is usually computed upon the basis of the kilowatt hours consumed. For convenience of comparison, the cost of energy at the lamp terminals, rather than at the station switchboard, is considered here. The energy charge will then cover the cost of delivering this power, such as depreci-

ation on station apparatus, poles, wires, etc., and the maintenance charges on the same. In order to compare direct and alternating current lamps operated from the same station, the cost of the rectifier outfit is included in the cost of the direct current lamps, and its depreciation charge is placed there also. The efficiency of a rectifier outfit may be taken as 87.5 per cent. The loss of energy must therefore be charged to the D. C. lamps. This charge is made by increasing the cost of energy in line 31, Table 15, inversely in the ratio of the efficiency to 100 per cent. These values are then used in Tables 17 and 18 and in line 31, Table 15. No accurate information is available as to the energy charge for the different stations of this State. Tables 15 and 16 have been computed with an energy charge of one cent per kilowatt hour. From Tables 15 and 16, Tables 17 and 18 have been computed for two schedules and an energy charge from one cent to five cents per kilowatt hour. Curves have also been drawn between energy charge and cost of operation per lamp per year for two different schedules. These are shown in Fig. 25 to 28. They show a ready comparison between the operating cost of the different lamps chosen. There are many factors which affect the cost of energy in different cities. Some of these are the following:

A Cost of coal delivered at the power plant;

B Available water power or natural gas;

C Cost of the land on which the plant is located and taxes on the same;

D Amount of power developed and the load factor;

E Available market for steam heat;

F Cost of delivering the power to the consumer, (under this item will come type of construction, whether pole lines or conduits and taxes or rentals on the same);

G Interest and depreciation on the equipment.

These items vary so widely in different localities that the writers have not attempted to make any definite statements.

21. *Schedules.*—There are numerous schedules in use in this country. The "all night and every night", or 4000 hours per year schedule is used in most cities having a population of 20,000 and over. In smaller cities and suburban districts, various moonlight schedules are in use, the average of these being about 2500 hours per year. This latter schedule is supposed to approximate moonlight every night in the year. The intensity of illumination at the first or last quarter of the moon is only about one-tenth of that

at full moon instead of one-half as might be supposed. Hence the moonlight schedule should be more nearly three-fourths of the all night schedule, instead of from one-half to five-eighths that amount. In small cities, where only a few circuits are necessary, and where a duplicate set of station equipment cannot be afforded, the all night and every night schedule may be approximated by allowing four nights each month near full moon when the lights on certain circuits may be turned off, allowing the necessary repairs to be made. Many other schedules are also in use but are not listed here. Calculations covering the cost of operation of lamps may be made in a manner similar to those for Tables 17 and 18.

Cost Tables

To illustrate how the values in Table 17 are derived the following problem is chosen.

Problem.—Assume a 6.6 amperes, alternating current, enclosed arc lamp to be operating on a 4000 hour schedule, the energy costs being three cents per kilowatt hour. What will be the cost of operating this lamp for one year?

From Table 4.—

For 1000 Hours and One Cent per Kilowatt Hour, (See Table 15):

Annual fixed charges	$3.20
Maintenance per 1000 hours	2.30
Energy per 1000 hours at one cent	4.25

For 4000 Hours at Three Cents per Kilowatt Hour,

Annual fixed charges	$3.20
Maintenance charges for 4000 hours $=4\times2.30=$	9.20
Energy for 4000 hours at three cents $=4\times3\times4.25=$	51.00
Total charge per lamp per year,	$63.40

A total charge of $70.00 per lamp per year would represent a fair profit to the central station.

An analysis of the three factors entering into the operating cost of street lamps may be made from Tables 15 to 18. It is at once apparent that none of the factors may be omitted safely for any of the lamps. The greater the number of hours burning per year, the less the importance of the fixed charge. On the other hand, the shorter the schedule, the more important the fixed charge, and the less the importance of the maintenance and energy charges. Another fact clearly shown is the effect upon annual cost of the length of life of the electrodes per trim. The

TABLE 15

COST AND OPERATING CHARACTERISTICS OF ARC LAMPS

	ITEMS	Open Arc D. C. Twin Carbon	Enclosed Arc. D. C.	Enclosed Arc A. C.	Enclosed Arc A. C.	Magnetite Arc D. C.	Magnetite Arc D. C.	Flame Arc D. C. Inclined Carbon	Flame Arc A. C. Inclined Carbon	Flame Arc D. C. Vertical Carbon Long Burning	Flame Arc A. C. Vertical Carbon Long Burning
1	Voltage across lamp terminals.........	50	72	72	72	80	80	55	55	70	70
2	Current..................................	9.6	6.6	7.5	6.6	6.6	4.0	10	10	5.5	5.5
3	Terminal watts........................	480	475	480	425	528	320	550	467	385	350
4	Maximum candle power value....... ..	1250	525	350	275	1600	730	2400	—	2575	1925
5	Maximum candle power angle with vertical..............................	45°	65°	40°	40°	80°	80°	0°-30°	—	70°	85°
6	Mean lower hemispherical candle power (H. S. C. P.)......	800	468	298	244	1170	457	1785	850	1800	1485
7	Watts per H. S. C. P....................	.6	1.01	1.61	1.71	.45	.70	.31	.55	.26	.24
8	Kind of glassware, inner globe.........	None	Opal	Opal	Opal	None	None	None	None	Clear	Clear
9	" " " outer "	Clear	Clear	Clear	Clear	Clear	Clear	Opal	Clear	Opal	Clear
10	Curve sheet, figure.....................	3 A	4 C	4 B	4 A	3 B	3 C	5 D	—	5 C	—
11	Life positive electrode, hours..........	17	125	100	110	1500	4000	17	17	70	100
12	" negative " "	17	125			100	175	17		70	100
13	" inner globe........................	—	400	400	400	—	—	—	—	300	300
14	" outer "	1000	2000	2000	2000	2000	2000	1000	1000	2000	2000
15	Cost of lamp, dollars....................	15.00	34.50	20.00	20.00	40.00	40.00	60.00	60.00	60.00	60.00
16	" " positive electrodes, dollars per trim..............................	.015	.021	.022	.022	.50	.45	.14	.14	.25	.25
17	Cost of negative electrodes, dollars per trim..............................					.06	.05				
18	Cost of inner globe, dollars each.......	—	.15	.15	.15	—	—	—	—	.50	.50
19	" " outer " " "	.50	.50	.50	.50	1.00	1.00	1.00	1.00	.75	.75
20	" " labor in trimming, etc., per trim	.06	.06	.06	.06	.06	.06	.06	.06	.06	.06

	A. FIXED CHARGES, ANNUAL										
21	Interest on investment at 6%	.90	2.07	1 20	1.20	2.40	2.40	3.60	3.60	3.60	3.60
22	Depreciation at 10%	1.50	3.45	2.00	2.00	4.00	4.00	6.00	6.00	6.00	6.00
23	Total annual fixed charge..............	2.40	5.52	3.20	3.20	6.40	6.40	9.60	9.60	9.60	9.60
	B. MAINTENANCE CHARGES PER 1000 HOURS										
24	Cost of electrodes, dollars..............	.89	.17	.22	.20	.93	.40	8.23	8.23	3.56	2.50
25	" " inner globes, dollars.............	—	.37	.37	.37	—	—	—	—	1.67	1.67
26	" " outer " "	.50	.25	.25	.25	.50	.50	1.00	1.00	.37	.37
27	" " trimming and inspection, dollars..............................	3.54	.48	.60	.55	.60	.34	3.54	3.54	.85	.60
28	Cost of rectifier renewals, dollars......	—	.50	—	—	.50	.50	.50	—	.50	—
29	" " repairs, dollars..................	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50
30	" " maintenance, total, dollars....	5.43	2.27	1 94	1.87	3.03	2.24	13.77	13.27	7.45	5.64
	C. COST OF ENERGY, PER 1000 HOURS										
31	At one cent per kilowatt hour..........	4 80	5.42	4.80	4.25	6.05	3.65	6.25	4.67	4.40	3.50

* Line 31 has been computed from line 3, considering the rectifier efficiency.

TABLE 16

COST AND OPERATING CHARACTERISTICS OF TUNGSTEN LAMPS

	ITEMS	For Suspension in Place of Arc					For Suspension at Side of Street				
1	Candle power horizontal	32	60	100	200	350	32	60	100	200	350
2	Terminal watts	38	71	118	236	413	38	71	118	236	413
3	Watts per mean horizontal candle power	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18
4	Mean hemispherical candle power (with reflector)	43	80	135	227	471	43	80	135	227	471
5	Watts per H. S. C. P.	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04
6	Maximum candle power, value	45	84	140	280	490	45	84	140	280	490
	Maximum candle power, angle with vertical	70°	70°	70°	70°	70°	70°	70°	70°	70°	70°
7	Curve sheet, figure										
8	Life of lamp, hours	1350	1350	1350	1350	1350	1350	1350	1350	1350	1350
9	Cost of lamp, dollars	1.00	1.00	1.20	2.44	3.75	1.00	1.00	1.20	2.44	3.75
10	" " labor for inspection and cleaning, annual	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40
11	Cost of fixtures, dollars	1.50	1.50	2.00	2.50	2.50	5.00	5.00	5.00	5.00	5.00
12	" " lamp and fixtures	2.50	2.50	3.20	4.94	6.25	6.00	6.00	6.20	7.44	8.95
	A. FIXED CHARGES, ANNUAL										
13	Interest on investment at 6%	.15	.15	.17	.30	.37	.36	.36	.37	.45	.52
14	Depreciation at 10%	.25	.25	.32	.49	.63	.60	.60	.62	.74	.87
15	Annual inspection charge	.40	.40	.40	.40	.40	.40	.40	.40	.40	.40
16	Total annual fixed charges	.80	.80	.89	1.19	1.40	1.36	1.36	1.39	1.59	1.79
	B. MAINTENANCE CHARGES PER 1000 HOURS										
17	Cost of lamp renewals, dollars	.74	.74	.83	1.80	2.80	.74	.74	.83	1.80	2.80
18	" " repairs	.05	.05	.05	.10	.10	.15	.15	.15	.15	.15
19	" " maintenance total	.79	.79	.88	1.90	2.90	.89	.89	.98	1.95	2.95
	C. COST OF ENERGY PER 1000 HOURS										
20	At one cent per kilowatt hour	.38	.71	1.18	2.36	4.13	.38	.71	1.18	2.36	4.13

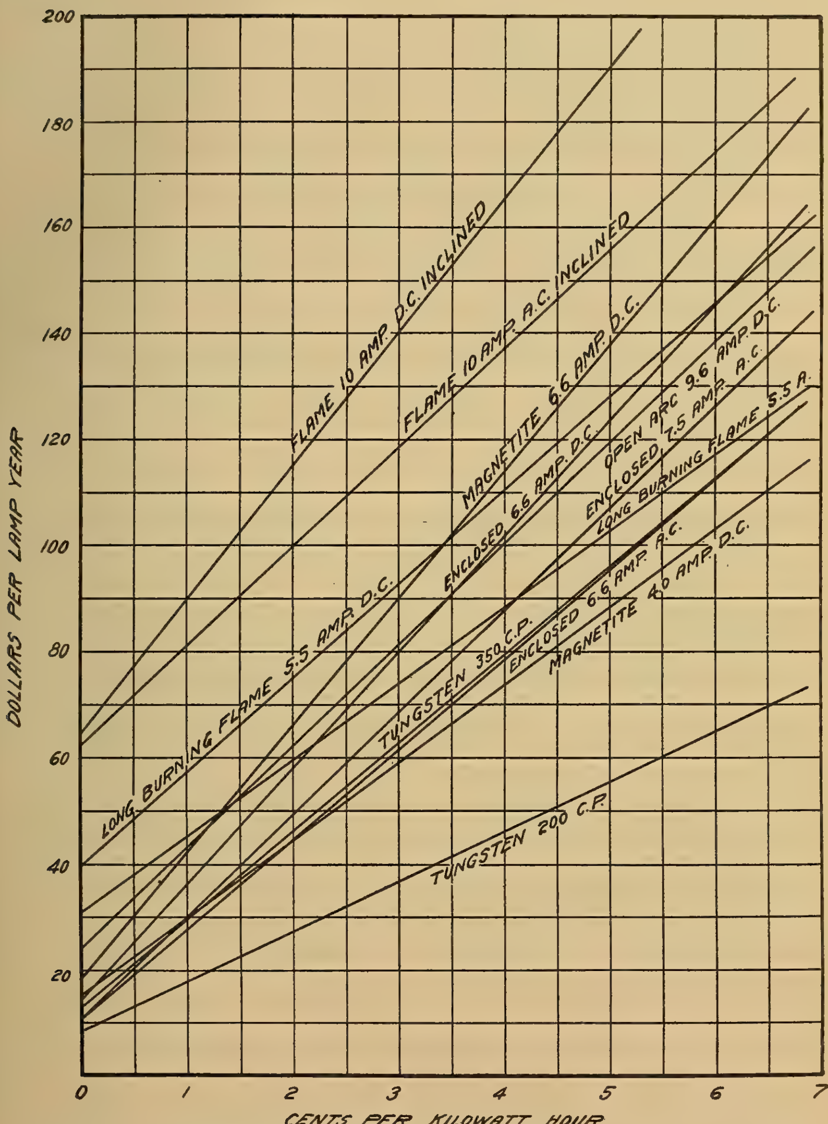

FIG. 25. CURVES SHOWING ANNUAL OPERATING COST OF ARC LAMPS ON 4000-HR. SCHEDULE

TABLE 17

Operating Cost on 4000-Hour Schedule

	Items	Open Arc Twin Carbon D. C.	Enclosed D. C.	Enclosed A. C. 7.5 Amp.	Enclosed A. C. 6.6 Amp.	Magnetite 6.6 Amp.
	For 1000 Hours from Table					
1	Annual fixed charge	2.40	5.52	3.20	3.20	6.40
2	Maintenance for 1000 hours	5.42	2.27	1.94	1.87	3.03
3	Energy per 1000 hours at 1 cent per K. W. Hr.	4.80	5.42	4.80	4.25	6.05
	Annual Cost 4000 Hour Schedule					
4	Annual fixed charge (1)	2.40	5.52	3.20	3.20	6.40
5	Maintenance, 4 × (2)	21.68	9.08	7.76	7.48	12.12
6	Energy at 1 cent, 4 × (3)	19.20	21.70	19.20	17.00	24.20
7	" " 2 cents, 2 × 4 × (3)	38.40	43.40	38.40	34.00	48.40
8	" " 3 " , 3 × 4 × (3)	57.60	65.20	57.60	51,00	72.60
9	" " 4 " , 4 × 4 × (3)	76.80	86.80	76.80	68.00	96.80
10	" " 5 " , 5 × 4 × (3)	96.00	108.50	96.00	85.00	111.00
	Total Annual Operating Cost					
11	Total cost per lamp, 1 cent per K. W. Hr. (4) + (5) + (6)	43.28	36.30	30.16	27.68	42.72
12	Total cost per lamp, 2 cents per K. W. Hr. (4) + (5) + (7)	62.48	58.00	49.36	44.68	66.92
13	Total cost per lamp, 3 cents per K. W. Hr. (4) + (5) + (8)	81.68	79.80	68.56	61.68	91.12
14	Total cost per lamp, 4 cents per K. W. Hr. (4) + (5) + (9)	100.88	101.40	87.76	78.68	105.32
15	Total cost per lamp, 5 cents per K. W. Hr. (4) + (5) + (10)	120.08	123.10	106.96	95.68	129.52

TABLE 17 (*Continued*)

4000–Hour Schedule

Magnetite 4.0 Amp.	Flame D. C. Inclined	Flame A. C. Inclined	Flame D. C. Vertical Long Burning	Flame A. C. Vertical Long Burning	Center Suspension Tungsten 32. C. P.	Center Suspension Tungsten 60 C. P.	Center Suspension Tungsten 100 C. P.	Center Suspension Tungsten 200 C. P.	Center Suspension Tungsten 350 C. P.	Side Suspension Tungsten 32 C. P.	Side Suspension Tungsten 60 C. P.	Side Suspension Tungsuen 100 C. P.	Side Suspension Tungsten 200 C. P.	Side Suspension Tungsten 350 C. P.
6.40	9.60	9.60	9.60	9.60	.80	.80	.84	1.19	1.40	1.36	1.36	1.39	1.59	1.79
2.24	13.77	13.27	7.45	5.64	.79	.79	.88	1.90	2.90	.89	.89	.98	1.95	2.95
3.65	6.25	4.67	4.40	3.50	.38	.71	1.18	2.36	4.13	.38	.71	1.18	2.36	4.13
6.40	9.60	9.60	9.60	9.60	.80	.80	.84	1.19	1.40	1.36	1.36	1.39	1.59	1.79
8.96	55.08	53.08	29.80	22.56	3.16	3.16	3.52	7.60	11.60	3.56	3.56	3.92	7.80	11.80
14.60	25.20	18.68	17.60	14.00	1.52	2.84	4.72	9.44	16.52	1.52	2.84	4.72	9.44	16.52
29.20	50.40	37.36	35.20	28.00	3.04	5.68	9.44	18.88	33.04	3.04	5.68	9.44	18.88	33.04
43.80	75.60	56.04	52.80	42.00	4.52	8.52	14.16	28.32	49.56	4.56	8.52	14.16	28.32	49.56
58.40	100.80	74.72	70.40	56.00	6.08	11.36	18.88	37.76	66.08	6.08	11.36	18 88	37.60	66.08
73.00	126.00	93.40	86.00	70.00	7.60	14,20	23.60	47.20	82.60	7.60	14.20	23.60	47.20	82.60
29.96	89.88	81.36	57.00	46.16	5.48	6.80	9.08	18.23	29.52	6.44	7.76	10.03	18.83	30.11
34.56	115.08	100.04	74.60	60.16	7.00	9.64	13.80	27.67	46.04	7.96	10.60	14.75	28.27	46.63
59.16	140.28	118.72	92.20	74.16	8.52	12.48	18.52	37.11	62.56	9.48	13.44	19.47	37.79	63.15
73.76	165.48	137.40	109.80	88.16	10.04	15.32	23.24	46.55	79.08	11.00	16.28	24.19	47.15	79.67
88.36	190.68	156.08	125.40	102.16	11.56	18.16	27.96	55.99	95.60	12.52	19.12	28.91	56.59	96.19

TABLE 18

OPERATING COST ON 2500-HOUR SCHEDULE

	Items	Open Arc Twin Carbon D. C.	Enclosed D. C.	Enclosed A. C. 7.5 Amp.	Enclosed A. C. 6.6 Amp.	Magnetite 6.6 Amp.
	FOR 1000 HOURS FROM TABLE					
1	Annual Fixed Charge	2.40	5.52	3.20	3.20	6.40
2	Maintenance For 1000 hours	5.42	2.27	1.94	1.87	3.03
3	Energy Per 1000 hours at 1 cent Per K. W. Hour	4.80	5.42	4.80	4.25	6.05
	ANNUAL COST 2500 HOUR SCHEDULE					
4	Annual Fixed Charge (1)	2.40	5.52	3.20	3.20	6.40
5	Maintenance, 2.5 x (2)	13.55	5.67	4.85	4.67	7.57
6	Energy at 1 cent, 2.5 x (3)	12.00	13.55	12.00	10.62	15.10
7	Energy at 2 cents, 2.5 x 2 x (3)	24.00	27.11	24.00	21.24	30.20
8	Energy at 3 cents, 2.5 x 3 x (3)	36 00	40.70	36.00	31.86	45.30
9	Energy at 4 cents, 2.5 x 4 x (3)	48.00	54.30	48.00	42.48	60.40
10	Energy at 5 cents, 2.5 x 5 x (3)	60.00	67.70	60.00	53.10	75.50
	TOTAL ANNUAL OPERATING COST					
11	Total Cost Per Lamp, 1 cent Per K. W. Hr. (4) + (5) + (6)	27.95	24.74	20.05	18.49	29.07
12	Total Cost Per Lamp, 2 cents Per K. W. Hr. (4) + (5) + (7)	39.95	38.30	32.05	29.11	44.17
13	Total Cost Per Lamp, 3 cents Per K. W. Hr. (4) + (5) + (8)	51.95	51.89	44.05	39.73	59.27
14	Total Cost Per Lamp, 4 cents Per K. W. Hr. (4) + (5) + (9)	63.95	65.49	56.05	50.35	74.37
15	Total Cost Per Lamp, 5 cents Per K. W. Hr. (4) + (5) + (10)	75.95	78.89	68.05	60.97	89.47

TABLE 18 (*Continued*)

2500-Hour Schedule

Magnetite 4.0 Amp.	Flame D. C. Inclined	Flame A. C. Inclined	Flame D. C. Vertical Long Burning	Flame A. C. Vertical Long Burning	Center Suspensions					Side Suspension				
					Tungsten 32 C. P.	Tungsten 60 C. P.	Tungsten 100 C. P.	Tungsten 200 C. P	Tungsten 350 C. P.	Tungsten 32 C. P.	Tungsten 60 C. P.	Tungsten 100 C.P	Tungsten 200 C. P.	Tungsten 350 C.H,
6.40	9.60	9 60	9.60	9.60	.80	.80	.84	1.19	1.40	1.36	1.36	1.39	1.59	1.79
2.24	13.77	13.27	7.45	5.64	.79	.79	.88	1.90	2.90	.89	.89	.98	1.95	2.95
3.65	6.25	4.67	4.40	3.50	.38	.71	1.18	2.36	4.13	.38	.71	1.18	2.36	4.13
6.40	9.60	9.60	9.60	9.60	.80	.80	.84	1.19	1.40	1.36	1.36	1.39	1.59	1.79
5.60	34.42	33.11	18.62	14.10	1.97	1.97	2.20	4.75	7.25	2.22	2.22	2.45	4.87	7.37
9.14	15.70	11.67	11.00	8.75	.95	1.77	2.95	5.90	10.32	.95	1.77	2.95	5.90	10.32
18.30	31.40	23.34	22.00	17.50	1.90	3.54	5.90	11.80	20.64	1.90	3.54	5.90	11.80	20.64
27.40	47.10	35.01	33 00	26.25	2,85	5.31	8.85	17.70	30.96	2.85	5.31	8.85	17.70	30.96
36.50	62.80	46.68	44.00	35.00	3.80	7.08	11.80	23.60	41.28	3.80	7.08	11.90	23.60	41.28
45.70	78.60	58.35	55.00	43.75	4.75	8.85	14.75	29 50	51.60	4.75	8.85	14.75	29.50	51.60
21.14	59.72	54.44	39.22	32.45	3.72	4.54	5.99	11.84	18.97	4.53	5.35	6.79	12.36	19.48
30.30	75.42	66.11	50.02	41.20	4.67	6.31	8.94	17.74	29.29	5.48	7.12	9.74	18.26	29.80
39.40	91.12	77.78	61.22	49.95	5.62	8.08	11.89	23.64	39.61	6.43	8.89	12.69	24.16	40.12
48.50	106.82	89.45	72.22	58.70	6.57	9.85	14.84	29.54	49.93	7.38	10.66	15.64	30.06	50.44
57.70	122.62	101.12	83.22	67.45	7.52	11.62	17.79	35.44	60.25	8.33	12.43	18.59	35.96	60.76

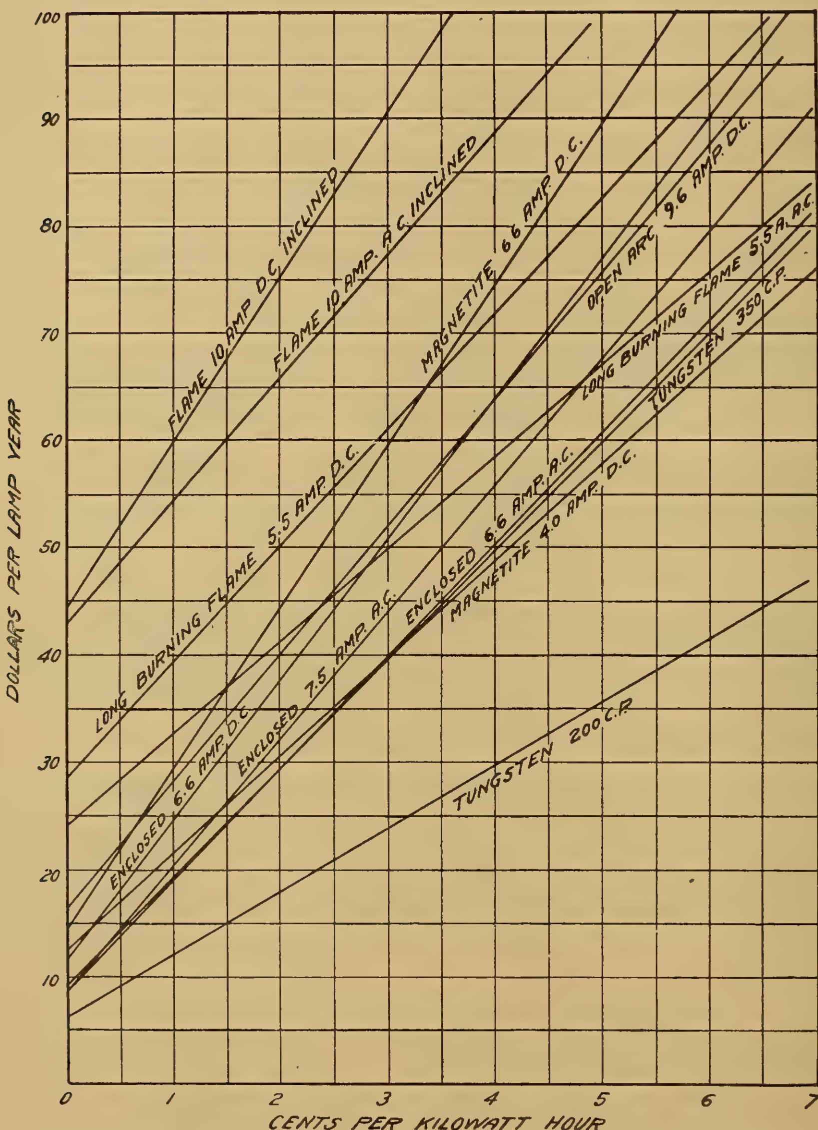

FIG. 26. CURVES SHOWING ANNUAL OPERATING COST OF ARC LAMPS ON 2500-HR. SCHEDULE

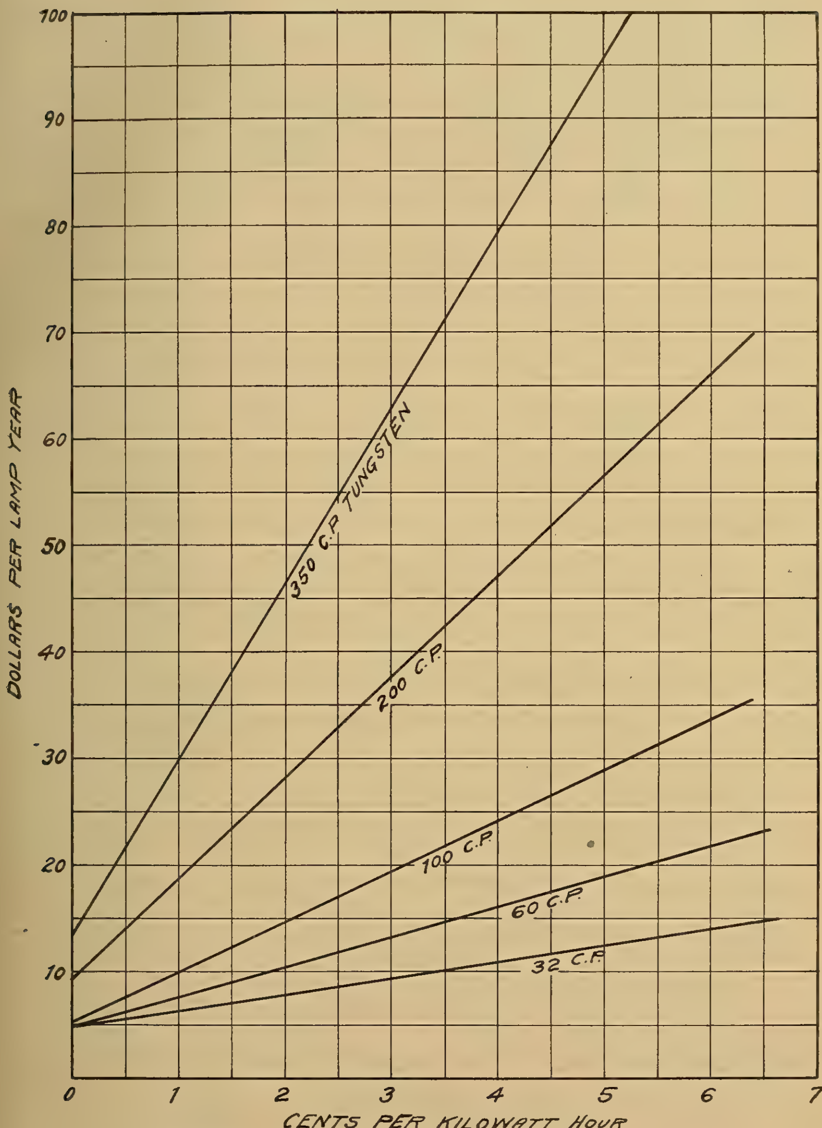

FIG. 27. CURVES SHOWING ANNUAL OPERATING COST OF TUNGSTEN LAMPS ON 4000-HR. SCHEDULE

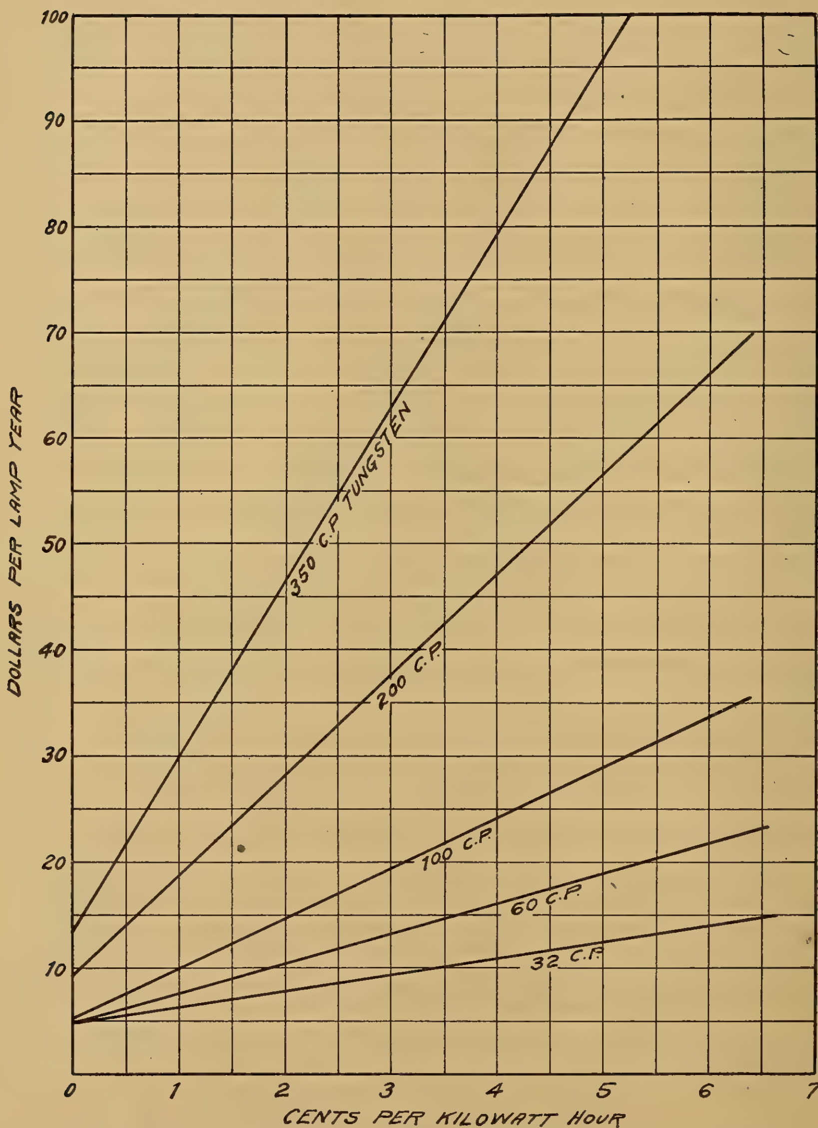

FIG. 28. CURVES SHOWING ANNUAL OPERATING COST OF TUNGSTEN LAMPS ON 2500-HR. SCHEDULE

item of labor and supplies is large for the open arc, and for short burning flame arcs. In fact, the maintenance charge on these lamps is nearly equal to the total operating charge on some of the other lamps. This is not offset by a proportionately greater amount of light. In the tungsten lamps, the energy charge is rather more important than in most of the arc lamps, but the fixed and maintenance charges are low on account of the long life of these lamps. Although the cost of each renewal is rather high, only very few renewals have to be made each year.

In the case of cities where a lighting system using series lights is already installed, the question arises as to the proper rate to be agreed upon in a new ordinance. This is usually computed on the basis of the lights at present in use, or these lights replaced by the same number of lamps of some different type. In fixing the rate for these new lights, Tables 17 and 18 or Fig. 22 to 28 will be of direct application, since the rate will be in proportion to the cost per lamp per year. A comparison of the lamps on the basis of street distribution will show whether the new lamp is equivalent to the old one in illuminating power. It will be seen that with four cent energy, a saving of $4 per lamp per year may be expected by replacing 6.6 amperes A. C. lamps with 4.0 ampere magnetites. At the same time, the street illumination is improved. For 100 lamps, this means a saving of $400 per year on the 4000 hour schedule. The 200 C. P. tungsten light would give as good satisfaction as the 6.6 amperes A. C., on account of its steady light, and at a saving of $3200 per year for the 100 lights.

It has been shown, from the comparison of Tables 17 and 18, that the high cost of maintenance for short burning lamps makes their yearly cost very high on any schedule or cost of energy, in comparison with the long burning lamps. This can be compensated for only by the use of fewer lamps. It will be seen, however, that for residence districts, the illumination is so low at the minimum point that none of the lamps can be spaced at a greater distance than 400 ft., or one block. Hence the comparison of arc lamps for this class of service must be made upon this spacing. The illumination on such streets can be improved only by using smaller units spaced closer together. This is the special field for the tungsten lamps. To show this more clearly, the following comparison has been made.

Problem.—To light a shaded residence street one mile long, 400 ft. blocks, energy being assumed at three cents per kilowatt hour and the 4000 hour schedule. Using only the lower cost

lamps, Table 19, page 59, has been computed. It is seen that a good minimum and low maximum illumination can be produced by using tungsten lamps. There will be required 14 lamps at 400 ft. or 27 lamps at 200 ft. or 53 lamps at 100 ft. This is assuming that poles are spaced conveniently for this arrangement. It will be seen that nearly as good illumination is produced by 60 C. P. tungstens, spaced 100 ft. apart, as by any of the arc lights, except the 6.6 amperes magnetite, and at a cost of only $713 per year, compared with $3220 for the 6.6 A. C. enclosed. Probably the 200 C. P. tungsten would be selected at 200 ft. spacing, costing $1020, since the street would appear much more brilliantly lighted than with the 60 C. P. Other comparisons will doubtless suggest themselves, depending upon local conditions.

VII. Summary and Conclusions

From Tables 1, 2 and 3, it is seen that for uniform distribution of illumination on a surface from single light sources, their illumination not overlapping to any extent requires very low values of intensity at angles near the vertical, and to as wide an angle as 60°, except where the lights are suspended high over the surface to be illuminated. A comparison of the curves in Fig. 3, 4 and 5 for arc lamps with these tables shows that none of these distribution curves can be expected to give uniform low illumination. Curves of the shape of those shown by B or C, Fig. 3, or A or C, Fig. 5, will give wider uniform distribution than the other curves shown. Curve A, Fig. 3, shows that a bright ring of light must be expected at a distance from the base of the lamp equal to the light with a dark circular area within, and a rather abrupt dark area without. For curves in Fig. 4, the lamps must be spaced a little farther apart than their height. A considerable amount of light between 60° and 90° is available for overlapping. Curves B and D, Fig. 5, and the curves on Fig. 6 show that these lamps must be used spaced rather close together. Inclined carbon flame lamps can be used successfully only for very bright illumination, on account of the intensity at the zero angle. For dim uniform illumination, the tungsten light is the proper one to chose, on account of the lower intensity available in the small units and its excellent distribution curve for this purpose. This point is very strikingly brought out by comparing the illumination curves in Fig. 22.

A lamp throwing a strong light at a wide angle must be surrounded by an opal globe in order to lower its intrinsic brilliancy. The lower the light is hung over the center of the street, the

TABLE 19

Annual Cost of Lighting One Mile of Street for .02-ft. Candle Horizontal Illumination

ITEMS	Illumination in Foot Candles						Annual Cost								For .02 ft. Candle Minimum					
	Spacing 400′		Spacing 200′		Spacing 100′		1 Lamp	Per Mile of Street 4000 Hr. Schedule			1 Lamp	Per Mile of Street 2500 Hr. Schedule			Distance Apart	Lights per Pole	Foot Candle		Cost	
	Max.	Min.	Max.	Min.	Max.	Min.		14 Lamps 400′	27 Lamps 200′	53 Lamps 100′		14 Lamps 400′	27 Lamps 200′	53 Lamps 100′			Max.	Min.	4000 Hour	2500 Hour
Enclosed D. C. 6.6 Ampere	.36	.003	.372	.023	—	—	79.80	1120	2150	4230	51.89	727	1400	2750	200	1	.372	.023	2150	1400
Enclosed A. C. 7.5 Ampere	.336	.0016	.34	.012	—	—	68.56	960	1820	3635	44.05	617	1190	2330	200	2	.68	.024	3640	2380
Enclosed A. C. 6.6 Ampere	.28	.0014	.28	.009	—	—	61.68	865	1660	3270	39.73	556	1083	2100	200	2	.56	.02	3300	2166
Magnetite D. C. 6.6 Ampere	.80	.009	.85	.069	—	—	91.12	1275	2460	4830	59.27	830	1600	3140	400	2	1.8	.018	2550	1660
Magnetite D. C. 4.0 Ampere	.56	.004	.564	.033	—	—	59.16	827	1595	3130	39.40	552	1065	2090	200	1	.564	.033	1595	1065
Flame D. C. 10.0 Ampere Inclined	3.81	.008	3.82	.07	—	—	140.28	2000	3850	7450	91.12	1275	2460	4830	400	2	7.62	.016	4000	2550
Flame D. C. 5.5 Ampere Long Burning	1.26	.012	1.27	.084	—	—	92.20	1290	2490	4880	61.22	857	1650	3240	400	2	2.52	.024	2580	1714
Tungsten 200 C. P. Center Suspension	.362	.003	.365	.023	.366	.068	37.11	520	1000	1968	23.64	331	638	1250	200	1	.365	.023	1000	638
Tungsten 350 C. P. Center Suspension	.37	.002	.37	.025	—	—	62.56	875	1690	3311	39.61	555	1070	2100	200	1	.37	.025	1690	1070
Tungsten 32 C. P. Side Suspension	—	—	.06	.002	.06	.01	9.48	133	256	503	6.43	90	174	341	100	2	.12	.02	1006	682
Tungsten 60 C. P. Side Suspension	—	—	.11	.003	.11	.021	13.44	188	363	713	8.89	126	243	477	100	1	.11	.021	713	477
Tungsten 100 C. P. Side Suspension	—	—	.181	.005	.183	.034	19.47	273	525	1032	12.69	178	342	673	100	1	.183	.034	1032	673
Tungsten 200 C. P. Side Suspension	.362	.003	.365	.023	.366	.068	37.71	528	1020	2000	24.16	338	652	1280	200	1	.365	.023	1020	652
Tungsten 350 C. P. Side Suspension	.575	.002	.6	.02	—	—	63.15	883	1700	3350	40.12	561	1080	2120	200	1	.6	.02	1700	1080

Height 25 ft. for Arcs and 350 C. P. Tungstens.
Height 20 ft. for all others.

more is this important. For very bright lights like the flame arcs, this must also be observed, even when the lights are hung high as they are still in the line of vision and effective at a greater distance. The sentiment of the general public is beginning to demand this consideration more and more each year.

VIII. APPENDIX

Definitions

Candle-Lumen.—The total flux of light from a source is equal to its mean spherical intensity multiplied by 4π (12.57). The unit of flux is called *lumen.* A *lumen* is the $\frac{1}{4\pi}$ th (0.0796) part of the total flux of light emitted by a source having a mean spherical itensity of one candle-power. A *hefner-lumen* is a 0.90 lumen.

Candle-power.—The luminous intensity of sources of light is expressed in *candle-power.* The unit of candle-power should be derived from the standards maintained by the National Bureau of Standards at Washington, D. C. The *hefner* is 0.90 of this unit. In practical measurements, seasoned and carefully standardized incandescent lamps are more reliable and accurate than the primary standard.

Efficiency of Electric Lamps.—The efficiency of electric lamps is properly stated in mean spherical candle-power per watt, and preferably in lumens per watt at the lamp terminals.

Electrode.—Either of the terminals of an arc lamp between which the arc is formed. The electrodes are sometimes called the carbons.

Foot-candle.—Illumination is expressed in foot-candles. A foot-candle is the normal illumination produced by one unit of candle-power at a distance of one foot.

Mean Horizontal Candle-power.—The average intensity of light in a horizontal plane passing through the center of light source.

Mean Spherical Candle-power.—The average candle-power of a source taken over the surface of a sphere having its center at the source of light. It is numerically equal to the total light emitted divided by 4π (12.57).

Mean Hemispherical Candle power.—The average candle-power in a hemisphere having the center of its plane surface at the center of the light source. Usually the lower hemisphere is chosen.

Note.—The hefner is a unit used mostly in Germany. It is smaller than the candle-power. 1 hefner = .9 candle power.

Moonlight Schedule.—A schedule of burning hours for lamps, the lamps burning only when the moon does not shine; in this bulletin, 2500 hours burning per year.

Watts per Candle-power.—The specific consumption of an electric lamp is its watts consumption per mean spherical candle-power. "Watts per candle" is a term used commercially in connection with incandescent lamps, and denotes watts per mean horizontal candle-power.

Load Factor.—Load factor is the ratio between the average load and the maximum load for the given day.

Reading Distance.—Where standard photometric measurements are impracticable, approximate measurements of illuminants used as street lamps may be made by comparing their "reading distances", i. e., by determining alternately the distances at which an ordinary size of reading print can just be read, by the same person or persons, where all other light is screened. The angle below the horizontal at which the measurement is made should be specified when it exceeds 15°.

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W. F. M. GOSS
Director.

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BULLETIN NO. 52

AN INVESTIGATION OF THE STRENGTH OF ROLLED ZINC

BY

HERBERT F. MOORE

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

URBANA, ILLINOIS
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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 52 DECEMBER 1911

AN INVESTIGATION OF THE STRENGTH OF ROLLED ZINC

BY HERBERT F. MOORE, ASSISTANT PROFESSOR IN THEORETICAL AND APPLIED MECHANICS

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LIST OF TABLES

AN INVESTIGATION OF THE STRENGTH OF ROLLED ZINC.

I. Introduction.

1. *Uses of Zinc.*—Zinc is used as a constituent of brass and other alloys, as a protective coating for iron and steel plate and pipe, and as a preventive against rusting out of steam boiler tubes. It is used for making fruit jar covers and corrosion-resisting cans and boxes. In the form of wire, it is used for making shoe nails, and in the form of plates, it is used for making etchings (line cuts) for the reproduction of drawings. Zinc has a wide use as the electro-negative element in electric batteries. The ductility of zinc is an important factor in the manufacture of fruit jar covers, cans, battery zincs, or in other cases when it is to be bent or stamped into shape. Zinc is rarely used as a stress-carrying member of a machine or structure; in Europe thin zinc plates are sometimes used for roofing, and in a few cases electric cables have been suspended by strips of sheet zinc, which resist atmospheric corrosion better than do strips of steel plate.

2. *Purpose of Investigation and Acknowledgment.*—The occasional use of sheet zinc hangers for supporting electric cables called the attention of the Matthiessen and Hegeler Zinc Company of La Salle, Illinois, to the question of the strength of rolled sheet zinc, and it was found that few data were available. The matter was referred to the attention of the Engineering Experiment Station of the University of Illinois, and the general subject of the strength of rolled zinc seemed to be of sufficient importance to warrant making tests. A series of tests of strength of zinc was made in the Laboratory of Applied Mechanics of the University of Illinois, by the writer, under the general direction of Professor Arthur N. Talbot, head of the Department of Theoretical and Applied Mechanics.

In addition to tension tests of zinc, tests of rolled zinc under punching and shearing were made. In the course of the investigation, several tests of cast zinc were made, including torsion and compression tests, and also cold bending tests of rolled zinc. The results of all these tests are recorded in this bulletin.

3. *Existing Data on the Strength of Rolled Zinc.*—The principal tests on the strength of rolled zinc have been made by Bauschinger, Martens, and Meyer. Bauschinger's tests* showed the effect of rapidity of loading on the properties of zinc. The influence of duration of test on the strength of cast zinc was slight, but with rolled zinc, rapidity of testing increased the tensile strength. In tests lasting 6 min., the average ultimate tensile strength was 29 100 lb. per sq. in.; in tests lasting 81 min., the average ultimate tensile strength was 23 300 lb. per sq. in.

TABLE 1.

TENSION TESTS OF ROLLED ZINC; EFFECT OF THICKNESS AND OF DIRECTION OF ROLLING (MARTENS).

Thickness of Plate—inches	Ultimate Tensile Strength lb. per sq. in.	
	With Grain (Parallel to Direction of Rolling)	Across Grain (Perpendicular to Direction of Rolling)
0.019—0.222	26 600—14 200	31 800—20 200

TABLE 2.

TENSION TESTS OF ROLLED ZINC; EFFECT OF TEMPERATURE OF TEST SPECIMEN (MARTENS).

Temperature of Specimen Degrees Fahr.	Ultimate Tensile Strength lb. per sq. in.	Elongation after Rupture per cent
66—81	20 600	12.4
176	12 500	29.4
248	8 960	59.4
302	5 790	101.5
338	7 960	17.1
392	6 120	7.2

Martens'† investigations of the strength of zinc dealt with the influence on strength of thickness of plate, of direction of rolling, and of temperature. His results are summarized in Table 1 and Table 2. From these results, it would seem that about 300° F. is the most favorable rolling temperature for zinc plates, as at that temperature the strength is low and the ductility a maximum. All Martens' tests were made on zinc refined from Silesian ores.

*Mitteilungen aus dem mechanisches—technische Lab. der Technische Hochschule im Munchen, 1887, Heft 20, Seite 16.

†Mitteilungen aus mech—tech. Versuchstantalten in Berlin, Erganzungsheft IV.

An extensive series of tests of the strength of zinc plates was made by Dr. Oswald Meyer of Vienna*. The plates tested by him were all rolled at the zinc works of Cilli, in Austria. The plates contained from 0.021 to 1.04 per cent of lead, 0.03 to 0.912 per cent of cadmium, 0.02 to 0.03 per cent of iron, from a trace to 0.009 per cent of copper, and a trace of arsenic. Tests were made of (a) plates as received from the zinc works; (b) plates treated with nitric acid; and (c) plates subjected to heat treatment before testing. The results of Meyer's tension tests of plates as received from the zinc works, are shown in Table 3.

TABLE 3.

TENSION TESTS OF ROLLED ZINC (MEYER).

Tests of zinc as received from the rolling mill.
Thickness of plate tested varied from 0.044 in. to 0 051 in.

Item	With Grain	Across Grain	Average
Stress at first permanent set, lb. per sq. in.	1 990	2 420	2 130
Stress at limit of proportionality of stress to deformation, lb. per sq. in.	710	1 280	995
Yield point, lb. per sq. in.	11 400	13 640	12 500
Ultimate, lb. per sq. in.	30 400	36 800	33 600
Elongation, per cent	27.2	9.7	18.5
Reduction of area, per cent	43	17	30
Modulus of elasticity, lb. per sq. in.	12 850 000	14 500 000	13 620 000

The following features of Meyer's tests are worthy of note: Specimens cut across grain (perpendicular to the direction of rolling) are somewhat stronger and stiffer than specimens cut with the grain (parallel to the direction of rolling).

The ductility of specimens cut with the grain is greater than that of specimens cut across the grain. Stresses at elastic limit and yield point are very low, the yield point is not sharply marked, and the flow of metal under high stress goes on for a long time.

The tests of zinc plates treated with acid show that a 10 min. immersion in 5 per cent nitric acid did not appreciably lessen the strength or the ductility of the zinc plate.

A series of tests was made by Dr. Meyer on zinc plates subjected to the following heat treatment before testing in tension:

*Oesterreichische Zeitschrift fur Berg und Huttenwesen, Oct. 7 and 14, 1905.

The specimens were subjected for one hour to a temperature of 527° F., when upon testing they developed the properties indicated in Table 4 (average values).

The ultimate strength in tension and the ductility are decidedly lowered by this heat treatment. The "critical temperature" for zinc was found to be at about 300° F. and the heat transformation of the zinc took place very rapidly, one minute being seemingly sufficient to effect it. The tests of Martens and of Meyer point to the desirability of keeping the working temperature during the rolling of zinc within narrow limits. Meyer recommends 302° F. as a maximum rolling temperature.

Meyer also made tests to show the effect of alloying zinc with cadmium and with lead. He found that the addition of 0.2 per cent of cadmium improved the quality of zinc plate, but that the addition of 0.4 per cent of either cadmium or lead either produced no appreciable effect or injured the quality. The addition of both cadmium and lead lowered both the strength and the ductility of zinc plate.

The Matthiessen and Hegeler Zinc Company of La Salle, Illinois, report a series of tension tests of zinc plate made at their request in 1907. The thickness of the plate tested varied from 0.011 in. to 0.04 in., the average tensile strength for sixty specimens was 29 370 lb. per sq. in. The thinner plate showed slightly greater strength than the thicker plate.

II. Specimens, Tests, and Methods of Testing.

4. *Source of Supply of Zinc for Tests.*—The zinc tested at the University of Illinois came from the zinc works of the Matthiessen and Hegeler Zinc Company at La Salle, Illinois, and from the stock of a local hardware store. The zinc from the Matthiessen and Hegeler Company was smelted from ores from the Joplin, Missouri, district. Several specimens of cast zinc were furnished which were remelted spelter poured directly into moulds. Eighteen sheets of rolled zinc, each 18 in. by 20 in., were furnished, varying in thickness from 0.006 in. to 1.0 in. Three sheets of each thickness were furnished, and each of the three sheets was from a different heat. No special precautions as to heat treatment were taken either with the cast zinc or with the rolled zinc. As a check on the values obtained for the zinc plates furnished by the Matthiessen and Hegeler Zinc Co., tests were made on sheet zinc purchased in the local market (Champaign, Illinois).

TABLE 4.

TENSION TESTS OF HEAT TREATED ZINC (MEYER).

Stress at first permanent set..	2 480 lb. per sq. in.
Stress at limit of proportionality of stress to deformation	1 710 lb. per sq. in.
Yield point	11 100 lb. per sq. in.
Ultimate	17 500 lb. per sq. in.
Elongation	4.6 per cent
Reduction of area	5 per cent
Modulus of elasticity	15 950 000 lb. per sq. in.

5. *Test Specimens.*—Specimens of cast zinc were tested in tension in compression, and in torsion. Specimens of rolled zinc were tested in tension, in cold bending and in shear; the tests in shear included punching tests and tests in direct shearing. The specimens for tension tests of cast zinc were similar in form and dimension to the specimen shown in Fig. 1. The cast zinc was furnished in bars 1¼ in. square by 12 in. long and machined to the size shown in Fig. 1.

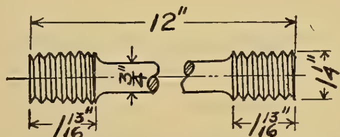

FIG. 1. TENSION SPECIMEN FOR CAST ZINC.

Fig. 2 shows the form and dimension for the specimens of cast zinc tested in torsion. The specimens of cast zinc tested in compression were circular cylinders 1 in. in diameter by 1½ in. long. They were machined all over.

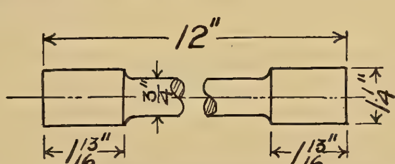

FIG. 2. TORSION SPECIMEN FOR CAST ZINC.

Fig. 3 shows the size of the plates of rolled sheet zinc furnished by the Matthiessen and Hegeler Company, and also shows the plan followed in cutting test specimens from the plates. The specimens similarly located in different plates were stamped with the same number, the plate being designated by a letter. The specimens for tension tests of rolled zinc were similar in form and dimension to the specimen shown in Fig. 4. The strips from which they were machined out were sheared from an 18 in. by 20

in. plate in all cases except some of the plates 1.0 in. thick, in which it was found impossible to shear strips from the plate without seriously injuring it. In these thick plates, the strips from which the tension specimens were machined were cut from the plates by drilling. The tension specimens of rolled zinc were machined on a shaper to the shape shown in Fig. 4.

The punching and shearing tests for rolled zinc were made on the portion of the 18 in. by 20 in. plates remaining after the tension test pieces had been cut out. The holes punched in the punching tests were separated by a distance at least $1\frac{1}{2}$ times the diameter of the punch used. The shearing tests were made on specimens cut from the portion of the plates left after the punching tests had been made. The shearing test specimens were cut 1 in. wide, and were sheared in double shear. Some specimens were sheared across the grain (perpendicular to the direction of rolling) and some with the grain (parallel to the direction of rolling).

Cold bending tests were made on small strips of zinc about 1 in. wide cut from any portion of the 18 in. by 20 in. plates remaining after the other tests had been made. For each plate tested in cold bending, one specimen was bent in a plane parallel to the direction of rolling (with the grain), and one specimen was bent in a plane perpendicular to the direction of rolling (across the grain).

The sheet of zinc bought in the local market differed in size from those furnished by the Matthiessen and Hegeler Company, but the general method of cutting specimens from it was the same as that just described.

6. *Testing Machines and Auxiliary Apparatus.*—Some of the tension tests of zinc were made on a Riehle 100 000–lb. testing machine, others on an Olsen 10 000-lb. testing machine, and still others on a Riehle 100 000–lb. testing machine fitted with an Olsen 1000–lb. spring balance for measuring load. The tension specimens for cast zinc were held in threaded-ended sockets; the tension specimens for rolled zinc were held by means of flat wedge grips. For measuring the elongation in the tension tests, the extensometer used in the routine testing of the Laboratory of Applied Mechanics for several years was used. The indicating pointer of this extensometer is operated by a drum round which is wrapped a fine insulated copper wire which by friction causes the rotation of the drum and the pointer over a dial as the specimen stretches. The extensometer read to $\frac{1}{10\,000}$ in. elongation.

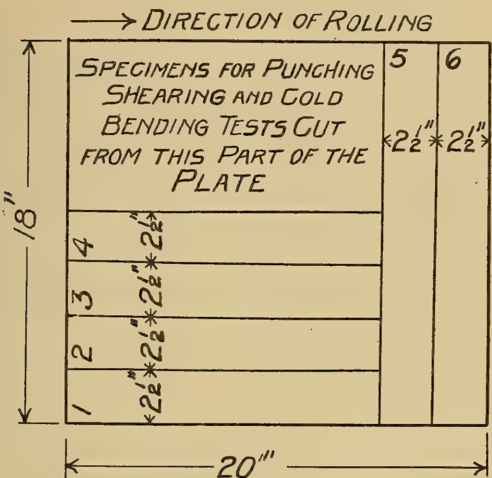

FIG. 3. ZINC PLATE SHOWING LOCATION OF SPECIMENS.

The torsion specimens of cast zinc were tested in a Riehle 10 000–lb. in. torsion testing machine of the pendulum type. The machine was hand operated during the tests, and the specimens held by self-centering and self-tightening toothed jaws. The angle of twist was measured by means of two dials and pointers similar to the dial and pointer on the extensometer used in the tension tests. The reading of each dial showed the twist of a section of the specimen with reference to the framework of the testing machine; the difference of the dial readings at the two sections gave the angle of twist of the specimen between the two sections at which the dials were attached.

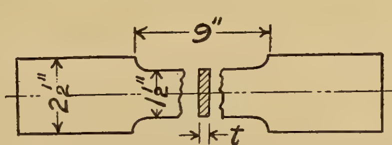

FIG. 4. TENSION SPECIMEN FOR ROLLED ZINC.

The compression specimens of cast zinc were tested in a 100 000-lb. Riehle testing machine. The ends of the specimens were carefully machined to a plane surface, and pressure was transmitted to the specimen through a spherical seated block. Compression was measured by means of a Ewing microscope compressometer reading to $\frac{1}{125\,000}$ in.

The punching tests of rolled zinc were made with hardened steel punches and dies mounted on the weighing table of a testing machine. The punches used were 0.505 in., 0.747 in., and 1.001 in. in diameter, respectively, and the corresponding dies were 0.61 in., 0.86 in., and 1.13 in. in diameter, respectively. The punches were flat faced. Most of the punching tests were made with a speed of punch so slow (0.1 in. per min.) that the weighing beam of the testing machine could be kept in balance by hand as the test progressed. Nearly all the punching tests were made on a Riehle 100 000–lb. testing machine fitted with an autographic apparatus which drew a diagram showing motion of punch as abscissas and load applied as ordinates. It was desired to run some punching tests at a higher speed than 0.1 in. per min., but as it was impossible to keep the weighing beam accurately balanced by hand in these tests, a steam engine indicator was attached to the testing machine weighing beam, so that the compression of the indicator spring measured the load on the punching tool*. Punching tests of very thin plates of zinc were made on an Olsen 10 000-lb. testing machine, and only the maximum load was recorded.

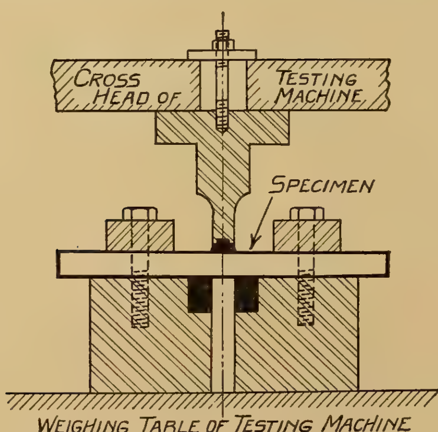

FIG. 5. APPARATUS FOR SHEARING TESTS.

Shearing tests of rolled zinc were made in a 100 000–lb. Riehle testing machine fitted with autographic recording apparatus. A hardened steel shearing tool cut the specimen in double shear. Fig. 5 shows the arrangement of the shearing tool. Most of the shearing tests were run at a low speed of tool (0.1 in. per min.) and the weighing beam of the testing machine was kept balanced

*For description of this apparatus see the Proceedings of the American Society for Testing Materials for 1908, p. 653.

by hand, while an autographic record was taken of the load and the motion of the shearing tool. Some shearing tests were made at a higher speed, and for these the steam engine indicator device for weighing loads was used as in the punching tests at high speeds.

7. *Data of Tension Tests.*—Table 5 gives the log of a sample tension test of zinc. The same general method was followed for all tension tests; both deformation under load and set after release of load were observed. During several of the tests, especially in the tests of cast zinc, a cracking noise was plainly audible under stresses as low as two-thirds of the ultimate. A set was detected in most tests after the removal of the first load applied, however low. After rupture the elongation over a gauge length originally measuring 8 in. and the reduction of area at the point of fracture were both measured when the rupture was inside the gauge length.

TABLE 5.

LOG OF SAMPLE TENSION TEST OF ROLLED ZINC.

Specimen H. 3. Dimension of Cross-section, 1.486 in. x 0.612 in.
Elongation after rupture 9.5% in 8 in.
Reduced cross-section 1.439 in. x 0.544 in.
Specimen tested with the grain.

Load lb.	Extensometer in.	Load lb.	Extensometer in.
1 000	0	8 000	0.0091
3 000	0.0015	1 000	0.0054
1 000	0.0006	8 750	0.0120
5 000	0.0031	1 000	0.0076
1 000	0.0011	9 500	0.0155
7 000	0.0066	1 000	0.0104
1 000	0.0035	19 530	rupture

In the tests of cast zinc and of the thinner specimens of rolled zinc the failure was, in general, sudden. In the very thin specimens of sheet zinc the failure occurred by tearing across, and in a few cases its course could be followed by the eye. Fig. 6 and 7 show typical stress-elongation diagrams of tension tests. Table 6 shows the summarized results of tension tests of zinc. There was considerable variation in strength shown by individual specimens of cast zinc, the ultimate strength ranging from 6 050 lb. per sq. in. to 12 220 lb. per sq. in. There was little variation in strength shown by individual specimens of rolled zinc. The extreme values were, in general, within 10 per cent of the mean. No

TABLE 6.

TENSION TESTS OF ZINC.

The values given are the average results for the number of specimens noted in the second column.

Thickness of Rolled Plate in,	Number of Specimens	Stress at First Noticeable Set lb. per sq. in.	Stress at Limit of Proportionality lb. per sq. in.	Stress at Ultimate lb. per sq. in.	Modulus of Elasticity lb. per sq. in.	Elongation in 8 in. per cent	Reduction of Area per cent	Speed of Pulling in. per min.	Specimens Tested with or across Grain
Cast	6	1600	2100	9 060	11 025 000	slight	slight	hand	——
1	5	0+*	0+	22 200	——	7.56	8.72	0.3	With grain
1	4	900	2900	21 340	10 450 000	4.85	6.58	0.5	With grain
1	3	900	4600	23 050	10 370 000	0.31	5.37	0.5	Across grain
0.6	12	0+	4200	21 490	11 033 000	16.63	22.25	0.5	With grain
0.6	5	0+	4000	23 550	10 780 000	3.33	5.80	0.5	Across grain
0.25	12	0+	5150	23 770	12 767 000	11.90	19.17	0.5	With grain
0.25	6	0+	5000	22 260	10 967 000	0.27	5.07	0.5	Across grain
0.10	6	0+	3600	22 280	10 650 000	18.75	——	0.5	With grain
0.10	6	——	——	24 870	——	21 00	——	3.25	With grain
0.10	6	0+	5500	33 620	13 700 000	14.3	——	0.5	Across grain
0.018	6	0+	5800	25 490	10 533 000	——	——	0.5	With grain
0.018	2	——	——	29 810	——	——	——	1.37	Across grain
0.018	3	——	——	23 840	——	——	——	0.2	With grain
0.018	6	——	——	34 960	——	——	——	0.5	Across grain
0.006	6	——	——	24 110	——	——	——	0.2	With grain
0.006	6	0+	5100	26 480	14 800 000	6.5	——	0.5	With grain
0.006	3	——	——	33 830	——	——	——	0.5	Across grain
0.006	3	——	——	38 440	——	——	——	2.5	Across grain
0.018†	3	——	——	24 200	——	——	——	0.5	With grain
0.018†	2	——	——	35 150	——	——	——	0.5	Across grain

*When the stress is given as 0+, a value below the lowest reading taken is denoted.

†Specimens from zinc bought in the local market; all specimens not specially designated were cut from zinc furnished by the Matthiessen and Hegeler Zinc Company.

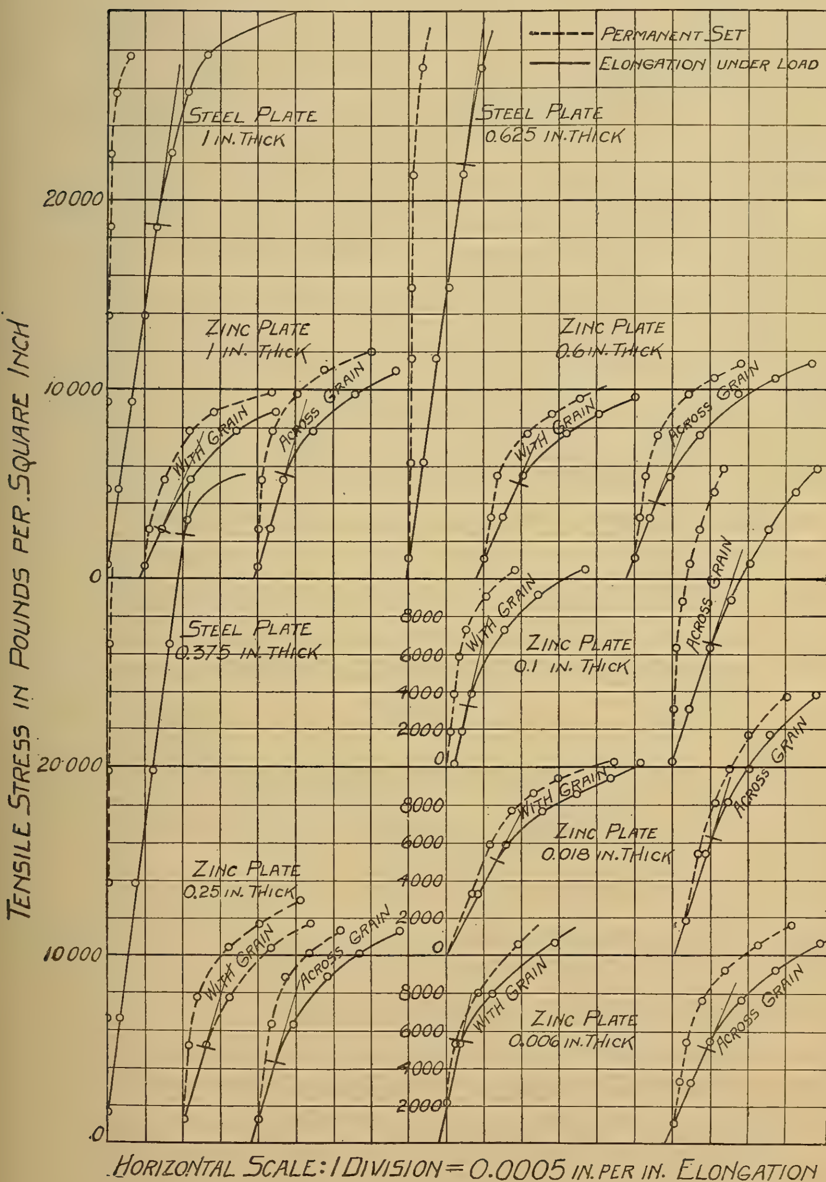

FIG. 6. TENSION TEST DIAGRAMS FOR ROLLED ZINC.

well-defined yield point could be determined in any tension test.

Fig. 8 shows two characteristic fractures of cast zinc in tension. The specimen at the left of the cut shows a much coarser grain than the one on the right; it also showed much lower tensile strength. Fig. 9 shows characteristic fractures of rolled zinc in tension. The specimen at the left is from a very thin plate, and it failed by tearing across. The specimen in the center is from a plate 0.25 in. thick, and it failed with very little elongation. The specimen at the right is from a plate 0.6 in. thick, and it showed great elongation. The necking down of the ductile specimen can be seen in Fig. 9.

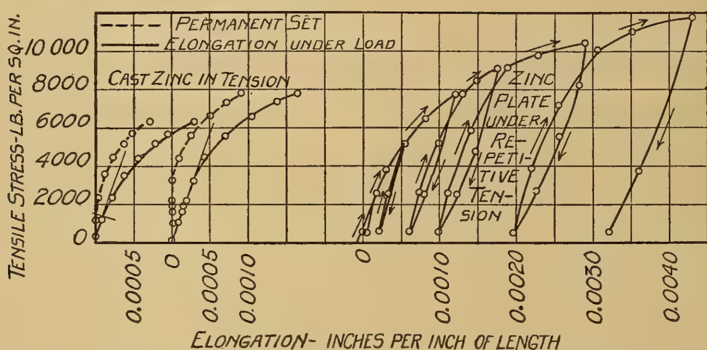

FIG. 7. TENSION TEST DIAGRAMS FOR ZINC.

8. *Data of Punching and Shearing Tests*.—The data for the punching and the shearing tests were all autographic except for punching tests in very thin plates. Fig. 10 shows typical diagrams for punching tests, and Fig. 11 shows typical diagrams for shearing tests of zinc plate. In Fig. 11 are also shown diagrams of shearing tests of steel plate.

The results of the punching tests are summarized in Table 7 and the shearing tests in Table 8. The variation of the extreme values for ultimate strength of individual specimens from the average values reported in Tables 7 and 8 was, in general, not greater than 10 per cent. The variation in the amount of energy required was somewhat greater. As one of the principal items of information desired was a comparison between zinc and steel as to maximum stress developed and amount of energy required in punching and shearing, punching and shearing tests were made on mild steel plates and the results are summarized in Tables 9, and 10.

Fig. 12 shows the appearance at several stages of the punching process of the "wad" of zinc as it is being pushed out ahead

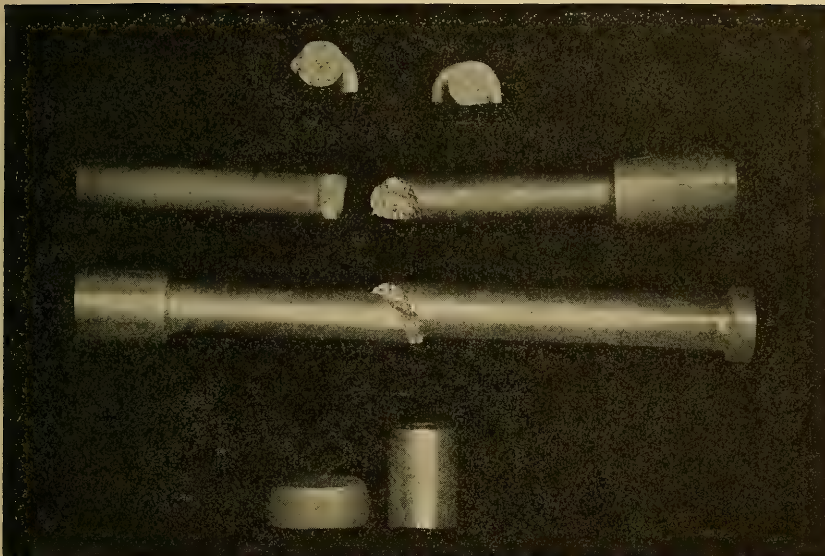

FIG. 8. SPECIMENS OF CAST ZINC AFTER TESTING.

of the punching tool. Fig. 12 also shows a shearing specimen which is about to fail. The distortion of the tool marks, originally straight, shows in a general way the distortion of the fibers of the specimen.

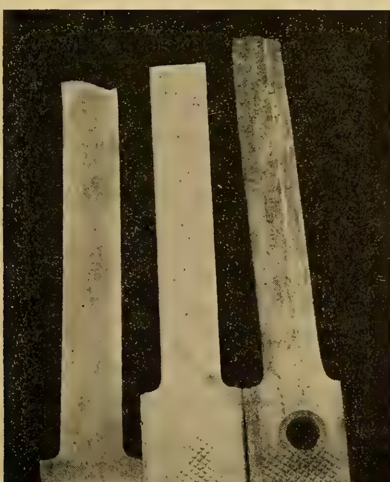

FIG. 9. SPECIMEN OF ROLLED ZINC AFTER TESTING IN TENSION.

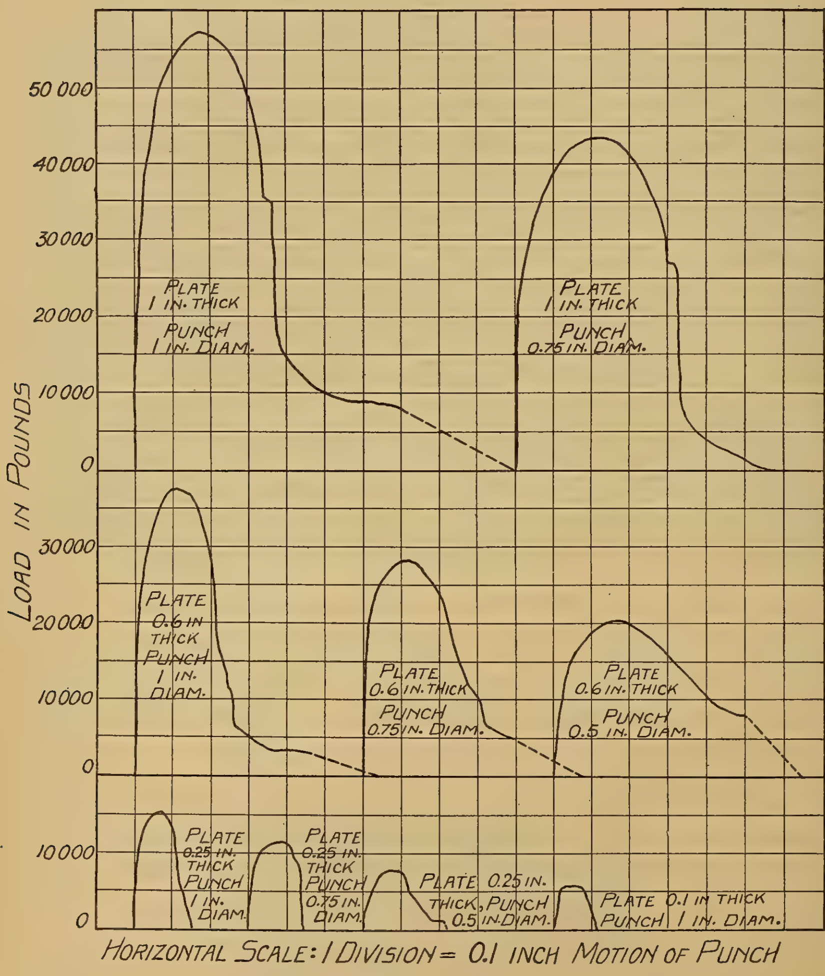

FIG. 10. PUNCHING TEST DIAGRAMS FOR ROLLED ZINC.

9. *Miscellaneous Tests.*—Torsion tests were made on six test pieces of cast zinc. The results of these tests are summarized in Table 11, and a typical stress-deformation diagram for torsion of cast zinc is shown in Fig. 13. In Fig. 8 are shown torsion test specimens after rupture. Attention is called to the character of the fracture, and its similarity in form to that of cast iron under torsion.

Compression tests were made on four short cylinders of cast zinc. The results of these tests are given in Table 12, and a typical stress-compression diagram for cast zinc is shown in Fig.

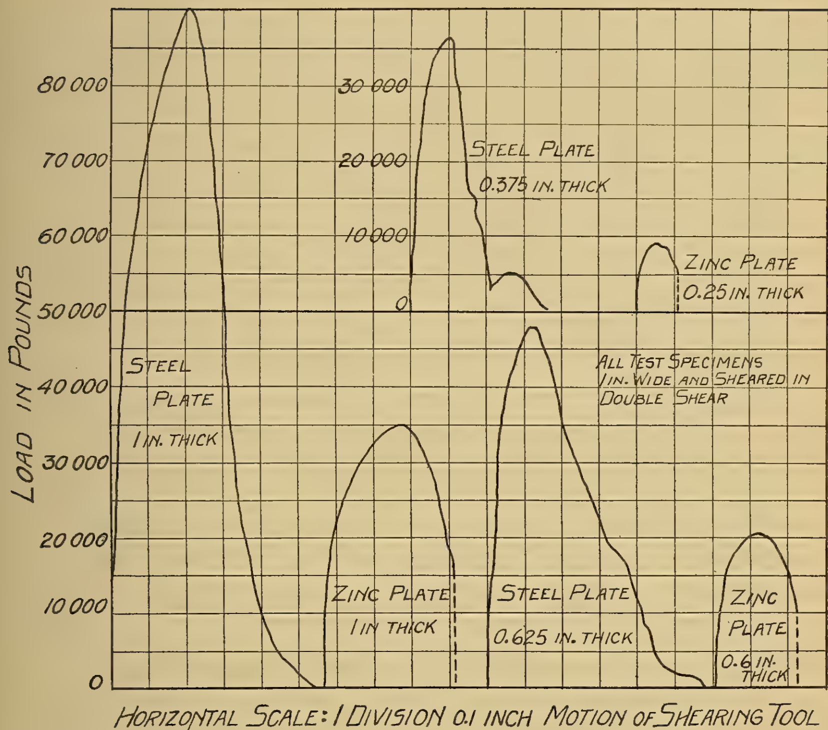

FIG. 11. SHEARING TEST DIAGRAMS FOR ROLLED ZINC AND FOR STEEL.

14. Fig. 14 also shows a stress-compression diagram for zinc under steadily increasing repetitive loading. Both in the repetitive loading test in compression and in tension (see Fig. 7) there was an appreciable loss of energy during the release and the reapplication of a load. This loss of energy, "mechanical hysteresis" as

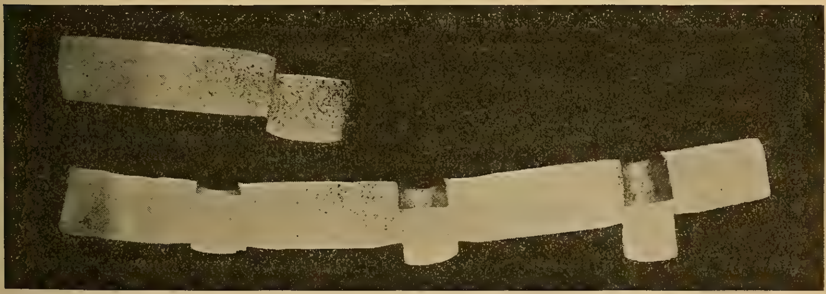

FIG. 12. PUNCHING AND SHEARING TEST SPECIMENS AFTER TESTING.

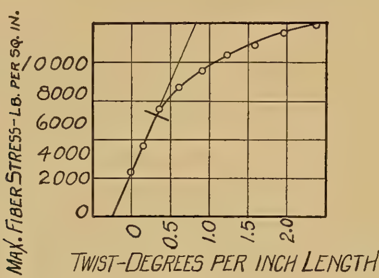

FIG. 13. TORSION TEST DIAGRAM FOR CAST ZINC.

it is called, is shown by loops in the diagrams of tests under repetitive load.

The behavior of cast zinc under compression is worthy of note. Judged by fractures in tension and in torsion tests, cast zinc is a brittle material, and under compression a shattering failure might be expected—such a failure as cast iron exhibits in compression. What actually happened to the compression test pieces of cast zinc was a gradual flattening out, such as occurs with soft steel. No maximum load could be determined. Cast zinc is evidently a plastic but not a ductile metal. Fig. 8 shows a cylinder of cast zinc before testing, and beside it another originally of the same size, after compression under 100 000 lb.

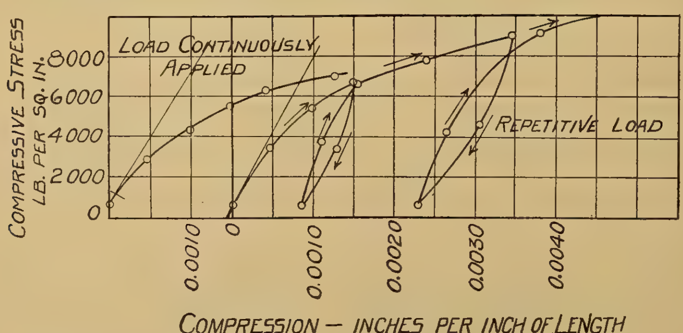

FIG. 14. COMPRESSION TEST DIAGRAMS FOR CAST ZINC.

Cold bending tests were made on specimens from all rolled plates except those 1 in. thick and those 0.6 in. thick. One test was made on specimens from a plate 0.6 in. thick. From each plate tested one specimen was bent in a plane parallel to the direction of rolling and one specimen bent in a plane perpendicular to the direction of rolling. Table 8 shows the results of the cold bending tests.

TABLE 7.

Punching Tests of Zinc.

The values given are the average results for the number of specimens noted in the second column.

Nominal Thickness of Plate in.	Number of Specimens Tested	Diameter of Punch in.	Speed of Punching in. per min.	Ultimate Shearing Stress Developed lb. per sq. in.	Energy Required to Punch in. lb. per sq. in. per in. thickness
1.00	9	0.75	0.10	18 750	7 260
1.00	9	1.00	0.10	19 060	6 650
0.60	6	0.50	0.10	19 740	10 590
0.60	6	0.75	0.10	19 350	8 630
0.60	6	1.00	0.10	18 990	6 400
0.25	6	0.50	0.10	20 000	10 550
0.25	6	0.75	0.10	19 700	8 740
0.25	6	1.00	0.10	19 390	8 030
0.25	6	1.00	0.50	19 400	8 350
0.25	6	1.00	1.60	20 850	8 380
0.10	6	1.00	0.10	18 230	5 650
				Av. 19 400	8 110
0.018*	9	1.00	0.10	12 930	——
0.018*	6	1.00	0.10	13 380†	——
0.018*	9	1.00	0.10	10 880	——

*In the very thin plates, shearing took place unevenly around the circumference of the punching tool.

†Zinc bought in the local market.

TABLE 8.

Shearing Tests of Zinc.

The values given are the average results for the number of specimens noted in the second column.

Nominal Thickness of Plate in.	Number of Specimens Tested	Speed of Shearing Tool in. per min.	Ultimate Shearing Stress Developed lb per sq. in.	Energy Required to Shear in. lb per sq. in. per in. thickness	Shearing with Grain or across Grain
1.00	6	0.10	16 700	4 850	Across grain
1.00	3	0.10	16 580	4 510	With grain
1.00	6	0.50	17 140	4 130	Across grain
1.00	6	1.60	17 770	4 330	Across grain
0.60	6	0.10	16 580	5 100	Across grain
0.60	3	0.10	17 380	4 620	With grain
0.60	6	0.50	15 480	4 640	Across grain
0.60	6	1.60	18 100	4 410	Across grain
0.25	6	0.10	18 860	6 690	Across grain
0.25	3	0.10	18 170	4 620	With grain
0.25	6	0.50	15 480	4 640	Across grain
0.25	6	1.60	17 040	5 730	Across grain
			Av. 17 100	4 850	

III. Results and Conclusions.

10. *Tensile Strength of Zinc.*—An examination of the results of tension tests shows that zinc, either cast or rolled, is imperfectly elastic under very low stresses. The results of Meyer's tests are in agreement with the results of the Illinois tests as regards this general conclusion, though in the Illinois tests permanent set was detected at lower stresses than in Meyer's tests. Whether the elastic limit be defined as the lowest stress under which a material is given a permanent set, or as the lowest stress at which Hooke's law (proportionality of stress to deformation) is found to be inexact, the elastic limit of zinc is very low and very poorly defined. It is doubtful if the elastic limit as determined either by Meyer or in the Illinois tests has any special physical significance. Though Meyer reported a yield point of zinc in the tables of results of his tests, in the accompanying discussion he stated that the yield point was poorly defined. In the Illinois tests no well-defined yield point could be detected, and none was reported.

The ultimate tensile strength of cast zinc depends on the temperature of pouring, and other factors, and varies between wide limits. Thin plates of rolled zinc are relatively stronger under tension than thick plates. From the results of the Illinois tests for plates under 0.05 in. thick, 24 000 lb. per sq. in. would seem to be about the value to be used for the ultimate tensile strength of rolled zinc. For plates over 0.05 in. thick, 21 000 lb. per sq. in. would seem a reasonable value to use. Meyer's tests and those of the Matthiessen and Hegeler Zinc Company show slightly higher values. Rolled zinc is somewhat stronger in tension across the grain than in tension with the grain. A higher tensile strength of rolled zinc was obtained by increasing the rapidity of application of load. Rapidity of testing may, in part, account for the fact that higher values of tensile strength were found by Meyer and by Matthiessen and Hegeler than were found at Illinois. Neither of the first two reports the speed of testing. The speed used in the Illinois tests was lower than is sometimes used in commercial testing.

From the results of the various tension tests of zinc herein quoted, it would seem that the modulus of elasticity of zinc is about 11 500 000 lb. per sq. in.

11. *Shearing Strength of Rolled Zinc.*—The values of shearing strength of rolled zinc reported in this bulletin were determined with the purpose of throwing some light on the problem of what

TABLE 9.

Punching Tests of Steel Plate.

For purposes of comparison with punching tests of zinc plates.

The values given are the average results for the number of specimens noted in the second column.

Nominal Thickness of Plate in.	Number of Specimens Tested	Diameter of Punch in.	Speed of Punching in. per min.	Ultimate Shearing Stress Developed lb. per sq. in.	Energy Required to Punch in. lb. per sq. in. per in. thickness
0.375	2	0.75	0.10	50 000	30 350
0.375	2	1.00	0.10	50 080	19 880
				Av. 50 040	25 200

TABLE 10.

Shearing Tests of Steel Plate.

For purposes of comparison with shearing tests of zinc plates.

The values given are the average results for the number of specimens noted in the second column.

Nominal Thickness of Plate in.	Number of Specimens Tested	Speed of Shearing Tool in. per min.	Ultimate Shearing Stress Developed lb. per sq. in.	Energy Required to Shear in. lb. per sq. in per in. thickness	Shearing with Grain or across Grain
1.000	2	0.10	44 700	11 580	Across grain
0.625	2	0.10	37 200	17 750	Across grain
0.375	3	0.10	46 970	18 850	Across grain
0.375	3	0.50	43 000	16 740	Across grain
0.375	2	1.60	44 500	19 630	Across grain
			Av. 43 270	16 910	

TABLE 11.

Torsion Tests of Cast Zinc.

Number of specimens tested..	6
Maximum fiber stress at limit of proportionality of stress to angle of twist....	5 450 lb. per sq. in.
Computed maximum fiber stress at rupture.	15 260 lb. per sq in.
Modulus of elasticity in shear (Torsion)	4 570 000 lb. per sq. in.

TABLE 12.

Compression Tests of Cast Zinc.

Number of specimens tested..	4
Fiber stress at limit of proportionality of stress to compression	1 620 lb. per sq in.
Modulus of elasticity..........	6 900 000 lb. per sq. in.

sizes of punches and shears should be used in working with zinc plates. Two factors which are important in their influence on the design of punches and shears are the maximum force to be exerted during the process of punching or shearing and the energy required to complete the punching or shearing action. The maximum force to be exerted determines the strength of frame, ram, gearing, and other parts of the punch or shear; the energy required determines, in large measure, the weight of the flywheel, size of belt, and power required for power-driven punches and shears, or the power and amount of water or air required for hydraulic or pneumatic punches or shears. The maximum force to be exerted during the punching or shearing action is in general proportional to the area actually sheared under the action of punching or shearing tool; the mean force during the action is approximately proportional to the maximum force and consequently to the area under the action of the punching or shearing tool. (This may be seen from the general similarity of shape of the punching and shearing diagrams for different thicknesses of plate; see Fig. 10 and 11). The energy required to punch or shear any plate will then be approximately proportional to the area to be sheared (by punching or shearing tool) multiplied by the distance traveled by the tool during the punching or shearing action, i. e., by the thickness of the plate. The significant features of the punching or shearing tests of rolled zinc were, then, the maximum shearing stress developed measured in pounds per square inch, and the energy required for punching or shearing measured in inch-pounds per square inch of surface sheared per inch thickness. An examination of the results of the punching and shearing tests of zinc plate shows an average value of shearing stress developed of 19 400 lb. per sq. in. for the punching tests, and of 17 100 lb. per sq. in for the shearing tests. The average value of the energy required was 8 110 in. lb. per sq. in. per in. thickness for the punching tests, and 4 850 in. lb. per sq. in. per in. thickness for the shearing tests. Evidently the frictional resistance of the metal pushed out (the "wad") is greater in punching than in shearing, as is shown by the slightly greater stress developed, and by the markedly greater energy required.

An examination of the results of the punching tests shows that the larger the punch for any given thickness of plate the less the unit-energy required to punch the plate. In both punching and shearing tests, the maximum stress developed was slightly increased under increased speed of punching or shearing tool.

TABLE 13.

COLD BENDING TESTS OF ROLLED ZINC.

Nominal Thickness of Plate in.	Specimen Bent with Grain or across Grain	Action under Cold Bending
0.006	Across Grain	Bent double and hammered flat without cracking.
0.006	With Grain	Bent double and hammered flat without cracking.
0.018	Across Grain	Bent double and hammered flat without cracking.
0.018	With Grain	Bent double and hammered flat without cracking.
0.018*	Across Grain	Bent double and hammered flat without cracking.
0.018*	With Grain	Bent double and hammered flat without cracking.
0.100	Across Grain	Two specimens bent double and hammered flat without cracking; one specimen cracked when hammered flat.
0.100	With Grain	All specimens bent double and hammered flat without cracking.
0.250	Across Grain	Specimens cracked when bent through 90–120 degrees.
0.250	With Grain	One specimen bent double and hammered flat without cracking; two specimens cracked after bending through 180 degrees.
0.600	Across Grain	One specimen tested, broke short off when bent through about 30 degrees.
0.600	With Grain	One specimen tested, cracked after bending through 180 degrees.

* Zinc bought in local market.

Both punching and shearing tests were made with flat-faced tools, the object being to bring stress as uniformly as possible on all parts of the area under shear. By the use of beveled punching and shearing tools, the maximum force resisting shear would have been reduced.

An examination of the results of the punching and shearing tests of mild steel plate tested for purposes of comparison with rolled zinc shows the following average values: Shearing stress developed, 50 040 lb. per sq. in. for the punching tests, 43 270 lb. per sq. in. for the shearing tests. Energy required 25 200 in. lb. per sq. in. per in. thickness for the punching tests, and 16 910 in. lb. per sq. in. per in. thickness for the shearing tests*. In punching or shearing zinc plates, about 40 per cent as high a stress is developed as is developed in punching or shearing mild steel plates of the same size, and about 30 per cent as much energy is required.

*Other punching and shearing tests of mild steel give results not widely differing from these. See article by H. V. Loss in the American Engineer and Railroad Journal for March 1893 and results in Kent's Mechanical Engineers' Pocket Book.

12. *Ductility and Plasticity of Zinc.*—For the comparison of the ductility of different metals there is, unfortunately, no well-defined quantitative standard. In the series of tests described in this bulletin the elongation and the reduction of area after rupture in tension and the results of cold bending tests were all used to throw light on the ductility of the zinc tested. From the results of the tests, it is evident that zinc is much less ductile than wrought-iron or mild steel, and that it is less ductile across the grain than with the grain. For zinc plate which is to be stamped or bent into shape (for example in the making of zinc elements for dry batteries), a severe cold-bending test would seem to be of considerable value in determining the acceptability of a shipment of zinc plate.

The researches of Martens and of Meyer on the effect of heat treatment of zinc on its strength and ductility would indicate the desirability of measuring and of controlling the temperature in the rolling process and show the danger of rolling at too high a temperature. While the ductility of zinc is low as compared with that of steel, from the low and poorly defined elastic limit, from the loss of energy in "mechanical hysteresis" and from the behavior of compression test pieces it is evident that the zinc possesses a relatively high degree of plasticity.

13. *Summary.*—The following summary is given:

1. Zinc either rolled or cast has no well-defined yield point and its elastic limit is very low. Zinc possesses a relatively high degree of plasticity.

2. The ultimate tensile strength of thin rolled zinc plate (not more than 0.05 in. thick) is about 24 000 lb. per sq. in.

3. The modulus of elasticity of zinc in tension is about 11 500 000 lb. per sq. in.

4. The stress per square inch of area sheared developed in punching or shearing rolled zinc plates is about 40 per cent of the stress developed in punching or shearing mild steel plates.

5. The energy per square inch of area sheared per inch thickness of plate required to punch or shear rolled zinc plates is about 30 per cent of the energy required to punch or shear mild steel plates.

6. The ductility of rolled zinc is much less than that of mild steel, and the ductility of zinc plate with the grain is greater than the ductility across the grain.

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This action, so far as it concerns the bulletins of the Engineering Experiment Station, has for its purpose a threefold object:

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W. F. M. GOSS
Director.

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INDUCTANCE OF COILS

BY

MORGAN BROOKS

AND

H. M. TURNER

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 53 JANUARY, 1912

INDUCTANCE OF COILS

BY MORGAN BROOKS, PROFESSOR OF ELECTRICAL ENGINEERING AND
H. M. TURNER, ASSISTANT IN ELECTRICAL ENGINEERING

CONTENTS

INDUCTANCE OF COILS

I. Introduction

This bulletin deals with the self-inductance of closely-wound cylindrical coils of wire without iron cores, and presents tables and charts for obtaining without effort, the approximate inductance or reactance of coils of all dimensions. A given length of conductor has a definite resistance, but may have as many different values of inductance as there are different shapes of coils into which it can be wound, although the inductance becomes definite when the dimensions of the winding are fixed. From a large number of calculations and of tests of actual coils of many shapes, material has been gathered from which a universal formula for the self-inductance of coils has been derived, making it a simple matter to compare the relative value of different winding proportions. Regardless of the size of conductor, it is found that the shape for producing the maximum inductance from any length of wire is neither a long solenoid nor a flat disk, but a compact coil not unlike the ordinary wire bundle as received from the factory. While tables showing the resistance of copper wire are found in all electrical reference books, similar data for the inductive reactance of coils, if found at all, are often incorrect, or so presented as to be unavailable for occasional use. It is hoped that the information here given will prove convenient and useful.

II. Precision Formulas

Valuable information upon the mutual and self-inductance of lines and coils will be found in the bulletins of the Bureau of Standards, Washington, D. C., and special reference is made to Vol. V, No. 1 (1908), where many formulas are compared. For the various shapes of coils, a bewildering choice of formulas is offered; several for short single-layer windings, several for disks and rings, and others for intermediate shapes. The use of one of these precision formulas for a type of coil for which it is not intended may result in serious error, and their delimitation is not always clearly stated, so that it may be necessary to try two or more of them, and compare results.

Most theoretical formulas assume that the winding is composed of a thin conducting tape, whose edges lie close together, though electrically insulated, producing what is mathematically known as a current sheet, although a few are based upon the usual square or round wires employed to produce the ampere

turns of practical work. There are small differences, sometimes amounting to one per cent or more between these conceptions of inductance, and the Bureau of Standards has done an admirable work in reconciling the differences, and in providing correction formulas for bringing the results into strict harmony, establishing the accuracy of certain formulas, as shown by illustrative formal examples to a point of agreement within one part in a million.

The very precision of these elaborate formulas is a barrier to their usefulness to the busy engineer, by whom coil dimensions may not be known accurately. No simple formula of wide range is given, or suggested, and the only formula recommended for long solenoids requires the use of elliptic integrals for its solution, and even then is limited to a single-layer winding. Diversity in units and notation, and the absence, from the examples, of wires of ordinary gauge diameters place further difficulties in the utilization of the formulas, which are often insurmountable.

Uniformity in the winding of commercial coils is impossible, when, in the matter of gauge sizes alone, a variation of one per cent from tabulated diameter is allowable, and when the thickness of insulations is never known definitely. Compactness of winding is ordinarily subject to uncertainty, and the exact predetermination of the mean radius of a coil extremely difficult. The cumulative effect of these uncertainties is such that for many purposes an approximate formula is quite accurate enough.

III. Advantage of Coils without Iron Cores

There is a demand for coreless reactance coils, since they eliminate the uncertainty of action incident to coils with iron cores. If reactance is needed for protecting alternating current apparatus against the dangers of short circuits, the immediate response of a coil, not dependent upon the dilatory magnetizing of iron, is required. While a few turns of wire around iron seem to give the same reactance as many turns without iron, the coil with iron is relatively sluggish. Two coils of same measured reactance, but differing in design, may behave very differently because one has a closed, the other an open magnetic circuit; and even the same ferric coil may behave differently under similar conditions because of a difference in the residual magnetism. There is a further advantage in non-ferric coils, in the absence of core loss. Such coils also find an increasing use in telephony and in wireless telegraphy, where certainty of action, and alertness at

FIG. 1*a*

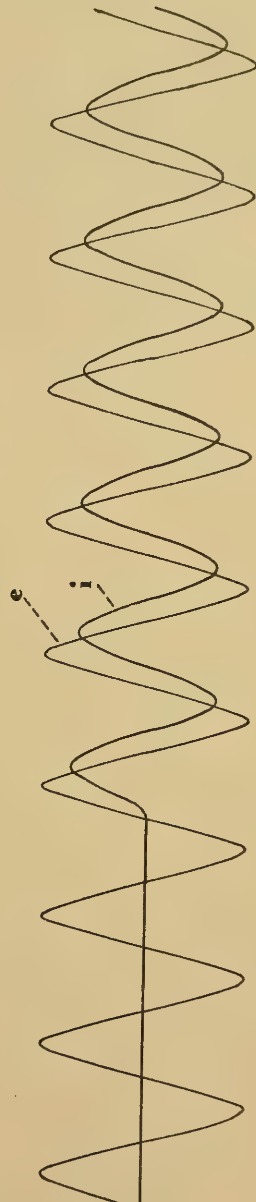

FIG. 1*b*

high frequencies are important. The data here given are just as applicable to small coils for telephone use as to those of large size for power use.

IV. Comparison with Iron-cored Coils

The reactive difference between iron-cored and coreless coils, when thrown suddenly into action, is well shown by the instantaneous photographic record of the oscillograph, as illustrated in Fig. 1*a*, 1*b*, 2*a*, and 2*b*. The same alternating electromotive-force is shown in all these reproductions; therefore, the scale of the figures is to be understood as a relative one merely. Fig. 1*a* shows a considerable current-surge, the first wave having many times the amplitude of those of normal current after a few cycles.

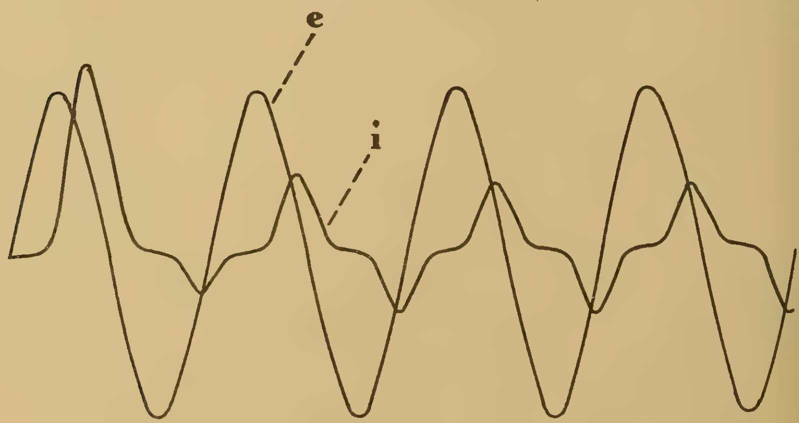

Fig. 2*a*

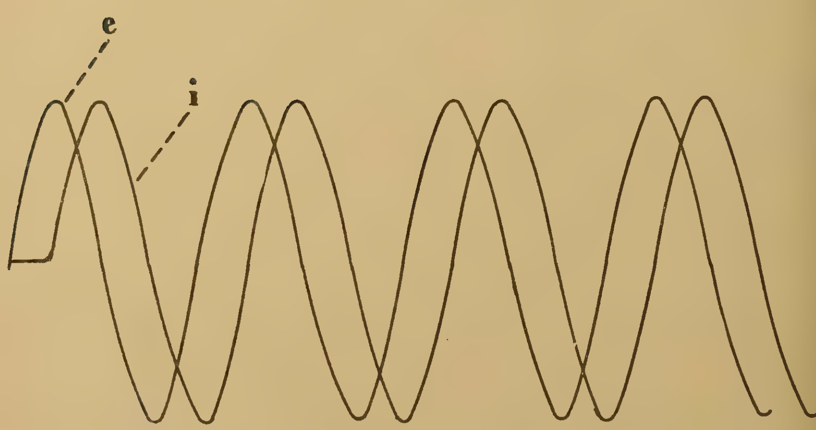

Fig. 2*b*

This records the action in a coil with iron core, to which the electromotive force was applied intentionally at the instant of its most rapid change, the moment when the normal current would naturally be a maximum. Fig. 1*b* represents the same action in a coreless coil, and it is seen that the surge, though present, is limited, the maximum effect never exceeding twice normal current wave values.

Fig. 2*a* and 2*b* represent the lesser action occurring in cored and coreless coils, respectively, when the electromotive force is applied at the instant of maximum, when its value is not changing, the very moment when the current is normally passing through zero, so that the current starts on its normal wave at once. In this case, there is little or no difference in the action of the coils, except that due to residual magnetism in the cored coil; but under the extreme condition of Fig. 1*a* and 1*b*, the advantage of the coreless coil is manifest, since there is relatively little difference between the best and the worst conditions. Nonferric coils have an increasing use for electrical work of all kinds, as their advantages are better appreciated.

V. Inductance and Reactance

A current in a coil of wire gives it the properties of a magnet, even when an iron core is not present. Magnetic properties of non-ferric coils are greater than is generally supposed. Within such a coil is produced a magnetic field or stress, called flux, (ϕ), proportional to the magneto-motive-force, (*mmf*), of the current, (i), flowing through the turns, (N), of the coil, and limited by the reluctance, ($\mathscr{R}$), of the flux path. Flux is expressed in maxwells, or lines of force, commonly called "lines". Magneto-motive-force is expressed in gilberts, where one gilbert is 4π current turns, ($4\pi Ni$), in absolute units, or as 0.4π ampere-turns ($0.4\pi Ni$) when i is in amperes, the practical unit of current, since the ampere is one-tenth the absolute unit; reluctance is expressed in oersteds, these units being in absolute values of the centimeter-gram-second system. Reluctance varies directly with length of the flux path, and inversely with its area, and also with its permeability when iron is included in the path. Length and area of flux path in a normal closed magnetic circuit of iron are easily determined with considerable precision, but in air or in any non-magnetic material, the dimensions of the flux path, while definite, cannot be expressed by any simple rule that applies to a variety of cases. In iron, the path has

a high permeability, varying with magnetization; in air and most substances, the permeability is low, but constant, and does not appear in equations for non-ferric coils, its value being taken as unity.

A steady current in a coreless coil produces a definite magnetic flux, since the magneto-motive-force and the reluctance are constants. If the current varies, the flux varies proportionally. In this case, another action arises, since experiment shows that a varying flux induces an electromotive-force, (e), in every turn of wire interlinking or surrounding the flux path. This electromotive-force is constant only while the flux is varying at a constant rate, being proportional to the rate-of-change of the flux, ($d\phi/dt$). Flux cannot long continue to change at a constant rate, but can pulsate or alternate and thus produce alternating electromotive-forces. An alternating current in a coil must be accompanied by an alternating magnetic flux, which will induce, in the turns of the coil, the counter-electromotive-force of self-induction, and in any other coil, an electromotive-force of induction, of equal value per turn, if similarly situated in respect to the flux path.

The coefficient of self-induction, or, briefly, the inductance, (L), of a coil is the constant ratio existing in non-ferric coils between the counter-electromotive-force, (e), and the rate of change of the current, (di/dt), on which it depends. It may also be expressed by the ratio of flux-turns, ($N\phi$), to inducing current, (i), as shown by the following equations expressing these laws;

$$\phi \text{ (in maxwells or lines)} = \frac{(mmf \text{ in gilberts})}{\mathscr{R} \text{ (in oersteds)}} = \frac{4\pi Ni}{\mathscr{R}} \quad \ldots\ldots\ldots (1)$$

$$e \text{ (in absolute units)} = -\frac{N\,d\phi}{dt} = -\frac{4\pi N^2 di}{\mathscr{R}\,dt} = -\frac{Ldi}{dt} \quad \ldots\ldots\ldots (2)$$

$$\mathscr{R} \text{ (reluctance)} = \frac{\text{length of flux path}}{\text{area of flux path}} = \frac{b+y}{\pi a^2} \quad \ldots\ldots\ldots (3)$$

$$\text{(from (2) and (3))}\ L = \frac{N\phi}{i} = \frac{4\pi N^2}{\mathscr{R}} = \frac{4\pi N^2 \pi a^2}{b+y} = \frac{(2\pi aN)^2}{b+y} \quad \ldots (4)$$

in which

a = mean radius of the coil in centimeters;
b = length of coil, also in centimeters;
πa^2 = the definite area of flux-path within the coil;
y = a quantity to be added to b to get equivalent return length of flux-path, outside the coil, as explained under "Flux-path".

The inductance coefficient, L, is seen to be a constant, depending upon constants of the coil winding, its linear dimensions and turns. The numerator, $(2\pi a N)^2$ is the square of the length of conductor of which the coil is made, and the denominator, $(b+y)$, is a coil length. If y could readily be defined for coils of different shapes, the determination of L would be a simple matter. It is evident that the square of a length divided by a length is of the dimension of length, and the inductance L, is, in absolute units, the centimeter. The practical unit is the henry, which is 10^9 centimeters, the length of the earth quadrant. The milhenry, the thousandth, and the microhenry, the millionth of this unit, are much used for the inductance of small coils.

VI. Flux-Path

The dimensions of the flux-path, if known, determine the reluctance. In a particular coil of four turns, the flux-path is graphically indicated by iron filings, as shown in a photograph reproduced from Simmons' "Outlines of Electrical Engineering," as Fig. 3, in which it may be seen how iron filings obey the ampere-

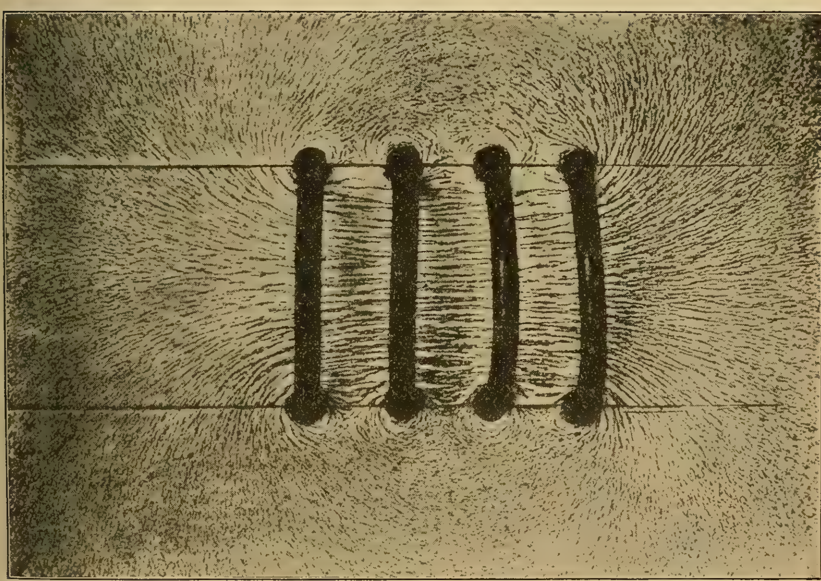

Fig. 3. Magnetic Flux Due to Ampere-Turns of Solenoid

turns of a steady current. The flux appears to be uniformly strong within the coil and to diminish rapidly without. The flux-path can be traced some distance beyond the coil before it be-

comes indefinite, just as smoke can be traced for some distance from a chimney mouth. As the path increases in area, its reluctance decreases; therefore, the reluctance of the return path outside the coil is relatively small. It is not zero, however, as sometimes assumed, since the path does not become indefinitely extended immediately upon leaving the coil. The flux-path within the coil may be assumed to have the area of an average turn of the coil, a definite value, and a length the same as the coil length. If the same area is assumed for the return path, evidently much too small, it is fair to assume a return length, y, also much too small, to correspond. It is found by trial that if $c + R$, the sum of the thickness of the winding and the outside radius of the coil, be substituted for y in the equation (4), a close approximation of L is obtained for long coils.

In a coil of but a single turn, the flux distribution must be somewhat as shown in Fig. 4, reproduced from Nipher's "Electricity and Magnetism". The flux is confined to a definite area only

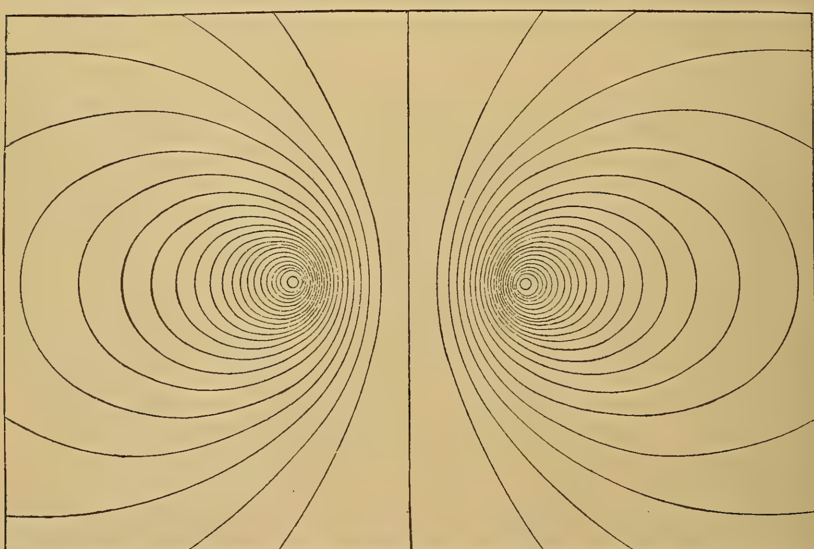

FIG. 4

where it passes through the ring, shown in section, but is not uniformly distributed even there, the lines being crowded more closely together within or near the conductor than toward the center of the ring. Flux paths may be represented by the smallest circle just around the conductor, or by any of the longer curves farther away. Indeed with reference to that portion of the

current energy at the conductor's center, the flux-path may lie wholly or partially within the metal of the conductor, and have a length that is less than the smallest circumference of Fig. 4. At very high frequencies, where "skin-effect" forces the current away from the center of a large conductor, increasing its absolute resistance, this internal flux path may be eliminated with consequent diminution of absolute inductance, an effect quite negligible for ordinary frequencies. In such a formula as (4), the definition of y, the equivalent length of the average flux line outside the coil is elusive for single turns, and other forms of equations have generally been derived for the inductance of rings.

In long coils, y is evidently subordinate to b, the coil length, and a slight error in its evaluation makes a smaller error in the value of inductance, but in ring coils such is not the case. In the general formula presented, (5) or (6), page 13, equation (4) is modified by factors, F' F'', which virtually modify $b + y$, giving an approximate value for all shapes of coils including rings.

VII. Development of Brooks' Universal Formula

There is an evident advantage in having a single formula apply to all forms of coils, provided it is reliable and approximately accurate. Where different formulas have to be used for long and for short coils, as has heretofore been the case, there is an uncertainty as to intermediate shapes, due to the discontinuity of the separate formulas. Professor Morgan Brooks, one of the writers of this bulletin, has developed such a universal formula, applicable to all shapes of closely-wound cylindrical coils. It is in agreement with the theoretical formula for very long solenoids, and represents approximately the best formulas for coils of other shapes of one or many layers, even including the most extreme cases of single turns of fine wire. Values calculated from precision formulas, as well as measured values for a great variety of coil shapes, indicate a possible variation not exceeding three per cent for this universal formula.

To the engineer who uses resistance tables without allowance for temperature, this degree of accuracy will be found sufficient, for it must be remembered that a difference of but 8°C. will change the resistance of copper wire three per cent. In fact, it is difficult to duplicate a reactance coil, and get results within three per cent without adjustment. In case greater precision is required, the universal formula will be found useful in

detecting any errors of calculation in using more elaborate precision formulas.

The universal formula is partly empirical, but is based upon the theoretically derived equation (4) which is directly applicable to long solenoids. The empirical factors are so devised that they cannot cause gross errors, no matter what the extreme of dimensions may be. It indicates, as no discontinuous formula can, the approximate percentage of increase or decrease of inductance that any modification of coil dimensions will produce. It points directly to the proportions for obtaining the maximum inductance in coreless coils. By its use the tables and charts accompanying this bulletin have been prepared; and by means of these any coil problem involving inductance values can be rapidly solved.

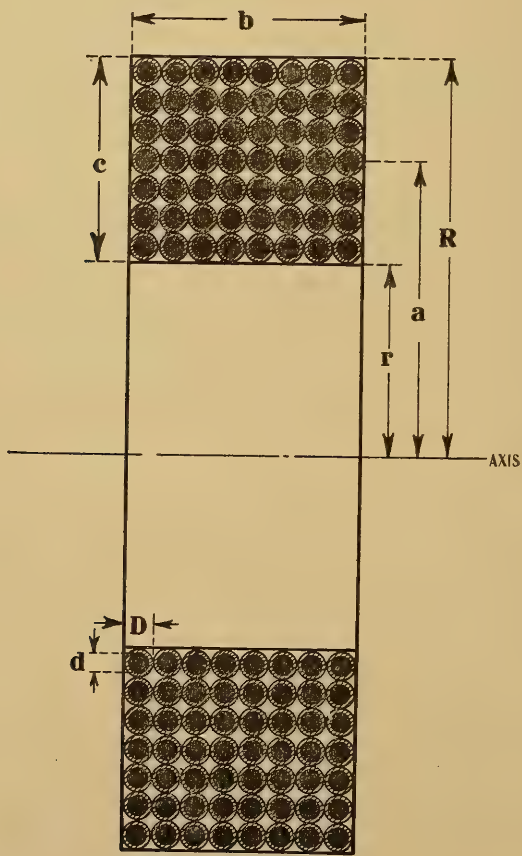

FIG. 5. COIL SECTION ILLUSTRATING NOTATION USED IN BULLETIN

VIII. Notation

Fig. 5 illustrates the notation used throughout this bulletin.

a = the mean radius of the winding;

b = the axial length of the coil; for single turns use d;

c = the thickness of the winding; for single turns use d;

r = the inner radius of the winding;

R = the outer radius of the winding;

d = the diameter of the bare wire;

D = the diameter of the wire over insulation.

The above dimensions are in centimeters, or in inches, as may be indicated. Mils are not used.

Cm indicates the length of the conductor in centimeters;

Ft indicates the length of the conductor in feet, and $Ft/1000$ = thousands of feet;

N is the total number of turns in the winding, whence

$Cm = 2\pi a N$, when a is in centimeters, and

$\frac{Ft}{1000} = \frac{2\pi a N}{12000}$, when a is in inches;

N' is number of turns for coils of prescribed maximum shape;

f = frequency of alternating current in cycles per second;

L = inductance, generally in henries, sometimes in milhenries or microhenries;

$X = 2\pi f L$ = ohms of reactance.

R = ohms resistance; for use on pp. 36 and 37 only.

Universal Formula for Coil Inductance

The formula derived by Professor Morgan Brooks is given in two forms, one (5) for dimensions in centimeters, the other (6) for dimensions in English units. Both give results in henries.

$$L \text{ (in henries)} = \frac{Cm^2}{b+c+R} \times \frac{F'F''}{10^9} \quad \ldots\ldots (5)$$

$$L \text{ (in henries)} = \frac{0.366\left(\frac{Ft}{1000}\right)^2}{b+c+R} \times F'F'' \quad \ldots\ldots (6)$$

In (6) the conductor length is in thousands of feet, and the coil dimensions in inches. 0.366 is the conversion factor. F' and F'' are empirical coil-shape factors dependent upon the relative, and independent of the absolute dimensions of the winding. Values of F' and F'' are as follows:

$$F' = \frac{10b+12c+2R}{10b+10c+1.4R}; \quad F'' = 0.5 \log_{10}\left(100 + \frac{14R}{2b+3c}\right)$$

These factors F' and F'' may appear formidable, but they are

easy to use. F' varies from unity for very long coils to 1.43 for the shortest of coils, the single turn. It is a factor that takes account of the varying thickness of coil windings, as well as of the relative importance of radius and length of coil in their influence upon inductance. F'' is negligible for long coils, falling below 1.01 for all coils whose axial length is the greatest dimension. F'' becomes active for short coils, especially for those of but a single turn.

TABLE 1

Shape-Factors, F' and F'' for Certain Coil Proportions

a	b	c	R	F'		F''		$F'\ F''$	
19	200	2	20	1.008		1.001		1.009	
15	200	10	20		1.015		1.001		1.016
19	100	2	20	1.015		1.003		1.018	
15	100	10	20		1.028		1.002		1.030
19	50	2	20	1.029		1.006		1.035	
15	50	10	20		1.051		1.004		1.055
19	20	2	20	1.064		1.013		1.078	
15	20	10	20		1.097		1.008		1.106
19	12	2	20	1.095		1.019		1.116	
15	12	10	20		1.129		1.011		1.141
19	8	2	20	1.125		1.026		1.154	
15	8	10	20		1.154		1.013		1.169
19	5	2	20	1.163		1.035		1.204	
15	5	10	20		1.182		1,015		1.200
19	4	2	20	1.182		1,040		1.229	
15	4	10	20		1.190		1.016		1.209
19	3	2	20	1.205		1.045		1.259	
15	3	10	20		1.203		1.017		1.223
19	2	2	20	1.235		1.054		1.301	
15	2	10	20		1.216		1.017		1.237
19	1	2	20	1.277		1.065		1.360	
15	1	10	20		1.232		1.018		1.254
19	0.5	2	20	1.302		1.073		1.397	
15	0.5	10	20		1.241		1.018		1.263
19	0.2	2	20	1.320		1.079		1.424	
15	0.2	10	20		1.246		1,019		1,270
19	0.1	2	20	1.326		1.081		1.433	
15	0.1	10	20		1.249		1.019		1,273

Square Winding-Section Coils

17.5	5	5	20	1.172	1.023	1.199
19.	2	2	20	1.235	1.054	1.301
19.5	1	1	20	1.292	1.097	1.418
19.75	0.5	0.5	20	1.342	1.163	1.561
19.9	0,2	0.2	20	1.387	1.29	1.79
19.95	0.1	0.1	20	1.407	1,42	2.00

The trend of values of F' and of F'' and of their product $F' F''$ may be seen from Table 1, in which will be found values for a variety of coil proportions. Approximate values for intermediate shapes may be estimated by interpolation.

Equations (5) and (6) may be relied upon for giving a close

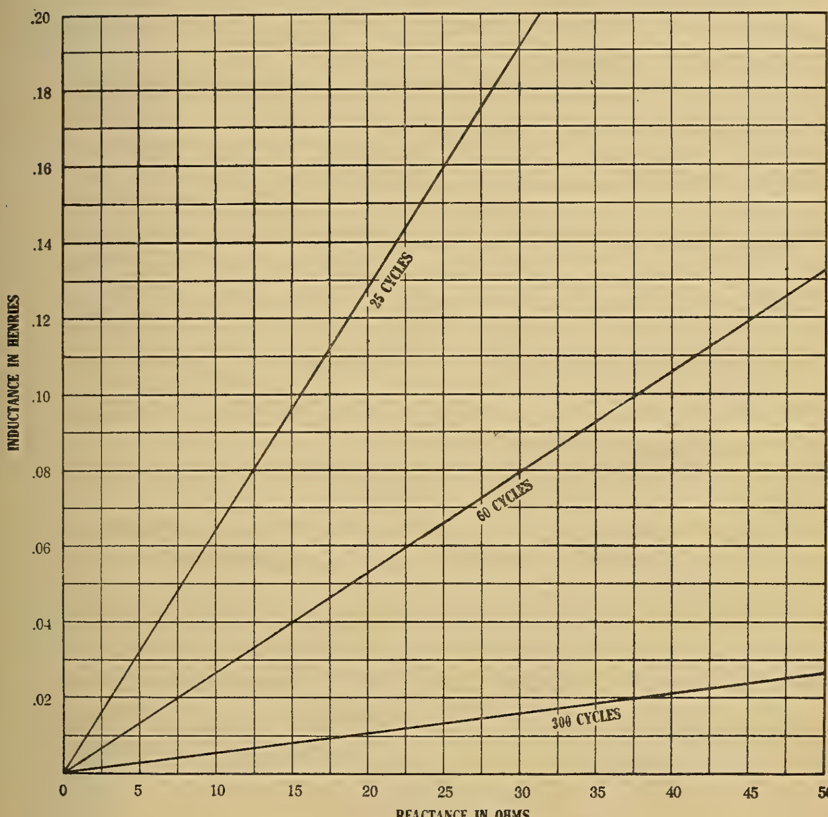

FIG. 6. REACTANCE CHARTS

approximation to the inductance of any closely-wound cylindrical coil, long or short, thick or thin, from the long solenoid to the single turn of fine wire. For spaced windings, they are not recommended, especially when the space between turns is great as compared with the diameter of the bare conductor. Equation (4), upon which these general formulas are based, assumes a perfectly smooth flux-path of uniform density within the coil. The photographic record of the iron filings, Fig. 3, shows some irregulari-

ties along the sides of the coil, as if the magnetic lines were trying to escape between the turns. If the turns were spaced farther apart, this effect would be increased, introducing another element into the equation.

IX. Reactance

When sine-wave alternating current is used, the value of the reactance in ohms that any coil will give is obtained from its inductance, L, in henries, by multiplying by $2\pi f$, which is 157, 377, and 1885 for frequencies of 25, 60, and 300 cycles per second, respectively. 300 frequency is often assumed in telephone circuits, although telephonic waves include many higher frequency harmonics. Fig. 6 is a chart for giving at once values of reactance, when inductance is known, for the frequencies above mentioned.

X. Application of Formula to Long Coils

For long coils, F' and F'' may be neglected for first approximations, when formula (5) becomes reduced to its first term,

$$L \text{ (in henries)} = \frac{Cm^2}{(b+c+R)\ 10^9} \qquad (7)$$

This corresponds to equation (4) with $c + R$ substituted for y as explained. The substitution is empirical, justified by the results obtained.

Even for approximations, (7) is not recommended unless the length, b, is at least twice the outside diameter, $2R$, when the error involved will scarcely exceed 4 per cent with a multilayer winding, and will be within 2 per cent for a single layer solenoid, becoming more accurate as the length of the coil increases.

In extremely long coils, c and R are so small as compared with b, that they too may be neglected, when the formula becomes reduced to $L \text{ (in henries)} = \dfrac{Cm^2}{b \times 10^9}$, the equivalent of the formula often given in text-books without reservation as suitable for solenoids. The Bureau of Standards refers to this "approximate formula for long solenoids" as having "a considerable error", but offers no substitute except the Lorenz single-layer formula, which involves elliptic integrals. It is, in fact, in error from 20 to 40 per cent according to thickness of winding, if the coil is twice as long as the outside diameter, and more than 5 per cent when coil length is as much as $20R$, its error being roughly ten times as great as that of (7), which is therefore recommended as a substitute for this common form.

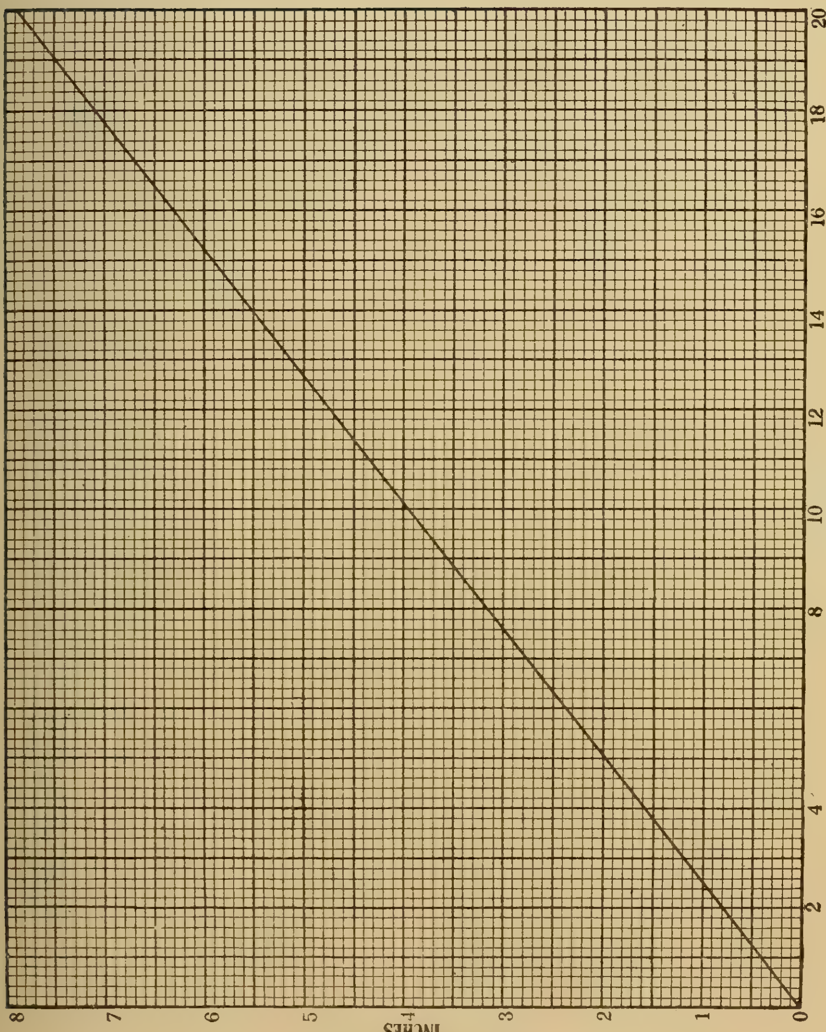

FIG. 7. CENTIMETER-INCH CHART

XI. Application of Formula to Short Coils

There is no sharp distinction between long and short coils, but coils whose axial length is less than their outside diameter may be classed as short. With a given length of conductor short solenoids yield more inductance than long, but equation (7) shows, as the common objectionable form does not, that this gain has a limit where the value of R begins to predominate, for the radius of the coil must increase with a material shortening of the winding. Experiment shows also that disks and rings have less inductance than ordinary compact shapes, using the same length of wire. However, equation (7) does not place enough emphasis upon the radius for short coils, and the empirical factors F' and F'' were introduced to correct this, and cause the general formula (5) to follow closely the varying influences of radius, length and thickness. A comparison with precision formulas for the several types of coils shows how well they perform their part.

XII. Stefan's and Kirchhoff's Precision Formulas

Stefan's formula for multilayer short coils has proved serviceable for comparisons, since it is stated to be precise without correction for very short coils, and within one per cent for coils whose length does not exceed radius. This formula, together with its table of constants, is reproduced from the Bureau of Standards bulletin. The examples of its use there given are confined to coils of relatively few layers.

TABLE 2

Constants for Stefan's Formula

$\frac{b}{c}$ or $\frac{c}{b}$	y_1	y_2	$\frac{b}{c}$ or $\frac{c}{b}$	y_1	y_2
.00	.50000	.1250	.55	.80815	.3437
.05	.54899	.1269	.60	.81823	.3839
.10	.59243	.1325	.65	.82648	.4274
.15	.63102	.1418	.70	.83311	.4739
.20	.66520	.1548	.75	.83831	.5234
.25	.69532	.1714	.80	.84225	.5760
.30	.72172	.1916	.85	.84509	.6317
.35	.74469	.2152	.90	.84697	.6902
.40	.76454	.2423	.95	.84801	.7518
.45	.78154	.2728	1.00	.84834	.8162
.50	.79600	.3066			

Stefan's formula for multilayer short coils is as follows:

L (in centimeters) =

$$4\pi a N^2 \left[\left(1 + \frac{3b^2+c^2}{96a^2}\right)\log_e \frac{8a}{\sqrt{(b^2+c^2)}} - y_1 + \frac{b^2}{16a^2}y_2\right] \ldots\ldots (8)$$

All dimensions are in centimeters. Result divided by 10^9 gives henries. For notation see page 13, except for values of y_1 and y_2, given in the table for certain ratios of b to c, or of c to b, so taken as to make the ratio a fraction. Naperian logarithms are employed in this formula. Multiply common logs by 2.3026 to get Naperian logs.

An excellent formula for the inductance of a single turn is that of Kirchhoff, which the Bureau of Standards says is extremely accurate, where $\frac{a}{d}$ is large, the error being less than one part in a million where $\frac{a}{d} = 250$, and is no more than one in 500 where $\frac{a}{d} = 5$. The formula is evidently more exact than any possible construction of a single turn in practice.

Kirchhoff's formula for single turns is as follows:

$$L \text{ (in centimeters)} = 4\pi a \left(\log_e \frac{16a}{d} - 1.75\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Transformed into a form to employ common logarithms this becomes,

$$L \text{ (in centimeters)} = 28.9 \times a \times \log_{10} \frac{2.8a}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The above have a, the mean radius, in centimeters. For English measure, it becomes

$$L \text{ (in microhenries)} = .0734 \times a \times \log_{10} \frac{2.8a}{d} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

where a is in inches. The logarithmic term being a ratio, values are in any units.

XIII. Modified Kirchhoff Formula For Many Turns

As an extremely simple formula, good for approximate work with short coils, the following modification of Kirchhoff's is submitted:

$$L \text{ (in milhenries)} = 0.29 \times a \times \left(\frac{N}{100}\right)^2 \times \log_{10}\left(\frac{5a}{b+c}\right) \ldots\ldots\ldots (12)$$

where coil dimensions are in centimeters, and

$$L \text{ (in milhenries)} = 0.736 \times a \times \left(\frac{N}{100}\right)^2 \times \log_{10}\left(\frac{5a}{b+c}\right) \ldots\ldots\ldots (13)$$

where coil dimensions are in inches.

These formulas (12) and (13) are not recommended for use where the value of the logarithmic term is less than unity; i. e., where $\frac{5a}{(b+c)}$ is less than 10, although it gives fair results

when this ratio is between 5 and 10. They apply equally well to single and multi-layer coils and to disks and single turns, covering substantially the field of both Stefan's and Kirchhoff's. It is discontinuous, as it would become negative for coils of relatively small radius. For single turns $b+c=2d$.

XIV. Comparisons

Table 3 gives the specifications of nineteen coils of widely varying proportions, of which a comparison of the value of induc-

TABLE 3

Description of Coils*

Coil	Layers	Turns	Length	Thickness	Mean Radius	Length of Conductor
1	28	784	3.2	3.20	6.19	30480
2	10	330	3.0	0.79	2.90	6003
3	14	462	3.0	1.15	3.08	8926
4	18	594	3.0	1.59	3.25	12111
5	20	660	3.0	1.65	3.28	13511
6	1	1	0.2	0.20	25.00	157
7	1	1	0.1	0.10	25.00	157
8	1	1	1.0	1.00	25.00	157
9	1	2	0.2	0.10	99.85	627
10	2	4	0.2	0.20	99.90	2507
11	1	4	0.4	0.10	100.00	2513
12	1	10	1.0	0.10	25.00	1571
13	1	20	2.0	0.10	25.00	3142
14	4	16	0.4	0 40	100.00	10053
15	1	50	5.0	0.10	20.00	6283
16	10	100	1.0	1.00	4.00	1257
17	1	1	2.0	2.00	10.00	63
18	1	1	1.0	1.00	25.00	157
19	20	400	1.0	1.00	10.00	25133

* All dimensions are in centimeters.

TABLE 4

Comparison

Coil	By Experiment	Prof. Brooks' Formula	Error per cent
1	75.754	76.10	0.4
2	5.543	5.578	0.64
4	18.476	18.989	2.77
5	24.110	23.511	—2.49
Bureau of Standards			
6†	.02058	.00204	—0.88
7*	.00202	.00204	1.22
8	.00041	.000416	—1.39
9	.0374	.0372	—0.53
10	.1435	.1410	—1.75
11	.1394	.1397	0.31
12	.1490	.1506	0.89
13	.5150	5220	1.64
14	2.070	2.050	—0.97
15	1.860	1.900	1.93
16	1.147	1.176	2.53
17	.000331	.000326	—1.51
18	.00133	.00131	—1.50
19	64.158	65.80000	2.55

† (round)
* (square)

tance obtained by use of the universal formula with those given by the Bureau of Standards is given in Table 4. It will be observed that there is no error greater than 3 per cent.

It will perhaps give greater confidence in the universal formula to show its fair agreement in the most extreme case, that of a single turn of large radius, even if such a winding is not required commercially. For this purpose, a comparison with the exact Kirchhoff formula (10) is invited, as illustrated in the tabular statement below. The nearly uniform variation of the approximate formula (5) from the standard for such single turns is 2.5 per cent, and the error cannot exceed the 3 per cent limit in the most extreme case, as proved in the foot-note on this page*. The comparative values derived from the modified Kirchhoff formula (12) are also set down, to indicate that its extension to coils of many turns does not prevent its use for single turns. It is seen that the empirical factors F' and F'' are so held in check in the shortest of coils, the single turns, as well as the extremely long coils, where they become unity, that it is not possible for them to cause the errors that often arise from empirical formulas when used beyond usual limits.

Comparison of formulas (5) and (12) with (10) for large single turns:

a	$(b=c=d)$	R	L (in microhenries) by			Variation from (10) in per cent	
			(10)	(5)	(12)	(5)	(12)
100	1	100.5	7.07	6.88	6.96	2.7	1.6
1000	1	1000.5	99.60	96 90	98.50	2.7	1.1
10000	1	10000.5	1285.00	1253 00	1275.00	2.5	0.8

*For extremely large circles of but a single turn equation (5) reduces rapidly to a form similar to Kirchhoff's formula (10) as follows: The transformation is obtained by using $2\pi a$ for Cm, its equivalent; by neglecting b and c in the denominator of the first term, and by using a for R; by omitting b and c as negligible in comparison with R, or a in the F' term, making the value of that term 1.428; and lastly by omitting the 100 in the F'' term, as negligible in comparison with the ratio of $14\frac{a}{5d}$, as it will be in the most extreme cases. (5) then becomes transformed as follows:

$$L \text{ (in centimeter)} = \frac{(2\pi a)^2}{a} \times 1.428 \times 0.5 \log 14\ a/5d$$

$$= 28.2 \times a \times \log 2.8\ a/d$$

This differs from (12) only in the coefficient, which is 28.9 in that equation, showing a difference of 2.5 per cent in the most extreme case possible, such as the third line of the tabular comparison above.

XV. Effect of Shape of Coil on Inductance

Fig. 8 is a photograph of two coils wound with the same length, 50 ft. of No. 8 wire, the larger being wound in 7 turns loosely upon a bicycle rim, the other wound into a small coil of 56 turns. The small coil has about 2.5 times the inductance that the larger has. The measured values are .239 milhenries and .085 milhenries for the small and large coils. Calculated values are not very accurate for these coils, since they are not closely wound.

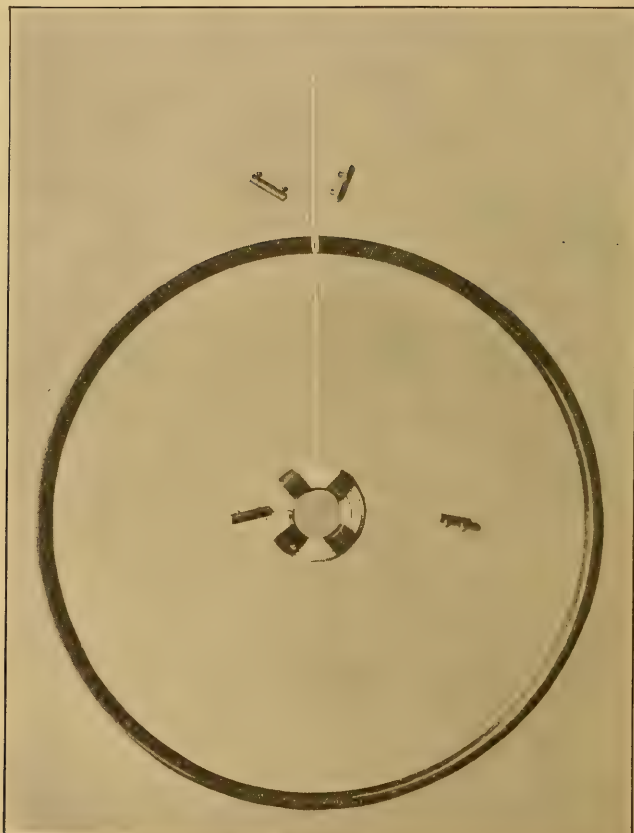

Fig. 8. Photograph of Two Coils Wound to Illustrate Influence of Shape on Inductance. (See Sec. XV.)

Table 5 has been prepared to illustrate the variation of inductance due entirely to the shape of the coil into which a given conductor may be wound. The length of wire chosen, 52.4 ft.

is that required to produce 80 turns of an average diameter of 2.5 in. These 80 turns are shown disposed in various ways, as a single layer, as two layers, as four, eight and sixteen layers, this last being a very thick, short coil, with a very small hole. Any shorter coil with 80 turns would fill up the hole completely, so the succeeding lines show coils of less turns and greater mean radius, until the final stage of a single turn of the entire 52 ft. is reached. The values were calculated by the general formula 6 and, so far as applicable, by formula (13), which is easier to use for very short coils, such as disks and rings. An additional line in the table gives the calculated inductance for the same conductor in a straight line, showing that the inductance of a single turn is less than the straight value.

TABLE 5

ILLUSTRATING THE EFFECTS OF COIL SHAPE UPON INDUCTANCE

Inductance in milhenries of 52.4 ft. of magnet wire, 0.1 in. diameter outside (small No. 11 d. c. c.) wound into cylindrical coils of various shapes as indicated.

No.	Description	No. of Layers	Total Turns N	Mean Radius a	Length b	Thickness c	Outer Radius R	F' F''	Induc. L. Milhenries (6)	(13)	Per cent of Max.
1	Spaced solenoid...	1	80	1.25	16.	0.1	1.3	1.007	.058	....	17
2	Solenoid...........	1	80	1.25	8.	.1	1.3	1.015	.108	....	33
3	Double layer ditto	2	80	1.25	4.	.2	1.35	1.032	.186	....	56
4	Thick tube........	4	80	1.25	2.	.4	1.45	1.072	.279	....	84
5	Compact MAXIMUM..........	8	80	1.25	1.	.8	1 65	1.141	.331	....	100
6	Thick disk........	16	80	1.25	0.5	1.6	2.05	1.2	.289	....	87
7	Thick disk........	10	50	2.	.5	1.	2.5	1.20	.301	.303	91
8	Sq. section ring...	5	25	4.	.5	.5	4 25	1.28	.244	.239	74
9	Sq. section ring...	4	16	6.25	.4	.4	6.45	1.38	.190	.188	57
10	Flat ring..........	4	8	12.50	.2	.4	12.7	1.57	.119	.119	36
11	Thin disk..........	4	4	25.	.1	.4	25.2	1.78	.069	.070	21
12	Thin disk..........	2	2	50.	.1	.2	50.1	2.13	.042	.043	13
13	Single turn........	1	1	100.	.1	.1	100.05	2.46	.025	.025	8
14	Straight line......		0	Inf.					.030(XVI)		9

XVI. INDUCTANCE OF A STRAIGHT WIRE

L (in milhenries) $= Ft. \times 0.00014\,(1.35+\log_{10}\dfrac{Ft.}{d})$, where $Ft.$ is the length of the conductor in feet, and d its diameter bare in inches. It is assumed that the return conductor is at infinite distance.

XVII. ILLUSTRATIVE CALCULATIONS

Complete calculations are here given for the coil represented by line 10 of Table 5, values being taken from the table.

By (6) L (in henries)

$$= \frac{.366\,(.0524)^2}{.2+.4+12.7} \times \frac{2+4.8+25.4}{2+4.\ \ +17.8} \times .5 \log \left(100+\frac{178}{.4+1.2}\right)$$

$$= \frac{.366 \times .00275}{13.3} \times \frac{32.2}{23.8} \times .5 \log 211$$

$$= .0000757 \times 1.35 \times 1.162$$

$$= .000119 \text{ or } .119 \text{ milhenries.}$$

By (13) L (in milhenries)

$$= 0.736 \times 12.5 \times (.08)^2 \times \log \frac{62.5}{.6}$$

$$= .0589 \times 2.018 = .119$$

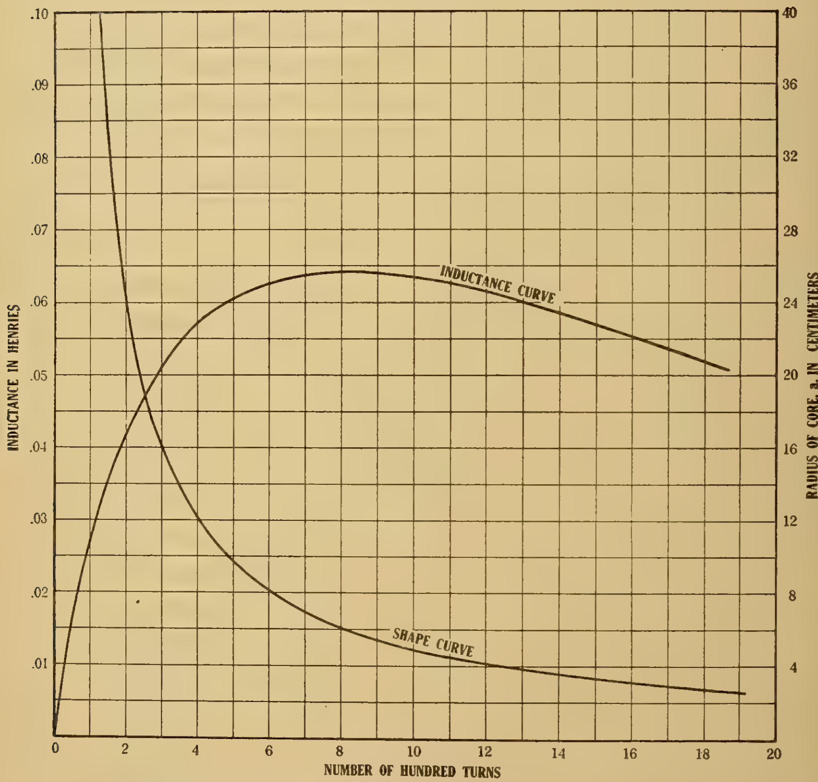

FIG. 9. CHART SHOWING THE VARYING INDUCTANCE OF 1000 FEET OF No. 16 D. C. C. WIRE. When wound into coils of varying mean radius, a, while the ratio of $\frac{b}{c}$ is maintained equal to 1.2.

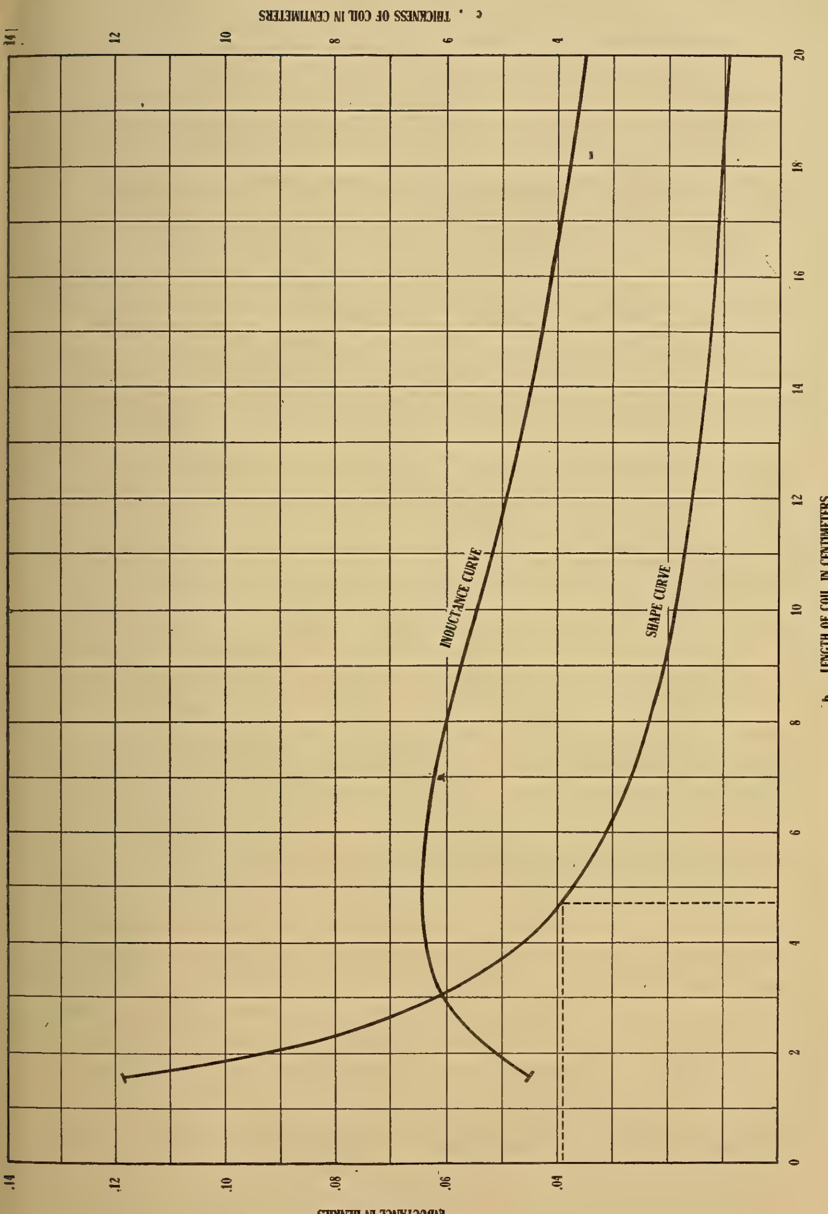

FIG. 10. CHART SHOWING THE VARYING INDUCTANCE OF 1000 FEET NO. 16 D. C. C. WIRE

When wound into coils of mean radius, $a = 5.85$ cm., length b, and thickness, c, varying reciprocally as indicated by Shape Curve. The inductance values correspond to values of length of coil b.

Values are calculated by equations (6) and (13), except line 14, for which see page 23. Line 1, spaced solenoid, is calculated by (6), since the spacing is not sufficient to vitiate the result. The formula is not designed for spaced windings. Line 5 is the maximum inductance, the coil being practically the prescribed maximum shape. Equation (18) gives .3315 milhenries as the possible maximum. Lines 4, 6 and 7 show that a considerable alteration of coil proportions does not seriously affect the inductance. Equation (13) is applicable only to short coils.

Fig. 9 and 10 show how the inductances of a given length of wire vary with certain factors. In Fig. 9, the ratio of b to c was kept constant ($b = 1.2\,c$), while the number of turns was varied. The shape curve in Fig. 9 gives the mean radius of the coil for a given inductance under the conditions imposed. In Fig. 10, the mean radius a was kept constant, while b and c were varied through a wide range; since their product was kept constant, an equilateral hyperbola (shape curve) will give shape of cross section of coil, i. e., the abscissa to any point on the inductance curve gives the axial dimension b and the ordinate to the shape curve at this point will give the thickness of the winding.

XVIII. Maximum Inductance in Coreless Coils, with Equations

For a given conductor, the numerator of the first term of equations (5) or (6), the metric and English forms of the universal formula, is constant, and the variation of inductance is determined by the shape of the coil, as indicated by the denominator of the first term, and by the shape factors, F' F''. In closely-wound coils, the values of a, b and c must vary, so that their product remains constant, since $2\pi a \times bc$ is the volume of the winding. The denominator, $b + c + R$, may be written $a + b + 1.5\,c$, since $R = a + .5\,c$, and its minimum value occurs when $a = b = 1.5\,c$ under the law stated. This would indicate the coil proportions for maximum inductance but for the varying values of F' and F''. These factors both increase if the coil is shortened and its radius slightly increased, while the denominator is but little affected. It is found by the method of approximations that the relative proportions of the coil which will produce the maximum inductance are in fact nearly as follows: $a : b : c = 1.5 : 1.2 : 1.0$. To the same scale, $r = 1$ and $R = 2$, showing that the outside diameter of the coil is twice its inner diameter, and the axial

length 0.3 its outside diameter, a compact shape, shown in Fig. 5.

A slight variation from these proportions, such as may be necessitated by the integral number of layers and of turns per layer that must exist in a smooth winding, will cause little loss from maximum inductance, while a great variation will reduce the inductance materially. Line 5 of Table 5, already referred to, is so nearly of prescribed proportions that the inductance is indistinguishable from maximum. Line 7 shows a reduction of inductance by 10 per cent due to a moderate change in the coil shape. Lines 4 and 6 show a still greater reduction of inductance although the change in shape would not appear to be greater than that of line 7 from the maximum. It will be noticed that in all the coils of this table, except line 1, which is a spaced coil, the product abc is unity. The effect of further modification of coil shape is graphically shown in Fig. 24 and 25, described under "XIX. Charts".

Since economy in the use of material for the production of inductive reactance requires the employment of approximately the maximum shape, the general formulas may be reduced to a much simpler form if the prescribed shape, where $a:b:c = 1.5:1.2:1.0$ is used. For these proportions $F' \times F'' = 1.14$, and $b + c + R = 4.2\,c$.

Equation (5) then becomes for maximum inductance:

$$L_{\max}\text{ (in henries)} = 0.2714\,\frac{Cm^2}{c \times 10^9} \quad\ldots\ldots\ldots\ldots (14)$$

where all dimensions are in centimeters. See also equation (17). The value of c is chosen as the dimension unit, since it is equal to r, the radius of the spool on which the coil must be wound. Its absolute value may be determined from the dimensions of the given conductor as follows:

The coil volume, $2\pi a \times b\,c$ becomes for maximum shape $3.6\,\pi\,c^3$, using the relative proportions given above. This volume in closely-wound coils must equal the conductor length multiplied by its outside diameter squared, or $Cm\,D^2$, the units being taken in centimeters. From this equality, c is determined.

$$c = \left(\frac{Cm\,D^2}{3.6\,\pi}\right)^{\frac{1}{3}} = 0.4455\,(Cm\,D^2)^{\frac{1}{3}} \quad\ldots\ldots\ldots\ldots (15)$$

all values in centimeters; or

$$c\text{ (in inches)} = 1.02\,(Ft. \times D^2)^{\frac{1}{3}} \ldots\ldots\ldots\ldots (16)$$

where conductor length is in feet, and outside diameter in inches.

Substituting (15) in (14), the maximum inductance is found in terms of conductor dimensions alone:

$$L_{\max}\text{ (in henries)} = 0.609\,Cm\left(\frac{Cm}{D}\right)^{\frac{2}{3}} \times 10^{-9}, \ldots (17)$$

the conductor dimensions being in centimeters. In English units this becomes:

$$L_{\max} \text{ (in henries)} = 97.3\, Ft. \left(\frac{Ft.}{D \text{ (inches)}}\right)^{\frac{2}{3}} \times 10^{-9} \quad (18)$$

$Ft.$ is the length of conductor in feet, and D is its outside diameter in inches.

A simple equation for maximum inductance, avoiding exponents, is obtained from (14). In every coil, $Cm = 2\pi a N$. Writing N' for the turns and $3c$ for $2a$ in a coil of maximum shape,

$$Cm = 3\pi c N' \text{ or } \frac{Cm}{c} = 3\pi N'.$$

Substituting in (14)

$$L_{\max} \text{ (in henries)} = .2714 \times Cm \times 3\pi N'\, 10^{-9} = 2.56\, Cm\, N'\, 10^{-9}. (19)$$

and

$$L_{\max} \text{ (in henries)} = 78 \times Ft.\, N'\, 10^{-9} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

for English units.

N' is the number of turns for the prescribed shape of coil only, and a small variation from that shape may affect the number of turns more than it does the inductance. Putting (17) = (19) N' is defined by

$$N' = 0.238 \left(\frac{Cm}{D}\right)^{\frac{2}{3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

This shows that the number of turns in a coil, wound for maximum inductance, is dependent upon the ratio of length to outside diameter of conductor, and independent of absolute dimensions, hence any units may be used in equation (21), providing the same units are used for numerator and denominator. The number of turns derived from (21) should be used in equations (19) and (20), since this number may differ from actual count of turns, unless the winding is very exact.

XIX. Charts

Since curves often present information that is to be obtained only with effort from formulas, it is a feature of this bulletin to offer charts for the determination of the inductance of any coreless cylindrical coil.

Unlike resistance, inductance depends not only upon the conductor dimensions but upon the thickness of insulation, and the shape of the coil winding, therefore to exhibit in simple chart form the information desired requires a comprehensive scheme. Since the inductance of any conductor will reach a maximum when wound into the best shape, and has a definite proportion of this maximum value when wound into any other specified shape, the use of the two curves may be made to determine the inductance of any cylindrical winding whatever; one giving the maximum value, the other the percentage of that maximum that the conditions prescribe. Their product is the desired inductance.

A curve from which the maximum may be taken will be found upon one of the several inductance charts covering the principal commercial sizes and insulations of magnet wire, as explained fully under section XX. A curve for the proportion of maximum value inherent in various possible shapes of coil will be found in one of the two shape-coefficient charts for long and short coils, as explained in detail in section XXII. These charts are so bound as to permit the use of the two selected charts simultaneously.

The charts may seem to be drawn on too small a scale for definite readings, but the use of a more open scale might lead to the assumption of greater accuracy than the method warrants. There should be no difficulty in deriving values accurate to 5%, with perhaps a little greater uncertainty for interpolated conditions and furnishing an excellent check upon calculated values. The use of logarithmic section paper will be seen to give the same proportionate accuracy throughout the range of the charts, and facilitates the extension of this range to any desired degree.

XX. Maximum Inductance Charts

Charts herewith presented permit direct readings of the maximum self-inductance that may be obtained from any commercial magnet wire for lengths between 100 and 10 000 ft., wound into coils without iron cores. They also furnish information as to the necessary dimensions of the coil, and of the number of its turns. Other charts give shape-factors in percentage of the maximum inductance that coils of other than the prescribed maximum shape will produce from a certain length of conductor.

The equations of the preceding section furnish means of calculating the information needed to produce these charts. When a definite size of wire is chosen with known insulation, inductance

is seen to vary as the $\frac{5}{3}$ power of the conductor length, as shown by equation (17) for closely wound coils of the prescribed maximum shape. Upon ordinary coordinate paper, such an exponential function produces a curve difficult to plot with exactness, while with logarithmic coordinate paper, the curves become straight lines, whose slope is determined by the exponent. Two points determine a line, and single points then determine other lines, since they must be parallel, having the same exponent.

The logarithmic paper chosen for the inductance charts has four "squares", whose horizontal and vertical divisions are similar to the upper scale of the ordinary slide rule, starting from unity and going to 100. In all charts, except the shape factor charts, horizontal divisions, or abscissas, represent thousands of feet of conductor length, the scale running from 0.1, or 100 ft., to 10, or 10 000 ft. Vertical divisions represent different quantities on the several series of charts.

The first series, Fig. 11 to 14, gives inductance directly in henries on its vertical divisions, or ordinates, the bottom line indicating .01 henry, the middle line, .1 henry, and the upper line 1 henry with intermediate readings between as on a slide rule.

Since inductance depends upon the compactness of the winding, the thinner the insulation upon the conductor the higher the inductance. Commercial magnet wires have various insulations, and different charts were found to be necessary for the most important of these. Fig. 11*a* and 11*b* are for double-cotton-covered wires; Fig 12*a* and 12*b* for double-silk-covered wires; other pairs of charts are 13*a* and 13*b* for single silk, and 14*a* and 14*b* for enamel insulated wire. In each pair of charts, one has lines representing even gauge numbers (B. & S. gauge) that are divisible by four; the other those even numbers that are not divisible by four. Odd numbers are not directly represented, but may be closely estimated as between the adjacent even sizes.

It will be noticed that the parallel lines representing sizes, 4, 8, 12, 16, etc., are not always in normal space relation. This is not due to an error in making charts, but to the fact that insulations are not progressive like gauge numbers. The thickness of insulation upon the several wire sizes was taken from the tables of the Standard Handbook, reproduced as Tables VI, VII, VIII, IX, X and XI, and the same thickness of cotton or silk is used for several sizes of wire, when there is a sudden change to another

thread, causing the apparent discontinuity in the charts. For paper-insulated magnet wire, the charts for single-silk suffice.

XXI. Charts for Dimensions and Number of Turns

To wind a coil, it is necessary to know the spool dimensions, and as the calculation involves cube-root, as shown by equation (16), it is believed that the series of dimension charts, Fig. 15*a* to 18*b*, will prove useful. These have horizontal dimensions representing lengths of wire in feet as before, and vertical dimensions, or abscissas, giving the value of c directly in inches. c is the thickness of the winding; $1.2c$ its length; $2c$ its inner diameter, these dimensions giving all spool dimensions. If any other shape than these maximum proportions be used, it will be necessary to provide somewhat more space for the winding, since more wire will be required for a given inductance.

It is often desirable to know the number of turns that a coil will have, and a third series of charts, Fig. 19*a* to 22*b*, gives this information for coils of prescribed maximum shape, employing the different insulations as before. Conductor lengths are abscissas as usual, and ordinates give directly the number of hundreds of turns, the range of the charts being from 100 to 10 000. For large conductors, it may be necessary to adjust dimensions slightly to accommodate an integral number of turns in the length or thickness of the winding. A moderate change in the number of turns, owing to such adjustment, makes little change in the inductance. Values of turns were determined by equation (21).

Fig. 23 has two curves upon it, to illustrate the use of the charts both for inductance and number of turns. The two lines concern No. 24 single-silk-covered wire. Thus to get 0.1 henry of inductance, 890 ft. of this wire are required (1.15 lb.), and the coil will have 1420 turns. The coil dimension, c, is obtained from Fig. 17*a*.

It may be necessary to find values beyond the range of the charts. The lines, being straight, may be extended as indicated upon Fig. 23. In the lower right-hand corner, is the extension of the inductance line, so that, for example, 8000 ft. will produce 4 henries of inductance. For the extension, inductance values are multiplied by 100, and readings are therefore in the whole number henries, hence 4 in this case. Equation (18) shows that L varies with the $\frac{5}{3}$ power of length of any given conductor, and it is convenient to know that $4^{\frac{5}{3}} = 10$ almost exactly, so that

the value for 8000 ft. could have been obtained from the chart unextended, by reading the inductance for 2000 ft. as 0.4 henry, and multiplying it by 10.

In a similar way the number-of-turns line can be extended. A backward extension is shown in the lower left-hand corner. Turns vary with the $\frac{2}{3}$ power of length, hence 8 times the length gives 4 times the turns for same shape of coil.

XXII. Shape-factor Charts

Two charts, Fig. 24 and 25, as well as small scale duplicates, Fig. 24*a* and 25*a*, are provided to indicate the percentage of maximum inductance that a coil of any shape will give. As there are three variables involved in a winding: radius, length and thickness, it is difficult to show this in a simple way. Parallel curves represent different winding sections. On Fig. 24, these are marked 1.2, 2, 5, 10, 20, 50, 100, and 500, for values of $\frac{b}{c}$. The larger numbers, such as 100, and 500 will generally represent single-layer windings of 100, or 500 turns; the smaller figures will generally be multilayer coils; thus, 5 might represent two layers of 10 turns each, 5 layers of 25 turns each, or any such ratio. The curve for $\frac{b}{c} = 1.2$ includes the maximum value 100 per cent where $\frac{a}{b} = 1.25$. Fig. 24 includes all coils whose axial length, b, exceeds thickness, c, while Fig. 25 includes square section coils, $\frac{c}{b} = 1$, and all coils where c exceeds b. Whole numbers are attached to the curves, the ratio being inverted to $\frac{c}{b}$ to avoid fractions in Fig. 25, which includes all disk-shaped coils. The curve $\frac{c}{b} = 1$ is for all single turns, as well as for those whose turns per layer are equal to the number of layers, assuming that round wire is used, and no allowance made for bedding.

Logarithmic paper is again used, as it is easy to read, and covers a wide range. It also avoids the employment of reversed curves. The shape-factor charts have horizontal divisions, or abscissas, representing ratios of $\frac{a}{b}$ throughout. In Fig. 24 $\frac{a}{b}$ runs from .01 to 100, and in Fig. 25 from .1 to 1,000. Ordinates

represent percentages from 1 to 100, most coil portions giving more than 10 per cent of the maximum inductance as would be expected. To find the percentage ratio of a coil whose section is not represented, such as $\frac{4}{1}$; its value may be estimated from the curve representing $\frac{5}{1}$.

The abrupt ending of the lines at the left indicates the point where the coil space is entirely occupied with wire, or where the hole ceases to exist. For example, in Fig. 24 the line for $\frac{b}{c} = 5$ ends at $\frac{a}{b} = 0.1$. These proportions are all relative, and value $c = 2$; $b = 10$; $a = 1$, fit the case at this terminal point. In order to have the mean radius, a, half the thickness of the winding, since 1 is half of 2, the winding must come down to the axis of the coil, and any further decrease of radius, which could be represented by a continuation of the curve to the left, is impossible. At this terminal point, any length of wire wound into a solid coil would have 62 per cent of the maximum inductance, and if the 5 to 1 section were retained, the same length of wire would have 86 per cent at $\frac{a}{b} = .8$, for which ratios the proportions $c = 1$, $b = 5$, $a = 4$, in which the coil has a central hole of diameter 7, give the same volume of winding as the solid coil above mentioned.

In Fig. 25, the chart for disk-shaped coils, the lines representing various winding sections intersect. If the abscissas were based upon the ratio of a to c, there would be no intersections. In disk coils, the value of c, the thickness is more important than that of b, the axial length, but it was thought best to use the same ratio for the abscissas in both charts, Fig. 24 and 25, to avoid confusion in readings.

To find the inductance of any coreless coil, the maximum inductance charts give direct readings of the inductance that it is possible to obtain from a given length of wire, while charts 24 and 25 show the percentage of that maximum that the actual shape of the coil permits; hence the product of the two readings gives the inductance of any coil whatever, whose dimensions are known. The same information, without calculation, is thus available, that the universal formulas (5) or (6) give.

XXIII. Illustrative Examples

To find the inductance, L, of 320 turns of No. 11 d. c.c. magnet wire, wound into a four-layer solenoid, whose dimensions are $a = 2.5$, $b = 8$, and $c = 0.4$ in. The length of conductor is therefore 419 ft., and since the charts do not directly indicate odd sizes, find the maximum inductance of 419 ft. of No. 10 wire from Fig. 11a, as 0.0095 henry, and of the same length of No. 12 from Fig. 11b as 0.0115, and assume the mean, 0.0105, as the value for No. 11 when wound for maximum inductance.

For the shape indicated, find the shape-factor by referring to Fig. 24 and as indicated by curve marked 20, which is $\frac{b}{c} = \frac{8}{0.4}$, and read the factor .57 as the ordinate of this curve at value $\frac{a}{b} = 0.31$. 57 per cent of .0105 is .006, or 6 milhenries, the inductance value sought.

It may be noted that this assumed coil has twice the linear dimensions of the coil of line 3 of Table 5, and therefore 8 times the weight and conductor length. Now for the same coil shape, inductance varies as the $\frac{5}{3}$ power of conductor length. The $\frac{5}{3}$ power of 8 is 32, and the tabular value of inductance of No. 3 coil, .186, multiplied by 32, gives 5.95 milhenries, agreeing with the chart value already found, and checking its essential accuracy.

As another example, let it be desired to design a coreless coil to produce 30 ohms reactance at 60 cycles, employing No. 10 d.c.c. magnet wire. Fig. 6 shows that 30 ohms at 60 cycles is obtained from .08 henry of inductance; Fig. 11a gives 1500 ft. of No. 10 for .08 henry, if wound into the maximum-inductance shape; and Fig. 15a gives the dimensions: $r = 2.75$, $a = 4.12$, $b = 3.30$ and $c = 2.75$ in., prescribing the coil and conductor dimensions. If the number of turns is desired, Fig. 19a gives $N' = 730$ as the number of turns.

If there is a restriction upon the coil dimensions, for example, that the inner radius, r, of the coil cannot be less than 8 in., and that the axial length of coil must be 1 in., the following approximation method will determine the greater amount of wire required. The restrictions make the coil disk-shaped, and Fig. 25 will indicate the percentage factor for the less advantageous shape. To use Fig. 25, a is required, and $a = r + .5\,c$, but c is to be determined also. As a first approximation, take $c = 4$, when $a = 10$, with b

given $= 1$; then $\frac{a}{b} = 10$, and $\frac{c}{b} = 4$. The curves of Fig. 25 represent directly $\frac{c}{b} = 2$ and $\frac{c}{b} = 5$. The values for $\frac{c}{b} = 4$ will fall between these curves, and as they intersect near the abscissa 10, no interpolation is required, and the shape-factor is read off as .75.

To obtain 0.08 henry will therefore require as much wire as would produce $\frac{.08}{.75} = .1067$ henry inductance if wound into the prescribed shape. Fig. 11*a* shows that 1750 ft. of No. 10 wire will be required, and Fig. 15*a* gives its coil dimensions as $a' = 4.35$, $b' = 3.48$, and $c' = 2.9$, from which $a'\ b'\ c' = 44$. The volume constant, abc, for the approximate dimensions assumed, is 40, showing that the assumed dimensions are 10 per cent too small to contain the 1750 ft. of wire. Since r and b are restricted, c alone can be increased, but as increasing c also affects a to a less degree, it is not required to increase c 10 per cent, but 8 per cent only, making the revised values as follows: $a'' = 10.16$, $b'' = 1$, $c'' = 4.32$, and their product, $a''b''c'' = 44$.

Referring again to Fig. 25, for determining any change in the shape-factor for $\frac{c}{b} = 4.32$, and $\frac{a}{b} = 10.16$, it is seen that no appreciable change has occurred, and the 1750 ft. of No. 10 wire are sufficient. If a noticeable difference has arisen, a second approximation, and further use of the charts as before would be advisable.

In using logarithmic charts by interpolation, it is well to note that it should be done by logarithmic differences, which vary slightly from ordinary proportion.

XXIV. Weights and Reactance-resistance Ratios

If it be assumed that equal weights of wire occupy the same space, true of bare wire, and approximately true of insulated wire, then equation (14) shows that equal weights of wire produce inductance in proportion to the square of the conductor length. Thus a given weight of No. 13 magnet wire would have twice the length and yield four times the inductance that the same weight of No. 10 would give. It is seen that inductance is more cheaply obtained from small wires, and that the smallest wire that it is safe to use should be employed

For equal weights, resistance also varies as the square of the length of the conductor, so the ratio of inductance to resistance is practically constant for a given weight of wire, regardless of its size, provided the coil is wound for maximum inductance. Therefore the time constant, the value of $\frac{L}{R}$, depends upon the weight of the coil for maximum shaped coils.

From another equation (17), it is seen that with definite sized wire, inductance increases with the $\frac{5}{3}$ power of the conductor length. Resistance increases directly with length, therefore the ratio of the two increases with the $\frac{2}{3}$ power of the increase in length, or of weight, since weight and length increase together. A definite ratio of reactance to resistance requires a nearly definite weight of wire, independent of the size of the conductor. Thus 0.5 lb. of wire will produce a reactance at 60 cycles equal to its resistance in ohms, while a coil weighing 1000 times as much, or 500 lb. will produce a reactance that is 100 times as great as its resistance, since $100 = 1000^{\frac{2}{3}}$. As a specific example, 0.5 lb. of No. 14 wire has a reactance and a resistance of about 0.1 ohm, while the 500 lb. will give 10 000 ohms of reactance, and 100 ohms of resistance. This ratio of reactance to resistance of 100, giving a lag angle of 89.5° if connected at its terminals to an alternating electromotive-force may be considered nearly pure reactance, as the resistance is negligible in comparison.

TABLE 6.

ESTIMATE OF COIL WEIGHTS FOR CERTAIN RATIOS OF REACTANCE TO RESISTANCE COILS TO BE CLOSE-WOUND IN THE PRESCRIBED SHAPE FOR PRODUCING MAXIMUM INDUCTANCE.

$\frac{X}{R}$	Lag Angle	WEIGHT OF COPPER IN LB.		
		25 Cycles	60 Cycles	300 Cycles
1.	45	2.	0.5	.045
2.5	68	7.5	2.	.18
6.	80	30.	8.	.72
10.	84	60.	16.	1.44
100.	89.5	1860.	500.	45.

This table is not applicable to spaced coils, which it may be necessary to use in order to obtain sufficient insulation between turns.

The table above gives the results of a rough estimate of the weights of wire required for the various ratios of reactance, $X = 2\pi fL$, to resistance, R, of maximum inductance coreless coils wound with wire of any size. The estimate is necessarily rough, since the weights will vary slightly, increasing for the finer sizes of wire.

XXV. Ratings of Coils

There is no established rating for inductance coils. Since the object of such coils is usually to offer as much reactance as possible, the volt-ampere or kilovolt-ampere rating should be adopted. The rating is then the value of I^2X, and the indefinite factor is the current, I, since it is a matter of judgment how much current a certain conductor may safely carry, depending upon the conditions of operation. If a definite density is selected, the rating is determined when the coil dimensions are known. This rating corresponds to the resistance rating in I^2R units, which are watts. For a definite current density, I^2R varies directly with weight, while I^2X varies with the $\frac{5}{3}$ power of weight, as explained in the preceding paragraph, the current being fixed by the size of the conductor. Hence the rating of a reactance coil increases much more rapidly than a resistance coil, as conductor length increases.

The cost per kilovolt-ampere diminishes with increasein size of coil. Thus, a coil of No. 14 wire having a weight of 16 lb. has a reactance of about 32 ohms at 60 cycles, and costs, at 20 cents per lb. for copper, \$3.20. The rating, I^2X, is 3.2 kilovolt-amperes, if 10 amperes be employed, making the cost \$1 per kilovolt-ampere. Using the same size wire in the 500 lb. coil, the reactance is 10 000 ohms, the I^2X rating 1000 kilovolt-amperes, and the copper costs but \$100, or 10 cents per kilovolt-ampere. At 25 cycles, the weight of coil is nearly one ton before the cost per kilovolt-ampere is reduced to 10 cents.

The diminishing cost of copper per kilovolt-ampere for reactance may be compared with the constant cost of copper per kilowatt for resistance. Using the same current density and price of copper per pound, the cost is uniformly \$10 per kilowatt of I^2R. The extravagance of resistance control of alternating current is evident.

The rapid gain in the rating of coreless reactance coils with weight thus points to their increased use in engineering work,

since in very large sizes they are relatively cheap, and have the great advantage of absence of core losses as compared with ferric reactances.

XXVI. Mutual Inductance

Mutual inductance may sometimes be ascertained through self inductance calculations. Mutual inductance may be defined as the inductive influence of one coil or circuit upon another. Thus if a coil be thought of as consisting of two half-coils, each half has mutual inductance with respect to the other half as well as its own self-inductance. The self inductance of the whole coil will then be equal to the sum of the mutual inductances of the half coils added to their own self inductances. If, however, the two halves be connected in opposition, the mutual inductances are opposed to the self-inductances, and the combined self-inductance is equal to the difference between the self and mutual inductances. Expressed in symbols these facts are as follows:—

(22) $L = L_1 + L_2 + 2\,M$, when the coils are in series.

(23) $L_0 = L_1 + L_2 - 2\,M$, when the coils are in opposition.

As thus expressed, these formulas are not confined to equal halves of a single coil, but are general, where L_1 and L_2 are the self-inductances of the two parts, and M their mutual inductance. L_0 is the combined inductance of the parts in opposition, while L is their self-inductance in series.

When L_1 and L_2 are such parts of one coil that together they make a close-winding, the formulas of this bulletin or the charts may be used in obtaining their values, when the above equations will give the mutual inductance of one part upon the other.

If a winding have a square section, the mutual inductance of real or imagined halves, side by side, is substantially equal to that of two halves one within the other. Mutual inductance can never exceed 25 per cent of L, the self-inductance of the whole coil. Highest values of mutual inductance occur when the two parts make nearly the maximum prescribed shape; although if a disk-shaped coil where $\frac{b}{c} = \frac{2}{4}$ is divided into two thinner disks, side by side, each having a section $\frac{b}{c} = \frac{1}{4}$; or a short thick tube having the section proportions $\frac{b}{c} = \frac{4}{2}$, and the two half coils are the inner and outer layers of this short tube,

the value of M will be found to be slightly greater than if a square section coil be divided into two halves either transversely or longitudinally.

It should be noted that 15 per cent to 20 per cent of a large value of L may give a greater value of M than 25 per cent of a small value of L; and that a minimum value of L_0 does not coincide with the high values of M. It is possible to extend the calculation of mutual inductances by self-inductance formulas to coils not contiguous by having or assuming a coil to fill the gap between,—a method described in the Bureau of Standards, Bulletin Vol. V. No. 1, page 20.

It seems to be advantageous in transformers to make the two coils constituting the primary and secondary windings have as high a mutual inductance as possible, independent of the action of the core, in order that the weight of the core may be reduced, especially in the very large transformers that are now being built.

In very large reactance coils without cores, where the actual flux density within the coil attains values approaching those with weak iron magnetic circuits, the mechanical stresses due to sudden short-circuits are but little appreciated. It will be necessary to build such coils substantially, and to use spaced windings for this purpose, even if not required for insulation. That such coils will have an increasing use is probable, especially when it is realized that the cost per kilovolt-ampere decreases so rapidly with increase of rating, making the cost of very large coils comparable with cored coils, to which they are in many respects so superior.

Owing to the increasing use of non-ferric coils, there is a demand for convenient data concerning their characteristics, for definite knowledge of their most effective shape, and for a simple method of determining their inductive reactance. It is hoped that the information herein presented will facilitate the construction and utilization of such coils in every branch of electrical engineering.

XXVII. Tables and Charts

TABLE 7

Diameters of Cotton Covered Wire*

(Standard Underground Cable Co.)

Size	Diameter in Mils					
B & S	Bare	S. C. C.	D. C. C.	T. C. C.	S. S. C.	D. S. C.
0000	460	469	478	487	462	464
000	410	419	428	437	412	414
00	365	374	383	392	367	369
0	325	334	343	352	327	329
1	289	298	307	316	391	293
2	258	267	276	285	260	262
3	229	238	247	256	231	233
4	204	213	222	231	206	208
5	182	191	200	209	184	186
6	162	170	178	186	164	166
7	144	152	160	168	146	148
8	128	135	142	149	130	132
9	114	120	126	132	116	118
10	102	107	112	117	104	106
11	91	96	101	106	93	95
12	81	86	91	96	83	85
13	72	77	81	87	74	76
14	64	69	74	79	66	68

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TABLE 8

Thickness of Cotton and Silk Insulation*

Size	Thickness of Insulation in Mils					
B & S	S.C.C.	D.C.C.	T.C.C.	S.S.C.	D.S.C.	Maker†
0000–5	4.5	9.0	13.5	1.0	2.0	R.S.U.
6–7	4.0	8.0	12.0	1.0	2.0	"
8	3.5	7.0	10.5	1.0	2.0	"
9	3.0	6.0	9.0	1.0	2.0	"
10–12	2.5	5.0	7.5	1.0	2.0	"
13–19	2.25	4.5	7.75	1.0	2.0	R
13–32	2.5	4.5	7.0	1.0	2.0	S.U.
14–15	3.0	5.0	7.0	...	...	G.E.
16–18	2.5	4.0	...	...	...	"
19–22	2.0	4.0	...	...	...	"
23–25	2.0	4.0	...	1.5	3.0	"
20–40	2.0	4.0	6.0	1.0	2.0	R
26–28	2.0	4.0	...	1.5	3.0	G.E.
29–34	2.0	4.0	...	1.2	2.5	"
32–36	...	...	...	0.87	1.75	S.U.
35–40	2.0	4.0	...	1.0	2.0	G.E.

*Reprinted by permission from Standard Handbook.

†R=Roebling's Sons Company.

S.U.=Standard Underground Cable Company.

G.E.=General Electric Company.

Note:—Diameter overall = diameter bare + 2 (thickness of insulation).

TABLE 9

Diameter of Small Sizes of Magnet Wire*
(General Electric Co.)

Size	Diameter in Mils					
B & S	Bare	S.C.C.	D.C.C.	S.S.C.	D.S.C.	Enamel
14	64	70	64			67
15	57	63	67			60
16	51	56	59			53.5
17	45	50	53.0			47.5
18	40	45	48			42
19	36	40	44			37
20	32	36	40			34
21	28	32.5	36.5			30.5
22	25	29.4	33.4			27.5
23	23	26.5	30.5	26	29	25
24	20	24.1	28	23	26	22
25	18	22	26	21	24	20
26	16	20	24	19	22	17.5
27	14	18	22	17	20	15.5
28	12.6	16.6	20.6	15.6	18.6	14
29	11	15.3	19.3	14	17	12.3
30	10	14	18	12.5	15	11.3
31	9	13	17	11.4	13.9	10.2
32	8	11.9	15.9	10.5	13	9.2
33	7	11.0	15	9.5	12	8.2
34	6.3	10.3	14.3	8.8	11.3	7.3
35	5.6	9.6	13.6	7.6	9.6	6.8
36	5	8.5	12	7	9.0	6.2
38	4.5			6	8	5.2
40	4			5	7	4.2

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TABLE 10

Weights of Small Sizes of Magnet Wire*

(General Electric Co.)

Size	Weight in Pounds per 1000 Feet.				
B & S	S. C. C.	D. C. C.	S. S. C.	D S. C.	Enamel
14	12.684	12.918			12.684
15	10.082	10.274			10.053
16	8.012	8.176			7.973
17	6.375	6.510			6.322
18	5.081	5.188			5.009
19	4.043	4.130			3.966
20	3.215	3.289			3.136
21	2.569	2.628			2.475
22	2.055	2.106			1.970
23	1.630	1.676	1.57	1.604	1.555
24	1.297	1.344	1.241	1.298	1.232
25	1.036	1.082	.991	1.040	.980
26	.828	.873	.791	.833	.777
27	.661	.703	.631	.666	.616
28	.524	.562	.499	.521	.485
29	.421	.457	.397	.416	.384
30	.336	.372	.315	.332	.303
31	.271	.307	.254	.267	.242
32	.215	.248	.203	.214	.192
33	.174	.201	.161	.172	.152
34	.141	.161	.130	.140	.121
35	.12	.137	.110	.119	.101
36	.099	.112	.089	.096	.081
38			.058	.065	.051
40			.037	.040	.031

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TABLE 11

SPACE OCCUPIED BY MAGNET WIRES*

Turns per Inch					
B.&S.	S.C.C.	D.C.C.	S.S.C.	D.S.C.	Enamel
14	14	13			14
15	15	14			16
16	17	16			18
17	20	18			21
18	22	20			23
19	25	22			27
20	27	25			29
21	30	27			32
22	34	30			36
23	37	32	38	34	40
24	41	35	43	38	45
25	45	38	47	41	50
26	50	41	52	45	57
27	55	45	58	50	64
28	60	48	64	53	71
29	65	51	71	58	81
30	71	55	80	66	88
31	76	58	87	71	104
32	84	62	95	76	120
33	90	66	105	83	130
34	97	69	110	88	140
35	104	73	130	104	160
36	117	82	140	110	190
38			160	120	
40			200	140	230

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TABLE 12

Space Occupied by Magnet Wires *

Turns per Sq. In.					
B. & S.	S.C.C.	D.C.C.	S.S.C.	D.S.C.	Enamel
14	204	182			222
15	252	223			278
16	318	281			350
17	400	356			443
18	495	435			567
19	625	516			692
20	775	625			865
21	948	752			1070
22	1150	896			1320
23	1420	1070	1480	1190	1600
24	1720	1270	1890	1480	2060
25	2060	1480	2270	1740	2500
26	2500	1740	2770	2060	3260
27	3080	2060	3460	2500	4160
28	3620	2360	4100	2890	5100
29	4270	2680	5100	3460	6600
30	5100	3080	6410	4440	7830
31	5920	3460	7690	5180	9610
32	7070	3970	9090	5920	11800
33	8260	4440	11000	6940	14800
34	9440	4900	12900	7820	17700
35	10800	5400	17380	10800	21600
36	13000	6940	20400	12300	26000
38			27700	15600	37000
40			40000	20400	54000

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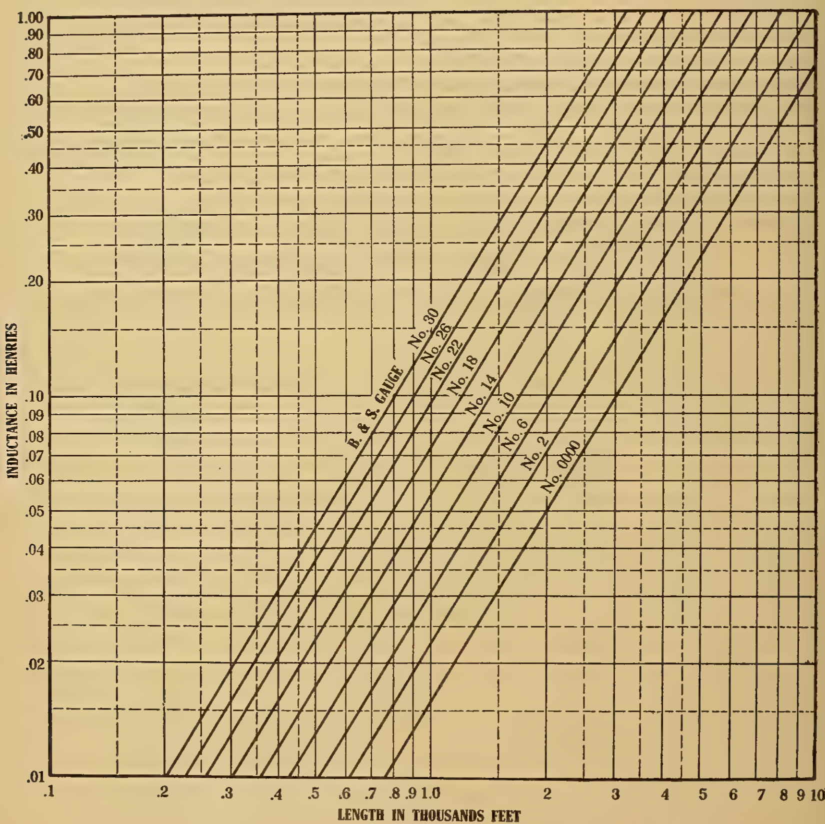


FIG. 11*a*

FIG. 11*a* AND 11*b* APPLY TO DOUBLE-COTTON-COVERED

Charts for Determining the Length of Magnet Wire When Wound into the Most Economical Shape for Producing a Given Inductance, L, in a

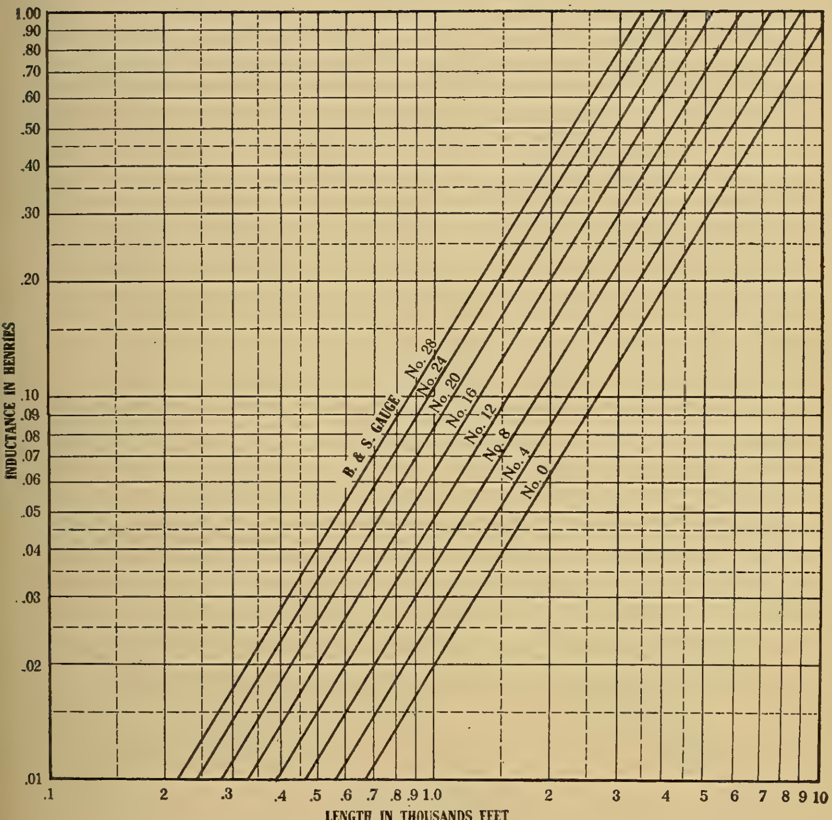


FIG. 11*b*

(D. C. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

Coil without Iron; or for Finding the Inductance in Henries of Any Length of Wire When Wound into the Prescribed Maximum-Inductance Shape. (See Section XX).

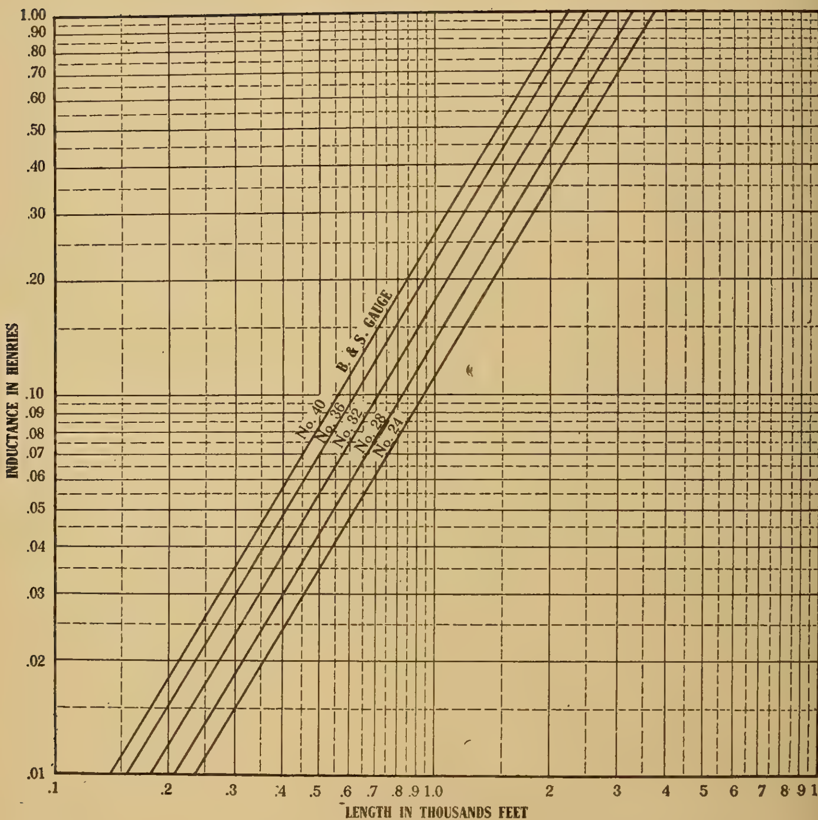


FIG. 12*a*

FIG. 12*a* AND 12*b* APPLY TO DOUBLE-SILK-COVERED

Charts for Determining the Length of Magnet Wire When Wound into the Most Economical Shape for Producing a Given Inductance, L, in a

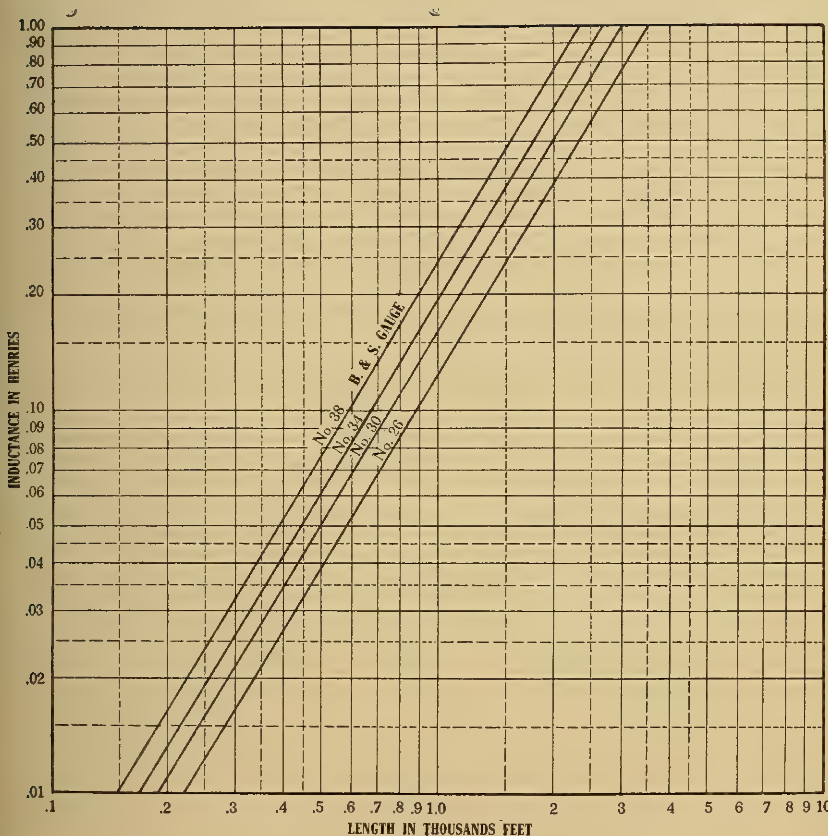


FIG. 12*b*

(D. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

Coil without Iron; or for Finding the Inductance in Henries of any Length of Wire When Wound into the Prescribed Maximum-Inductance Shape.

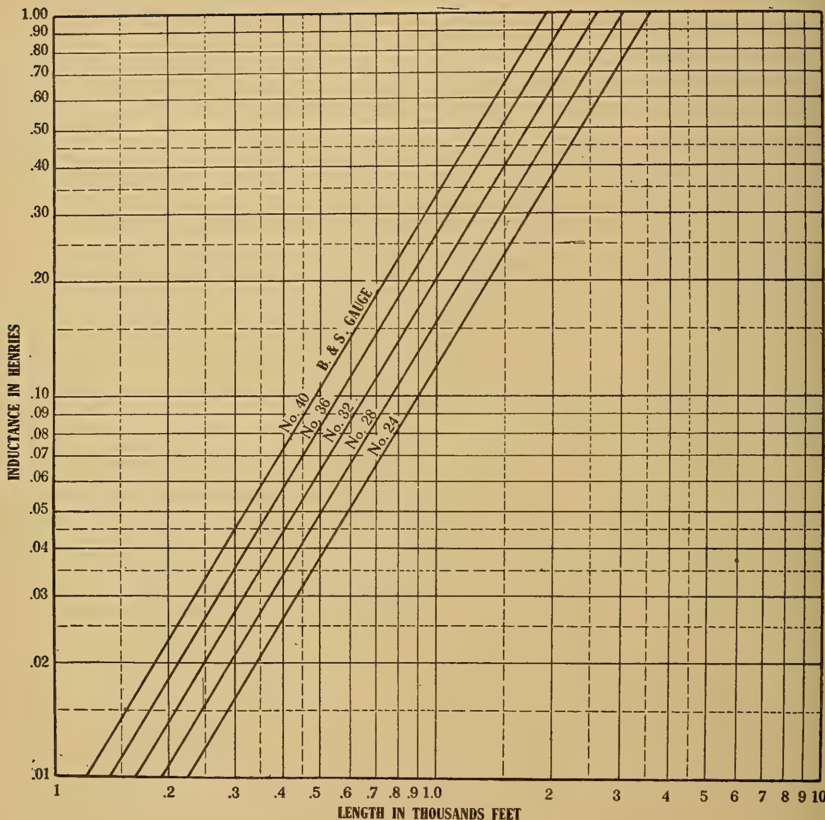


FIG. 13*a*

FIG. 13*a* AND 13*b* APPLY TO SINGLE-SILK-COVERED

Charts for Determining the Length of Magnet Wire When Wound into the Most Economical Shape for Producing a Given Inductance, L, in a

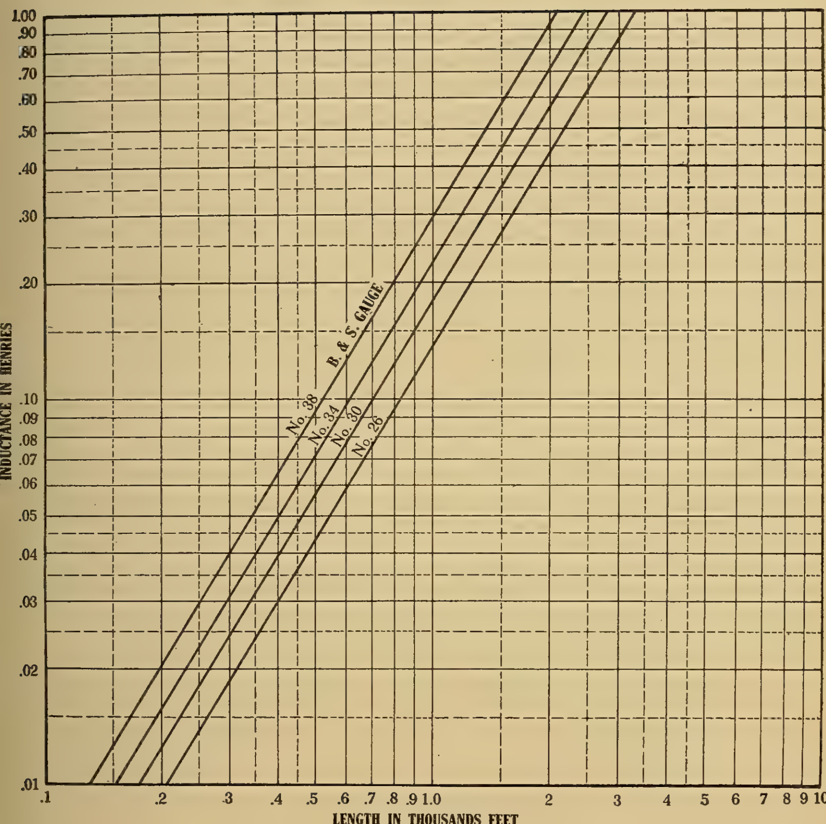


FIG. 13*b*

(S. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

Coil without Iron; or for Findng the Inductance in Henries of Any Length of Wire When Wound into the Prescribed Maximum-Inductance Shape.

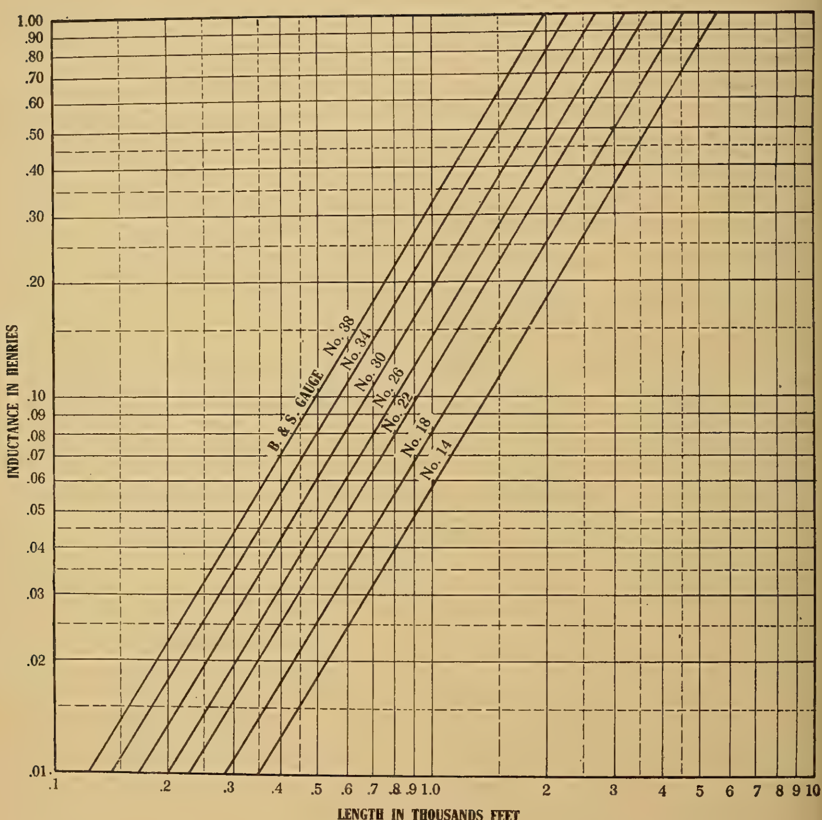


Fig. 14*a*

Fig. 14*a* and 14*b* Apply to Enamel-Covere

Charts for Determining the Length of Magnet Wire When Woun
into the Most Economical Shape for Producing a Given Inductance, *L*, in

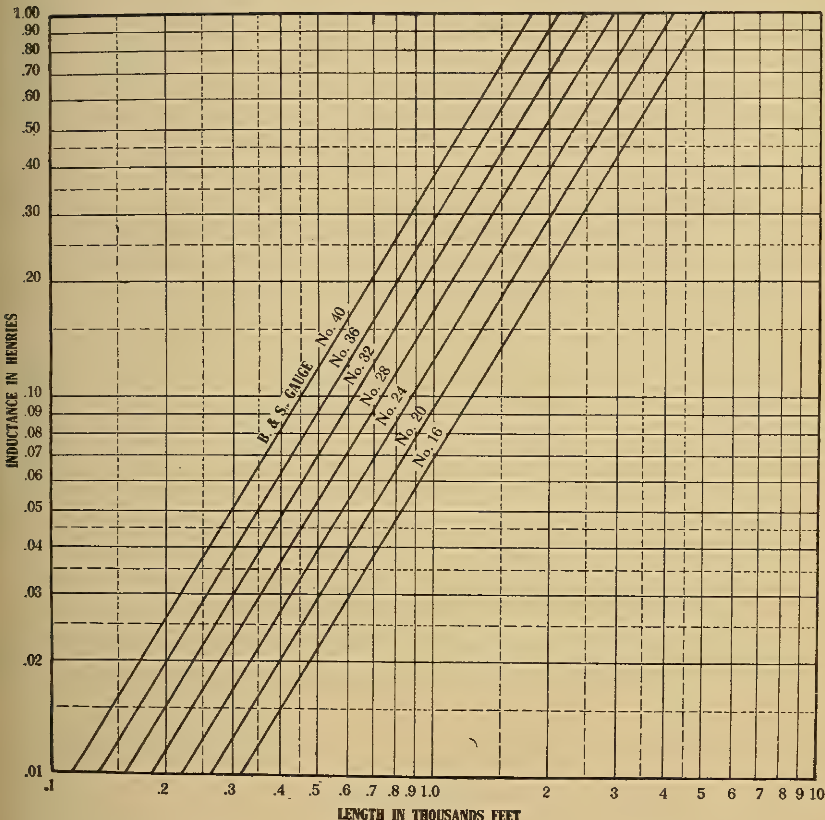


FIG. 14*b*

MAGNET WIRE OF EVEN GAUGE NUMBERS

Coil without Iron; or for Finding the Inductance in Henries of any Length of Wire When Wound into the Prescribed Maximum-Inductance Shape.

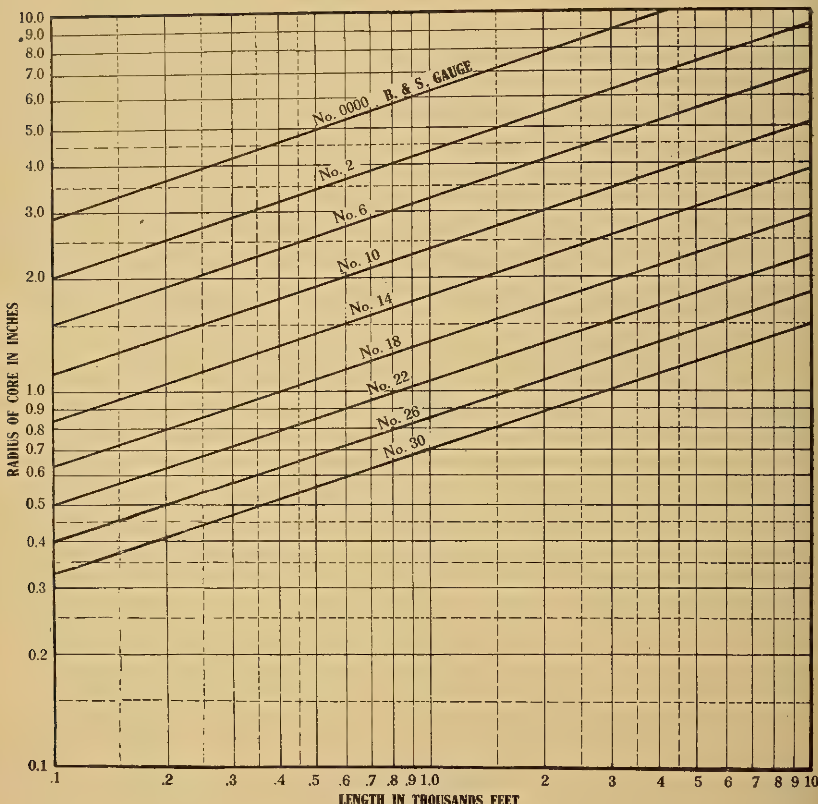


FIG. 15*a*

FIG. 15*a* AND 15*b* ARE CHARTS FOR INDICATING THE INNER RADIUS, *r*, OF COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE WOUND WITH

Other Coil Dimensions Are Then Proportional:

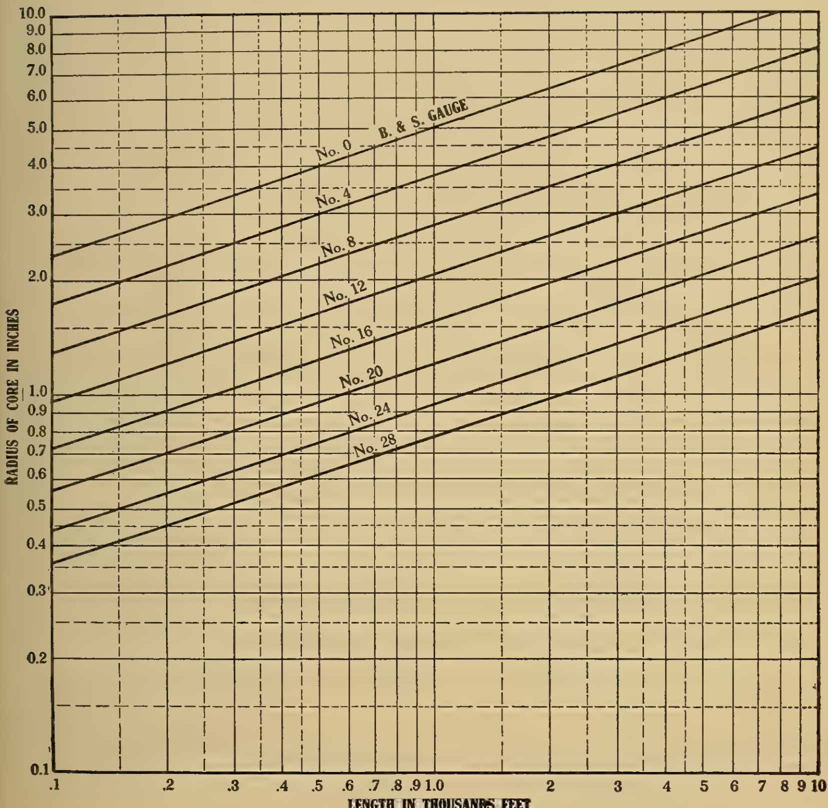


FIG. 15*b*

ANY LENGTH OF DOUBLE-COTTON-COVERED (D. C. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

$a = 1.5\ r$; $b = 1.2\ r$; $c = r$. (See section XXI.)

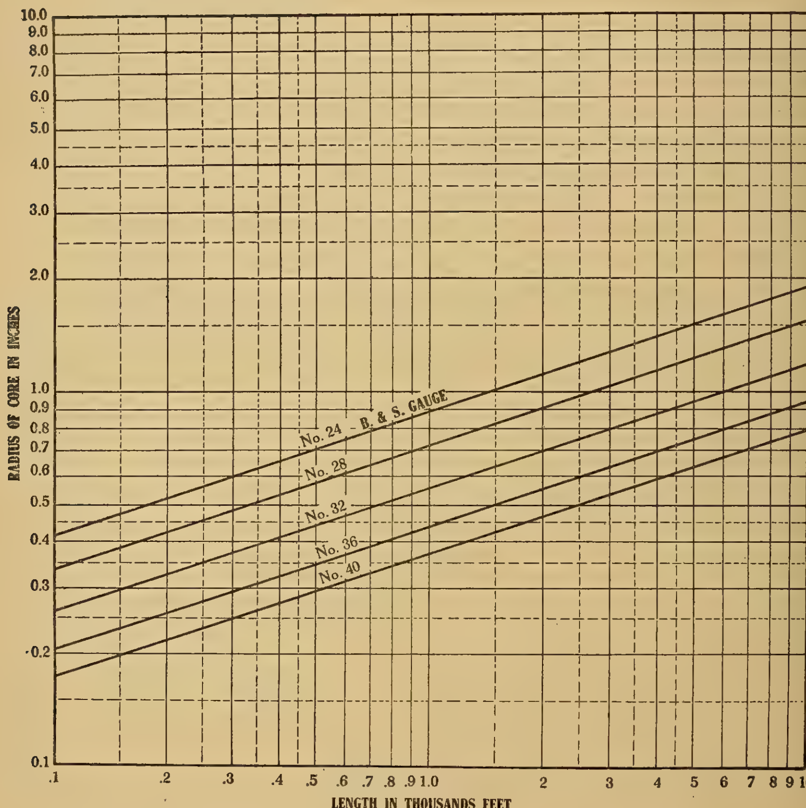


Fig. 16*a*

Fig. 16*a* and 16*b* are Charts for Indicating the Inner Radius, r, of Coils of Prescribed Maximum-Inductance Shape Wound with Any

Other Coil Dimensions Are Then Proportional:

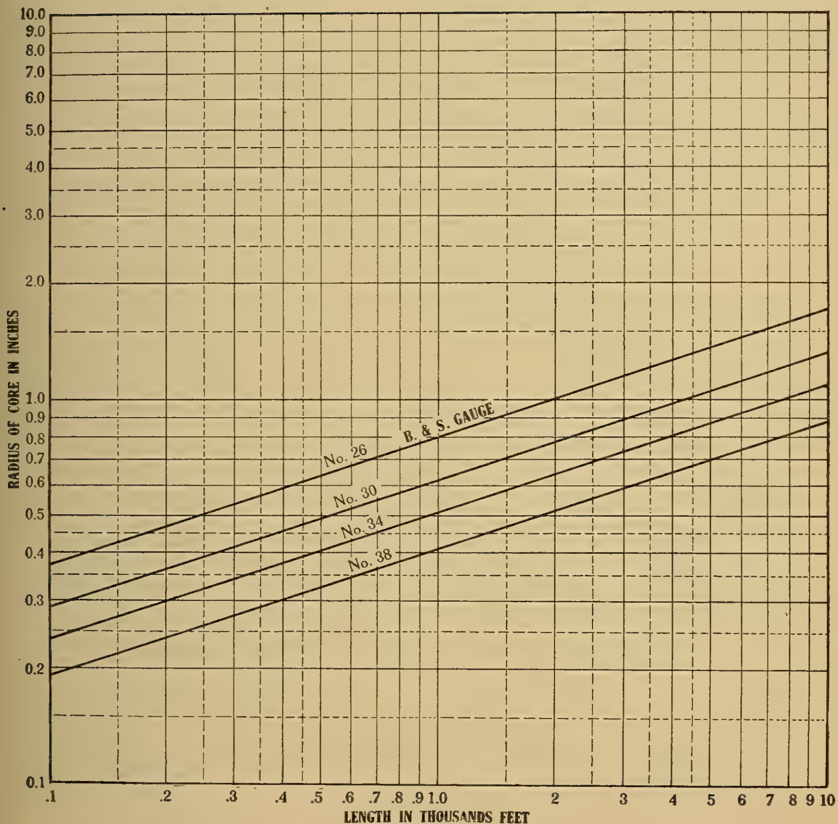


Fig. 16*b*

Length of Double-Silk-Covered (D. S. C.) Magnet Wire of Even Gauge Numbers

$a = 1.5\ r$; $b = 1.2\ r$; $c = r$. (See section XXI.)

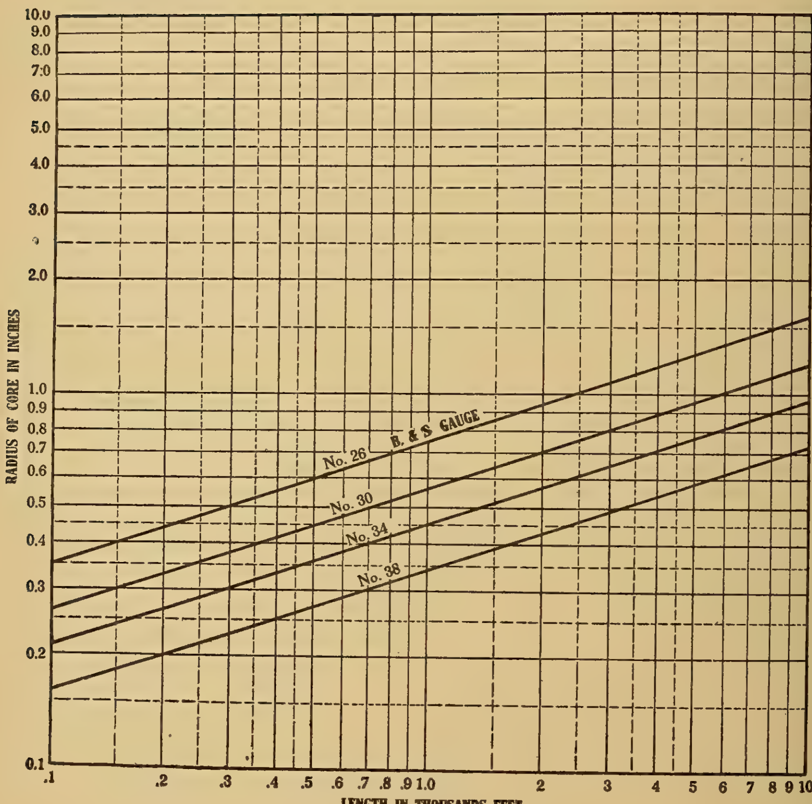


FIG. 17*a*

FIG. 17*a* AND 17*b* ARE CHARTS FOR INDICATING THE INNER RADIUS, *r*, OF COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE WOUND WITH ANY

Other Coil Dimensions Are Then Proportional:

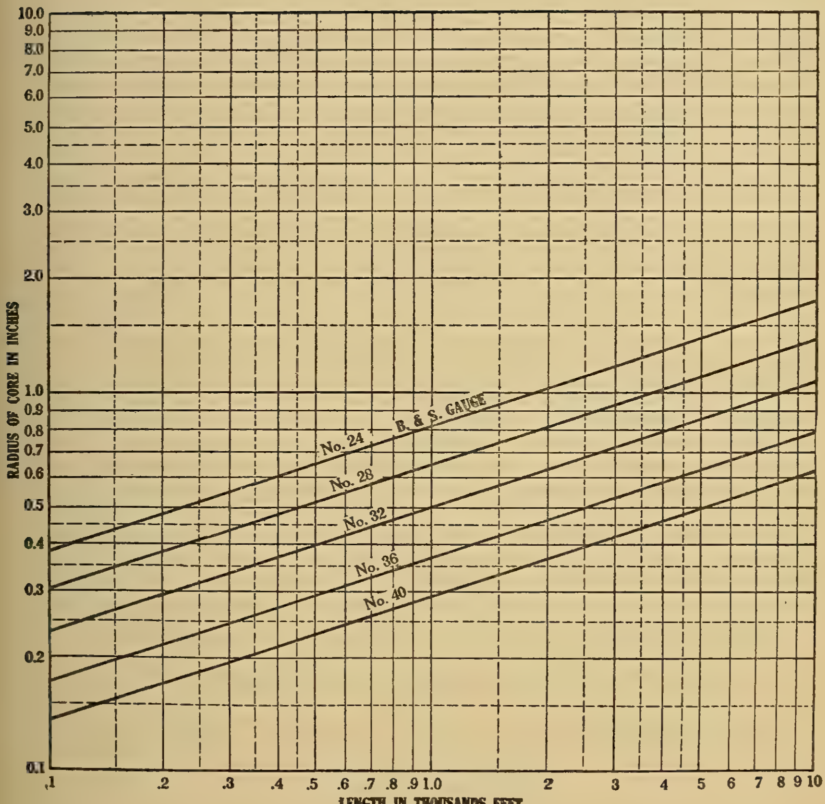


FIG. 17*b*

LENGTH OF SINGLE-SILK-COVERED (S. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

$a = 1.5\,r$; $b = 1.2\,r$; $c = r$. See section (XXI.)

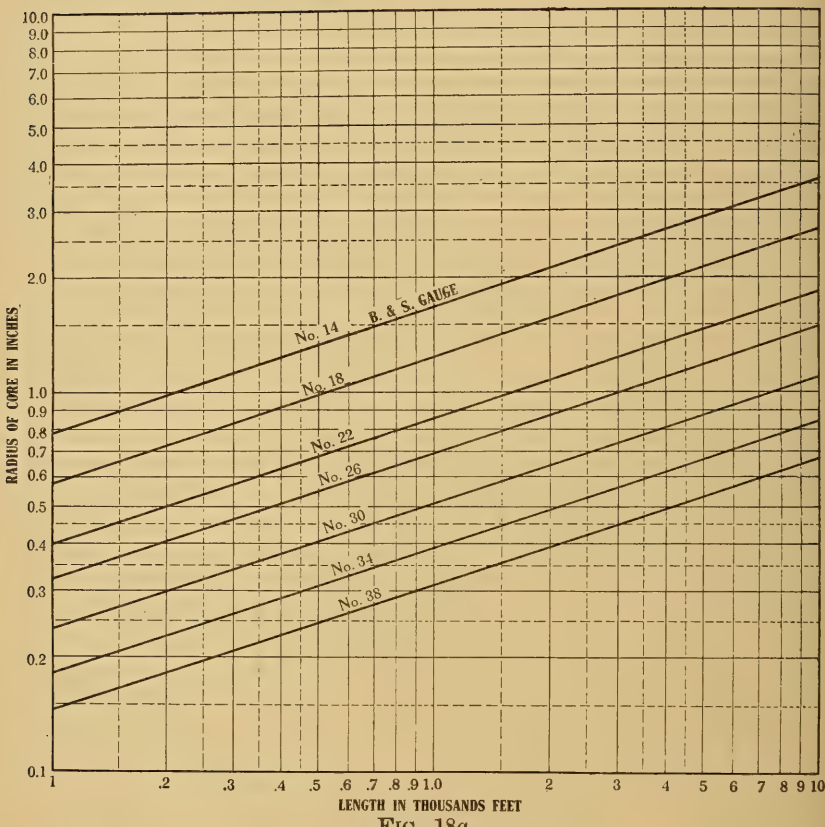


FIG. 18*a*

FIG. 18*a* AND 18*b* ARE CHARTS FOR INDICATING THE INNER RADIUS, *r*, OF COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE WOUND WITH ANY

Other Coil Dimensions Are Then Proportional:

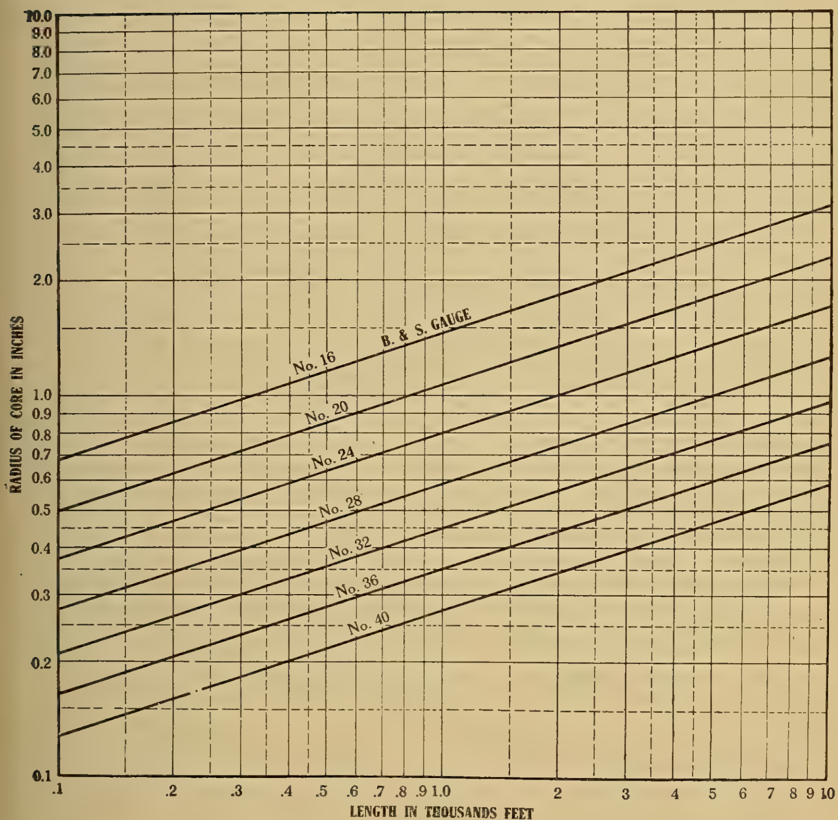


FIG. 18*b*

LENGTH OF ENAMEL-COVERED MAGNET WIRE OF EVEN GAUGE NUMBERS

$a = 1.5\,r$; $b = 1.2\,r$; $c = r$. (See section XXI.)

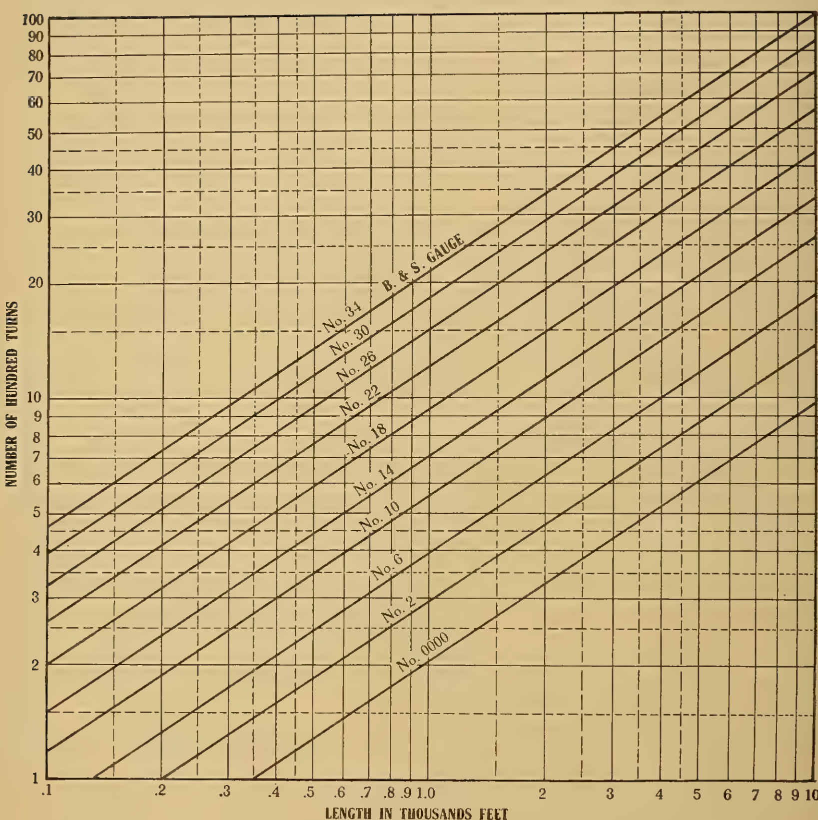


FIG. 19*a*

FIG. 19*a* AND 19*b* ARE CHARTS FOR INDICATING THE NUMBER OF HUNDREDS OF TURNS FOR COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE

Chart Values, Multiplied by 100, Give N′ in Formula (20).

The Indicated Number of Turns is Subject to Adjustment for Ob-

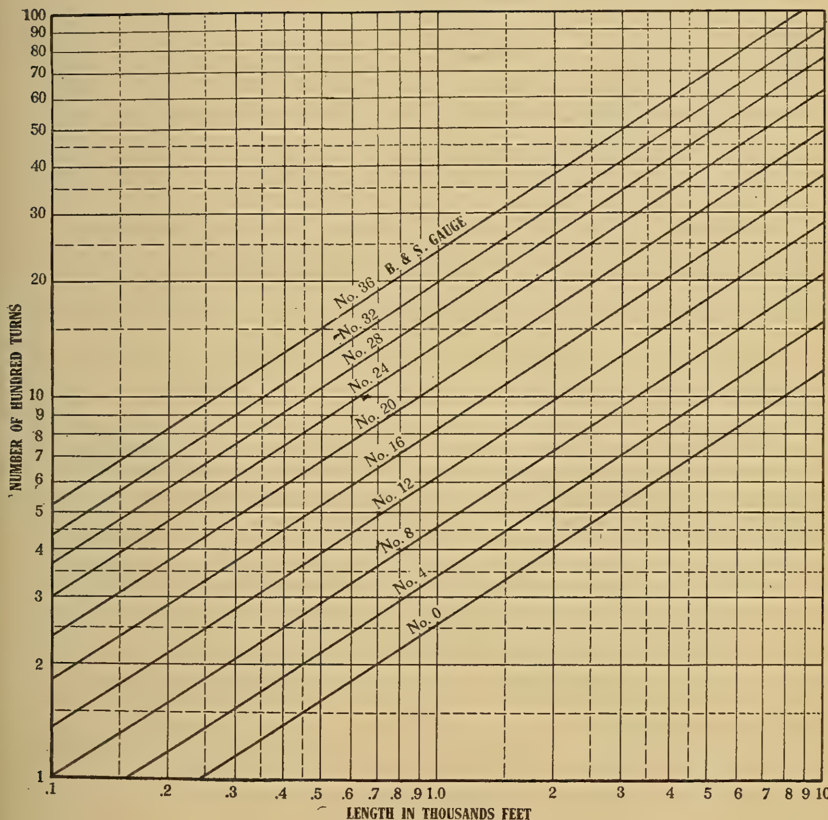


FIG. 19b

WOUND WITH ANY LENGTH OF DOUBLE-COTTON-COVERED (D.C. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

taining Complete Layers of the Winding, and Is Not Directly Serviceable for Coils of Other Shapes.

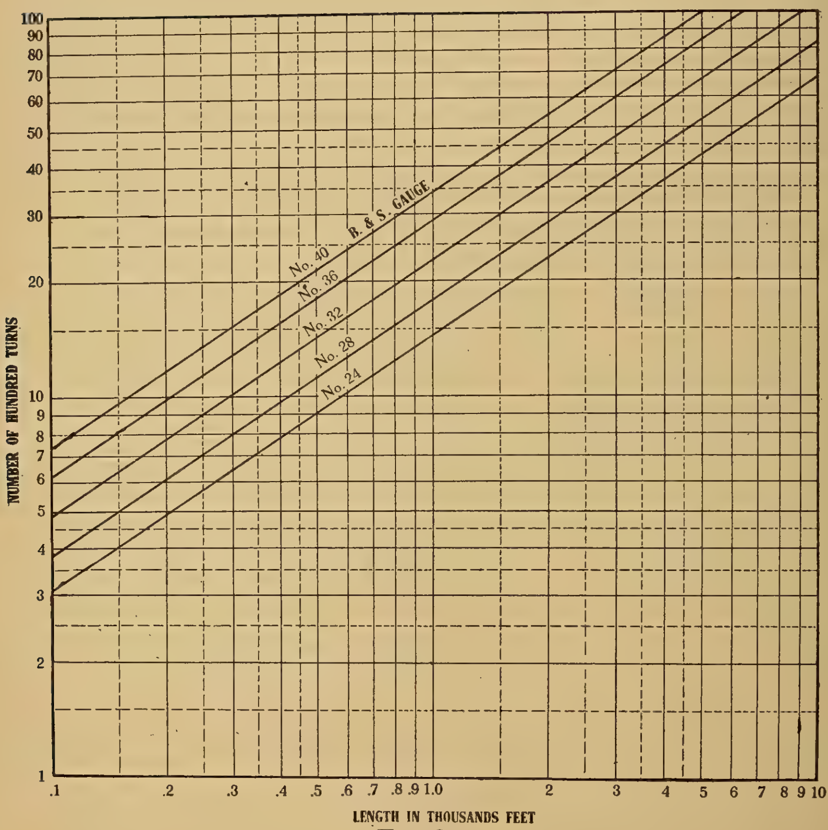


FIG. 20*a*

FIG. 20*a* AND 20*b* ARE CHARTS FOR INDICATING THE NUMBER OF HUNDREDS OF TURNS FOR COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE

Chart Values, Multiplied by 100, Give N' in Formula (20).
The Indicated Number of Turns Is Subject to Adjustment for Ob-

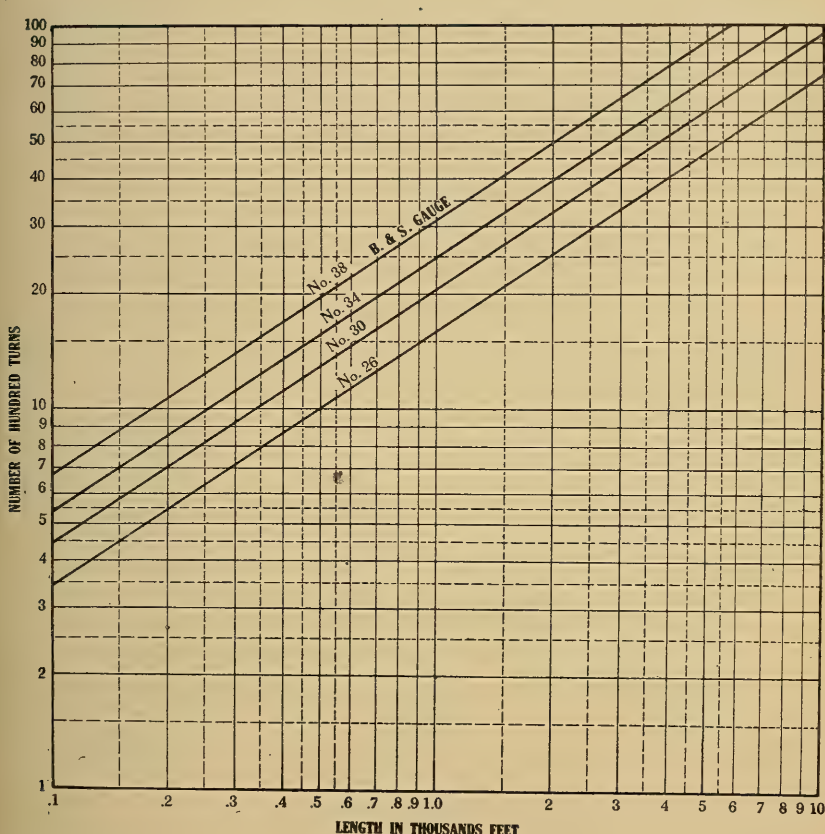


FIG. 20*b*

WOUND WITH ANY LENGTH OF DOUBLE-SILK-COVERED (D. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

taining Complete Layers of the Winding, and Is Not Directly Serviceable for Coils of Other Shapes.

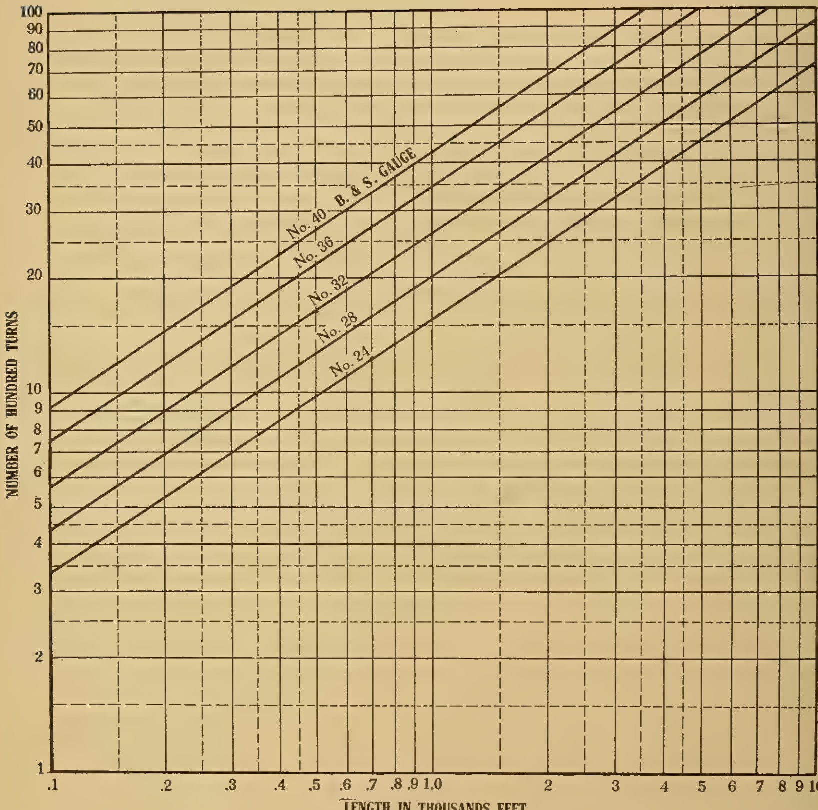


FIG. 21*a*

FIG. 21*a* AND 21*b* ARE CHARTS FOR INDICATING THE NUMBER OF HUNDREDS OF TURNS FOR COILS OF PRESCRIBED MAXIMUM-INDUCTANCE SHAPE

Chart Values, Multiplied by 100, give N′ in Formula (20).

The Indicated Number of Turns Is Subject to Adjustment for Ob-

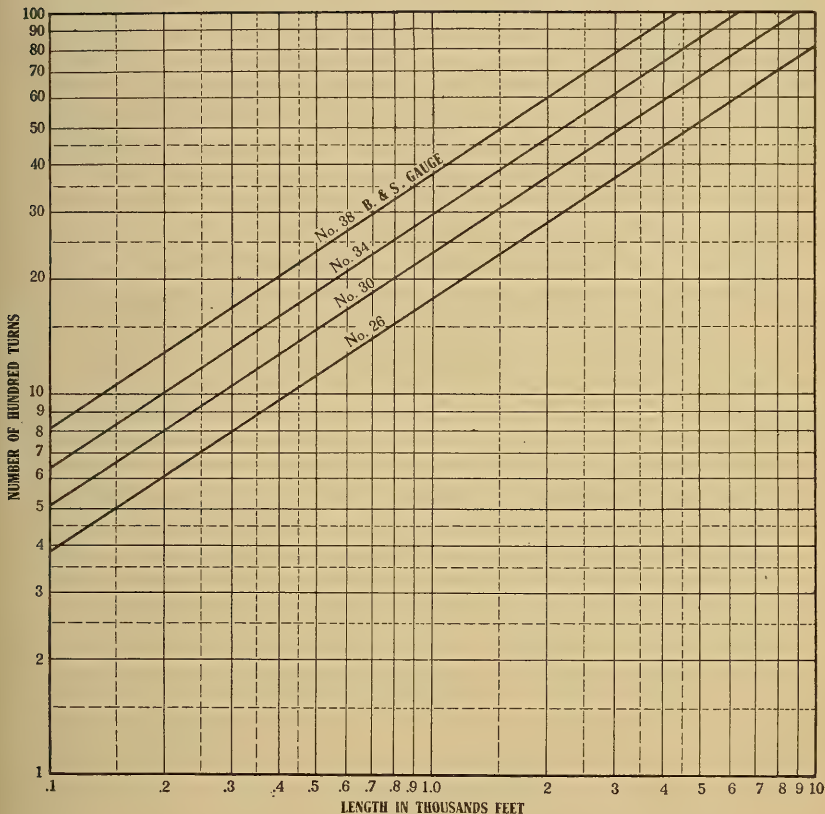


FIG. 21*b*

WOUND WITH ANY LENGTH OF SINGLE-SILK-COVERED (S. S. C.) MAGNET WIRE OF EVEN GAUGE NUMBERS

taining Complete Layers of the Winding, and Is Not Directly Serviceable for Coils of Other Shapes.

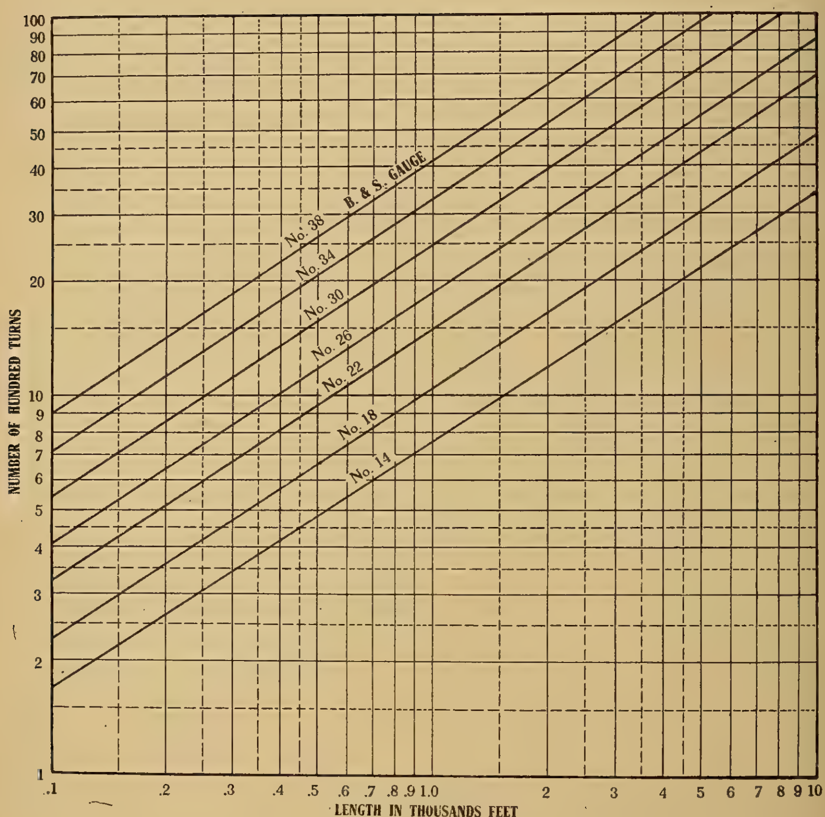


FIG. 22*a*

FIG. 22*a* AND 22*b* ARE CHARTS FOR INDICATING THE NUMBER OF HUNDREDS OF TURNS FOR COILS OF PRESCRIBED MAXIMUM–INDUCTANCE SHAPE

Chart Values, Multiplied by 100, Give N′ in Formula (20).

The Indicated Number of Turns Is Subject to Adjustment for Ob-

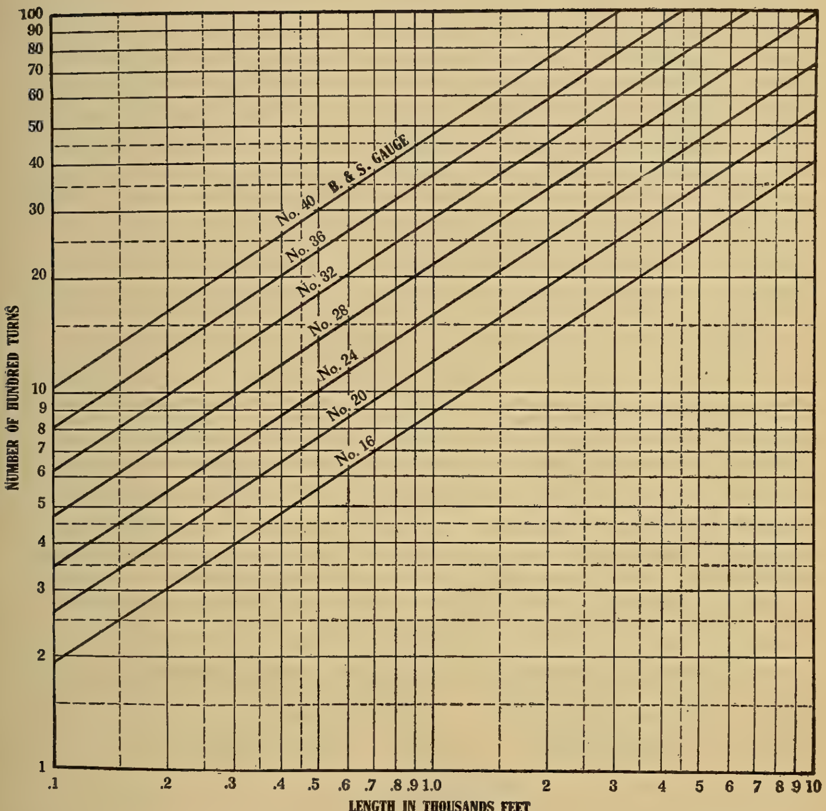


FIG. 22b

WOUND WITH ANY LENGTH OF ENAMEL-COVERED MAGNET WIRE OF EVEN GAUGE NUMBERS

taining Complete Layers of the Winding and Is Not Directly Serviceable for Coils of Other Shapes.

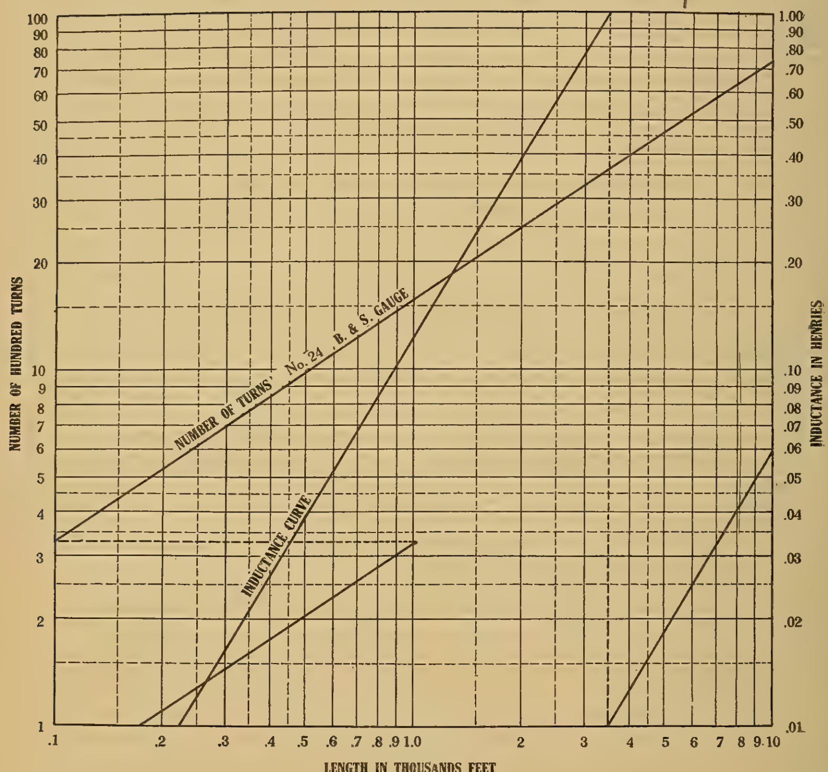

NUMBER OF HUNDRED TURNS
INDUCTANCE IN HENRIES
LENGTH IN THOUSANDS FEET
NUMBER OF TURNS No. 24 B. & S. GAUGE
INDUCTANCE CURVE
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
2
3
4
5
6
7
8
9
10

FIG. 23

FIG. 24

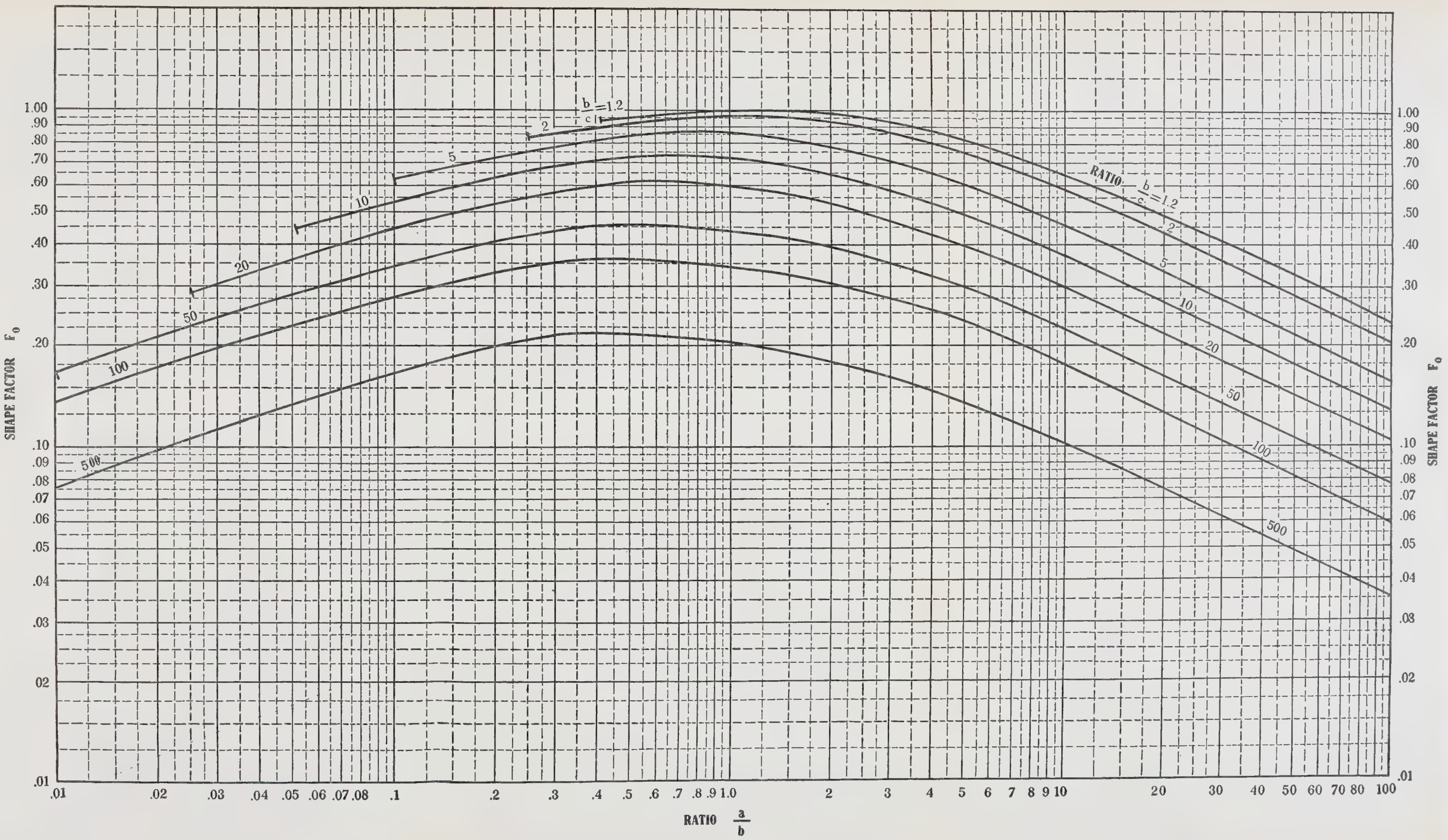
SHAPE-FACTOR CHART

SOLENOID TYPE

In coreless cylindrical coils, only those having the relative proportions, $a=1.5$, $b=1.2$, $c=1$, (see Fig. 5) give the maximum inductance, while other shapes employ the conductor less effectively.

Fig. 24 and 25 indicate the shape factor, or coefficient for reducing the maximum inductance possible from any conductor to its actual value when wound into other than the prescribed maximum shape.

Fig. 24 includes all cylindrical windings, whose axial length, b, exceeds radial thickness, c, mostly solenoids. Curves apply to specified values of the ratio, $\frac{b}{c}$, from 1.2 (which includes the maximum shape) to 500 for very thin shell windings. Points at left indicate long solenoids and thick tubes of relatively small radius; while points at the right give values for short tubes or bands of large radius. For ratios of $\frac{b}{c}$, not represented by curves, points may be interpolated. For given ratio of $\frac{a}{b}$ as abscissa, the desired factor is the ordinate of the curve or interpolated point for the given ratio of $\frac{b}{c}$. See section XXIII for illustrative examples.



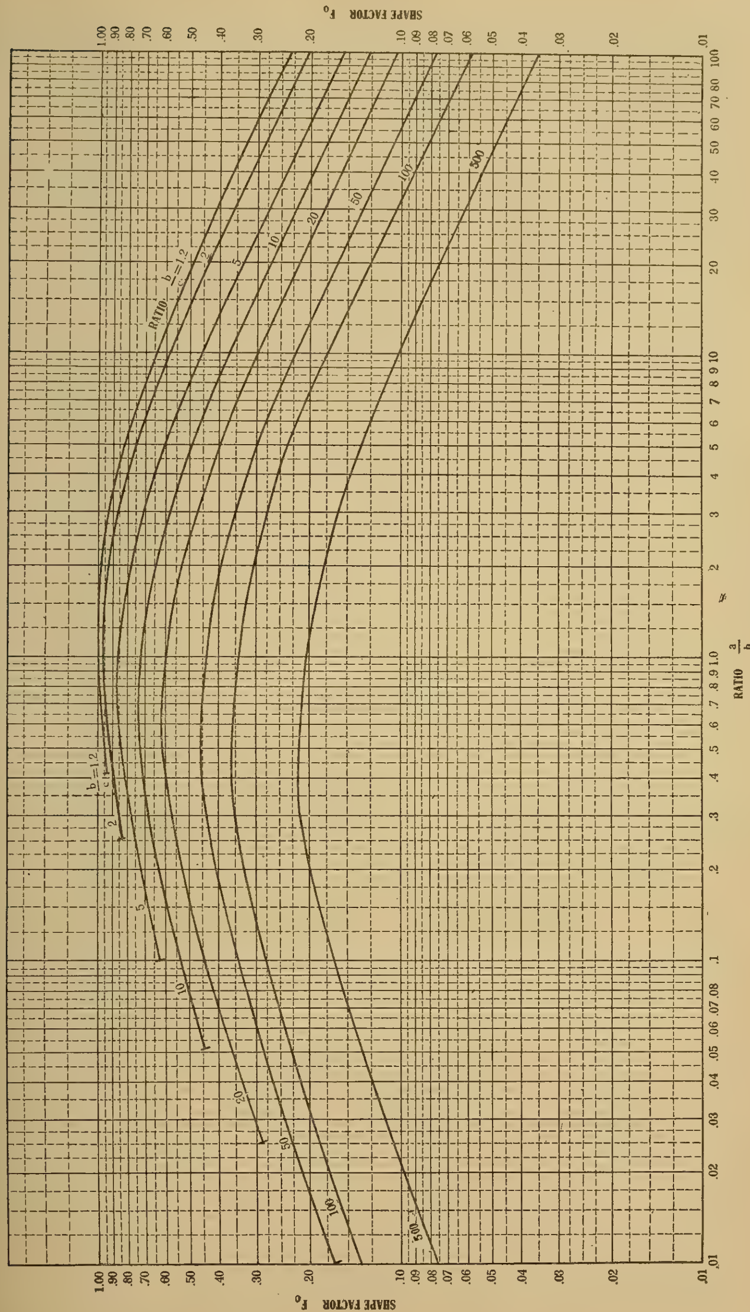


FIG. 24*a* See Fig. 24

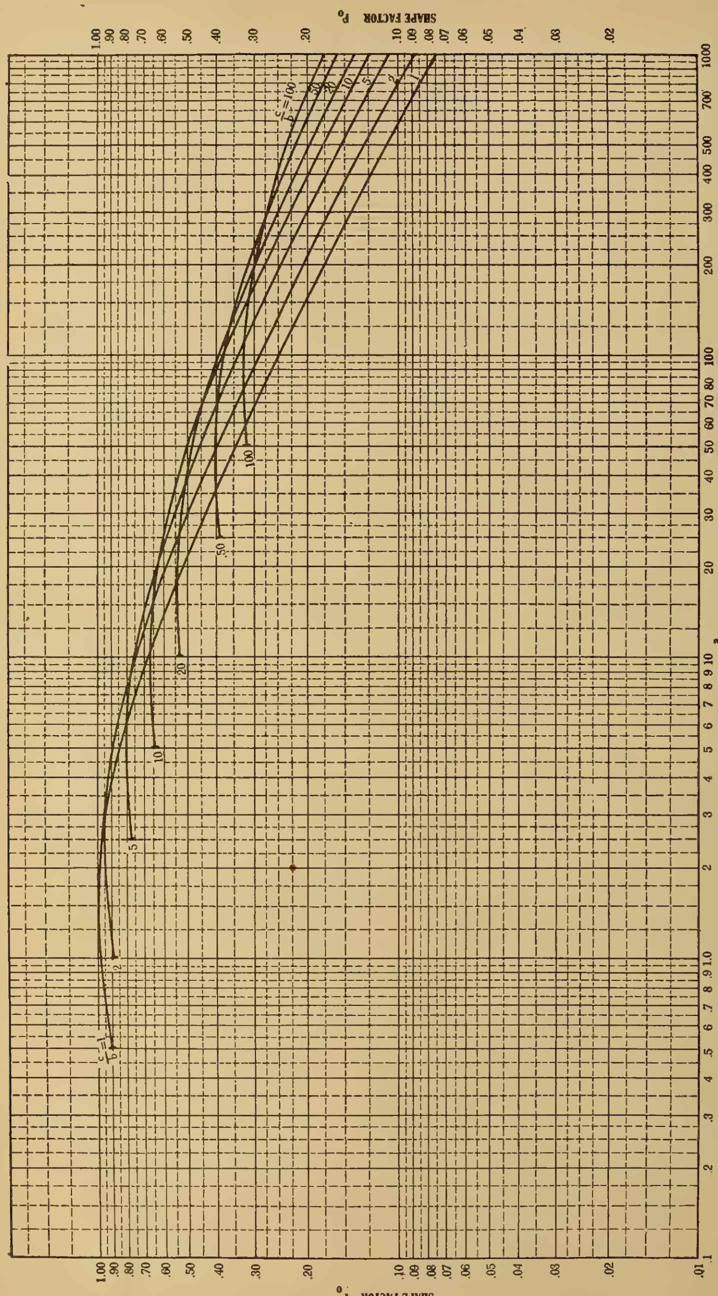


FIG. 25a See Fig. 25

Fig. 25

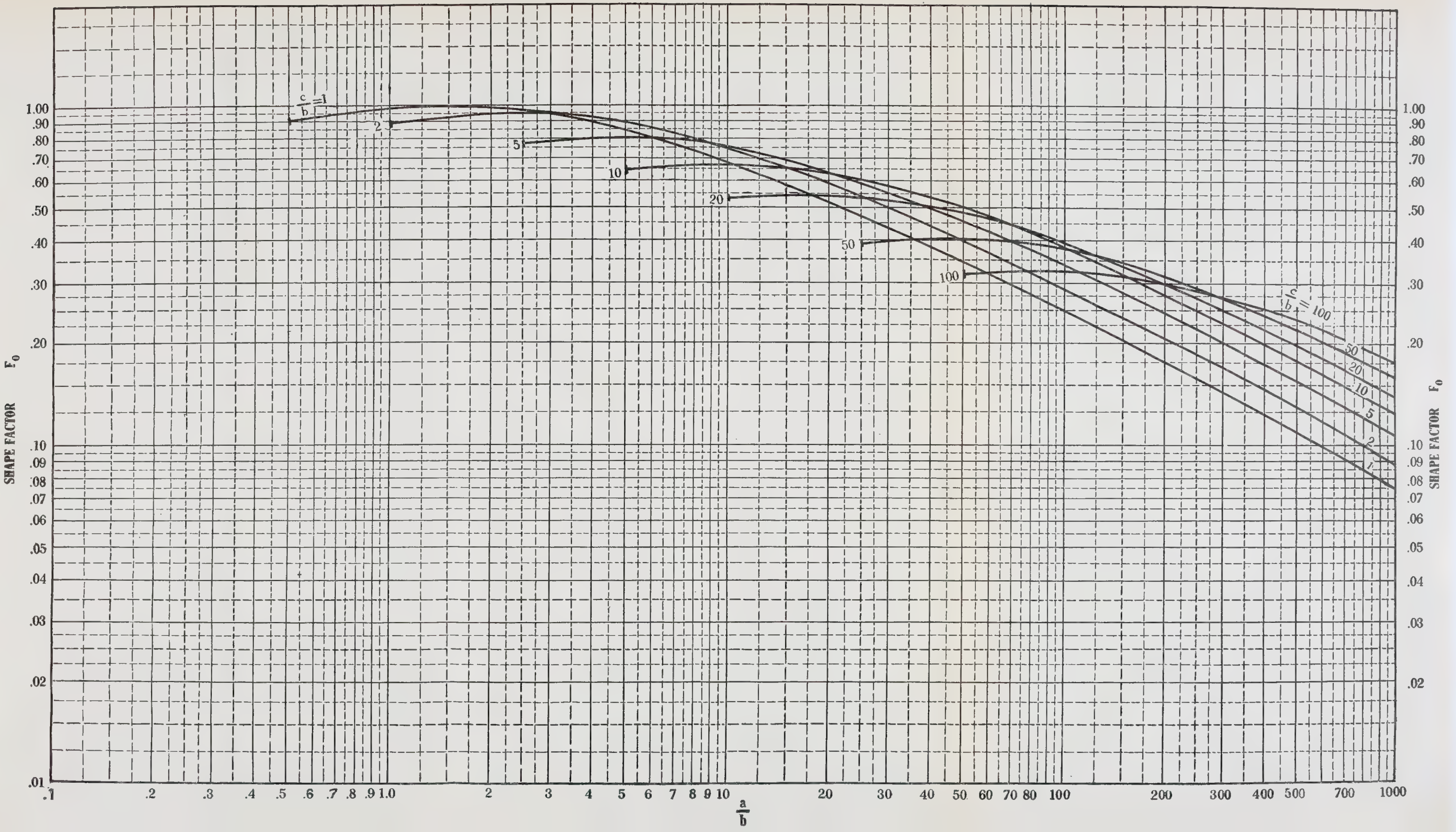
Shape-Factor Chart

Disk Type

In coreless cylindrical coils, only those having the relative proportions $a = 1.5$, $b = 1.2$, and $c = 1$, (see Fig. 5) give the maximum inductance, while other shapes employ the conductor less effectively.

Fig. 24 and 25 indicate the shape factor, or coefficient for reducing the maximum inductance possible from any conductor to its actual value when wound into other than the prescribed maximum shape.

Fig. 25 includes all cylindrical windings, whose radial thickness, c, equals or exceeds axial length, b, mostly rings or "pancake" coils. Curves apply to integral values of $\frac{c}{b}$ from unity for single turns and rings of square section, to 500 for very flat washer-shaped coils. Points at left indicate relatively small radius, or coils of little or no hole; at right indicate large radius and usually large hole. For ratios $\frac{c}{b}$ not represented by curves, points may be interpolated. For given ratio of $\frac{a}{b}$ as abscissa, the desired factor is the ordinate of the curve or interpolated point for the given ratio of $\frac{c}{b}$. See section XXIII for illustrative examples.



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This action, so far as it concerns the bulletins of the Engineering Experiment Station, has for its purpose a threefold object:

(1) To provide a greater degree of control in the distribution of bulletins.

(2) To make possible the establishment and maintenance of a trade circulation through the regular publishing houses.

(3) To regulate the distribution of the reserve or "out-of-print" supply.

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(2) EACH BULLETIN, HOWEVER, WILL STILL BE SUBJECT TO A FREE INITIAL DISTRIBUTION, AS HERETOFORE, ON THE BASIS OF EXISTING MAILING LISTS.

It will also be placed on sale with authorized agencies, both in this country and abroad.

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W. F. M. GOSS
Director.

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BULLETIN NO. 54

MECHANICAL STRESSES IN TRANSMISSION LINES

BY

A. GUELL

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

URBANA, ILLINOIS
PUBLISHED BY THE UNIVERSITY

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 54 JANUARY 1912

MECHANICAL STRESSES IN TRANSMISSION LINES

BY A. GUELL, RESEARCH FELLOW, ELECTRICAL ENGINEERING DEPARTMENT

CONTENTS

MECHANICAL STRESSES IN TRANSMISSION LINES

I. Introduction

1. *Preliminary.*—Owing to the fact that the mechanical stresses in transmission lines vary within unusually wide limits, many electrical engineers have overlooked the fact that otherwise well-designed and expensive electric power stations have been seriously crippled because the transmission line, though electrically strong, was mechanically weak. There are instances where it would be a commercial impossibility to design a transmission line that would withstand the local combined maximum wind pressure, ice load, and temperature range. If a telegraph or telephone line should be overturned by an exceptionally severe storm, accompanied by sleet on the wires, comparatively little inconvenience would result, owing to the fact that these small wires, though frequently numerous, carry harmless currents. The percentage of modern telegraph and telephone lines yearly destroyed is fairly small; and, for this reason, to build all the lines strong enough to resist the most exceptional wind pressure would be more expensive than to replace the few lines destroyed. Let a high-tension transmission line be overturned, however, and more serious results will follow. The transmission line is often the most expensive part of an electric power development; therefore, it should be built strong enough to resist the exceptionally violent local storms, and no expense should be spared to insure safety to life.

2. *Object of Bulletin.*—Transmission lines are, in general, subjected to stresses due to the following causes: (1) dead weight; (2) wind pressure; (3) changes of temperature; (4) tensional unbalancing. It is the object of this bulletin to discuss the first three of the above stresses.

3. *Acknowledgments.*—Acknowledgment is hereby made to Messrs. A. J. Bowie, Jr., (Electrical World, Vol. 48) and to J. Mayer

(Engineering News, Vol. 35) for the use of material from articles which appeared in the above mentioned magazines.

4. *Notation.*—The following notation will be used.

A = cross-sectional area of wire.
α = coefficient of linear expansion per degree F.
c = tension constant at lower temperature.
c_1 = tension constant at higher temperature.
E = modulus of elasticity of wire.
e = base of natural logarithms.
f = deflection at lower temperature.
f_1 = deflection at higher temperature.
ϕ = angle.
θ = angle.
H = tension at lowest point of wire (lower temperature).
H_1 = tension at lowest point of wire (higher temperature).
l = length of span.
L = length of wire at lower temperature.
L_1 = length of wire at higher temperature.
M,N = constants.
s = half length of catenary at lower temperature.
s_1 = half length of catenary at higher temperature.
T = tension at insulator at lower temperature.
T_1 = tension at insulator at higher temperature.
τ = temperature range.
u = constant.
w = resultant weight at lower temperature.
w_1 = resultant weight at higher temperature.
x = half length of span.
y = any ordinate; when maximum = f.

II. Stresses Due to Dead Weight

5. *Classification of Dead Weight Stresses.*—The dead weight consists of: (1) weight of poles or towers; (2) weight of one span of wire; (3) weight of snow or ice-coating, if any; (4) weight of foundation, if any.

Stresses (1) and (4) are, as a rule, easily taken care of by the bearing soil or sub-foundation, as they act vertically downwards, or nearly so. Stresses (2) and (3) have both a vertical and a horizontal component, but the horizontal component is the more important. Under

the heading, The Tension of Aerial Lines, it will be shown how great this horizontal component may become in extra long spans.

6. *Weight of Snow or Ice Coating.*—Snow or ice load becomes the more important, the smaller the diameter of the wire. This may easily be seen by considering two wires of the same length, but of different diameters. Assuming a coat of ice ½ in. thick, the increase in weight per unit length, on a wire 0.102 in. in diameter (10 B & S gauge) would be

$$\frac{\pi i}{4}[(1+0.102)^2-(0.102)^2]=1.204\ \frac{\pi i}{4}\ \ldots\ldots(1)$$

i = weight of ice in pounds per cubic inch

The increase in weight on a wire 0.409 in. in diameter (3-O B & S) would be

$$\frac{\pi i}{4}[(1+0.409)^2-(0.409)^2]=1.818\frac{\pi i}{4}\ldots\ldots\ldots(2)$$

The ratio of diameters is 1:4, but the ratio of increased load is 1:1.5, as may be seen from equations 1 and 2.

7. *Weight of Foundation.*—Any foundation may be regarded as having two components: (1) the bearing soil or sub-foundation; (2) the foundation proper, consisting of the materials forming a solid base for the superstructure. As geological conditions vary widely in different localities, an approximate knowledge of the characteristics of the soil under consideration is of importance.

(a) Bearing Soil.—Sometimes the bearing power of the soil must necessarily be increased. This can be accomplished by means of piles. Trautwine gives the following formula for determining the maximum load that a pile will stand:

$$W=\frac{0.023\sqrt[3]{H}\,w}{h+1}$$

where

W = ultimate load in tons of 2000 pounds;
H = fall in feet;
w = weight of hammer in pounds;
h = last sinking in inches.

This formula must be used with a factor of safety of from 1 to 10, depending upon local conditions.

Table 1 shows the safe bearing values of ordinary soils.

TABLE 1

SAFE BEARING VALUES OF SOILS

Kind of Material	Bearing Values tons per sq. ft. Maximum	Minimum
Rock, the hardest, in thick layers in native bed		200
Rock, the softest, easily worn by water or exposure to the weather		18
Clay, in thick beds, always dry	6	4
Clay, in thick beds, moderately dry	4	2
Clay, soft beds	2	1
Gravel and coarse sand, well cemented	10	8
Sand, compact and well cemented	6	4
Sand, clean and dry	4	2
Quicksand and alluvial soils	1	0.5

Table 2, compiled from various authorities, gives the crushing strengths and moduli of rupture of the principal materials used for foundations.

TABLE 2

CRUSHING STRENGTHS AND MODULI OF RUPTURE

Kind of Material	Moduli of Rupture lb. per sq. in.	Crushing Strengths lb. per sq. in.
Granite	1800	10400
Limestone, common varieties	1500	8670
Oolitic limestone	2338	6700
Sandstone (brownstone)	2160	8300
Concrete, 1 month, 1 part portland cement, 2 parts sand, and 4 parts broken stone	250	1500
Brick laid in portland cement, 1 to 2 mortar		2900
Brick laid in Rosedale cement, 1 to 2 mortar		1700

(b) *The Foundation.*—Stresses in foundations are due to two causes: (1) dead weight of poles or towers, plus weight of one span of wire, together with any snow or ice coating: (2) stresses due to wind pressure; tensional unbalancing caused by spans of different lengths, breaking of one, two, or all of the wires, changes in the direction of the line, changes of level, etc.

The total dead weight divided by the area of the foundation, plus the unit load produced by the maximum wind pressure, (see Stresses Due to Wind Pressure), plus a factor representing the unit load produced by tensional unbalancing, are represented by the following formula:

$$L = \frac{W}{A} + \frac{Ml}{2I} + U \qquad (3)$$

where

L = load on sub-foundation per unit area;
W = total dead weight;
A = area of foundation;
M = moment of the wind;
I = moment of inertia of the area of the base about a gravity axis perpendicular to the direction of wind;
l = distance of most remote point of base from this axis;
U = factor representing unit load produced by tensional unbalancing.

Considering a common case, in which the towers rest on square bases, the direction of the wind is at right angles to the line, and the line is straight, there is obtained for the maximum intensity of pressure on the footing:

$$L = \frac{W}{A} + \frac{l}{2I'}\sqrt{M^2 + N^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which:

l = length of one side of square base;
I' = moment of inertia of base about a gravity axis parallel to one side;
N = moment due to tensional unbalancing.

and the other symbols have the same meaning as above.

Sometimes it is found necessary to extend the base of a foundation in order not to overload the bearing soil. To show how to determine the safe projection of this base, a simple case will be considered. (Fig. 1).

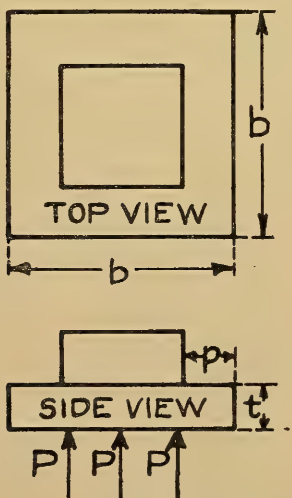

Fig. 1

Let P = the pressure in lb. per sq. in. at the bottom of the foundation;
R = the modulus of rupture of the material of the foundation in lb. per sq. in;
p = the projection in inches;
t = the thickness of the base in inches;
b = the breadth of the base in inches;

The projecting part, p, of the base may be considered as a cantilever beam uniformly loaded. Therefore, the bending moment of this beam may be represented by the formula

$$Ppb\frac{p}{2} = \frac{1}{6}Rbt^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Simplifying

$$p = t\sqrt{\frac{R}{3P}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

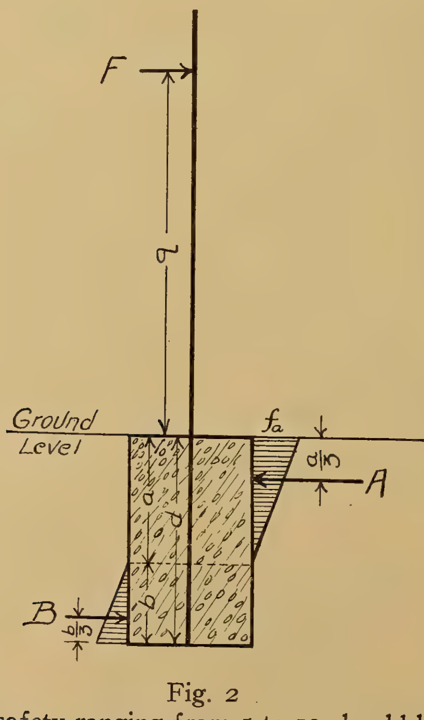

Fig. 2

A factor of safety ranging from 5 to 10 should be used in connection with this formula (6).

Fig. 1 may also be taken as representing one of the four separate masses of concrete needed, when the four posts of a tower are placed so far apart that it would be uneconomical to use a single concrete block to support the whole structure. It could also represent the slightly different case (on account of the earth on top) when the four blocks rest on a platform usually made of reinforced concrete.

The stresses in a foundation and the accompanying lateral earth pressures which are set up when the tower is subjected to lateral pressures will now be considered. Fig. 2 represents a longitudinal section of a prismatic concrete block of depth d which supports a tower which is subjected to lateral pressures. The longitudinal section of the block is taken in a plane parallel to the resultant lateral pressure F which acts at a distance q above the ground level. Neglecting friction of ground against the sides of the concrete block, the overturning moment of F is resisted by earth pressures having resultants A and B on opposite sides of the block. The intensity of earth pressure is assumed to vary in a straight line relation from zero at some section along the block to a maximum in one direction at the ground level (f_a), and to a maximum in the other direction at the bottom of the block. If the block is of uniform cross-section, the point of application of A will be one-third the distance from the ground level to the section of zero lateral pressure; similarly, the point of application of B will be one-third the distance from the bottom of the block to the section of zero lateral pressure. Using the notation of Fig. 2, we obtain the following equations for equilibrium. Taking moments about B,

$$F\left(q+d-\frac{b}{3}\right)=\frac{2Ad}{3},$$

$$B+F=A$$

also

$$\frac{A}{B}=\frac{a}{b}$$

and

$$a+b=d$$

From the above equations,

$$A=\tfrac{7}{8}F+\tfrac{3}{4}\,\frac{Fq}{d}\pm\frac{F}{8}\sqrt{17+36\frac{q}{d}+36\frac{q^2}{d^2}}$$

and

$$a=\frac{Ad}{2A-F}$$

From the straight line relation of the lateral earth pressures,

$$f_a=\frac{2A}{a\,w},$$

in which w is the width of the block perpendicular to the line of action of F. f_a should not exceed the safe bearing pressure for the soil around the block. The maximum bending moment on the block is

$$\frac{2Bd}{3} = \frac{2(A-F)d}{3}$$

and the maximum fiber stress due to bending equals this maximum moment divided by the section modulus of a cross-section of the block about an axis perpendicular to the line of action of F. This fiber stress should not be greater than about one-sixth the strength of the material of the block in cross-bending.

III. Stresses Due to Wind Pressure

8. *Lateral Stresses.*—The chief lateral stresses to which transmission line poles or towers are subjected are due to wind pressure acting on the wires, and on the poles or towers themselves; (the pressure on cross-arms and insulators may ususally be neglected).

The usual assumed wind pressures in England, Germany, Switzerland and the United States are: 56, 26, 21, and 30 lb. per sq. ft. of exposed area. These pressures are for wind acting normally to a flat surface. The effective area offered by a cylinder to the pressure of wind is theoretically two-thirds of the diameter multiplied by the length. In Switzerland, 0.7 of the diameter times the length is allowed; in England, 0.6.

9. *Wind Pressure on Flat Surfaces.*—Air pressure, especially on curved surfaces, is a very complex quantity, and, so far as the writer is aware, no exact mathematical treatment of the subject has been given. This is due to the fact that when a body immersed in a fluid is moving in the latter (or vice versa), there will generally exist, at every point of the surface of the body, a pressure different from that which would have existed without this movement. When there is not such motion, the resultant of all the elementary forces due to the static pressure of this fluid is equal to zero. If the body is in motion, this can easily be understood, remembering that the pressure at any point can be regarded as composed of the pressure due to the direct stream of air flowing normal to the surface in question, and the reflected air streams from other points of the surface near by.

Drs. Finzi and Soldati* give an account of a most interesting series of experiments which they performed in determining air pressures. The series of tests includes the determination of the normal pressure on planes, cylinders, spheres, and bodies of other shapes. The method of determination consisted in the use of a tube connected to

*Engineering (London) March 17, April 14, 1905.

small holes, normal to the surfaces, the other end of the tube being connected to a pressure gauge. Thus the pressure over the entire surface could be observed by point measurements, the pressure at any point being of course normal to the surface.

Previous experiments on wind pressure had been made by measuring the pressure as a whole (dynamometric method) without making any determination as to the distribution over the various regions of the body, and this very fact explains why eminent authorities obtained radically different results. But the experiments of Drs. Finzi and Soldati have proved conclusively that the wind pressure on any surface was not evenly distributed, and that, in all cases, the pressures vary directly with the first power of the velocity heads. In the consideration of flat surfaces normal to the wind, the pressure may be regarded as composed of two parts: (1) front pressure, (2) back pressure. The front pressure is greatest at the center of the figure, where its highest value is equal to that due to the velocity head. It falls off towards the edges. The gross front pressure for a circle is 75 per cent of that due to the velocity head; for a square, it is 70 per cent·, and for a rectangle whose length is very long compared with its width it is 83.3 per cent.

Back pressures are nearly uniform over the whole area except at the edges. In the above-mentioned experiments, the striking fact is brought out that the back pressure is dependent on the perimeter of the plane, and will vary between negative values of 40 and 100 per cent of the velocity head, its value being expressed by the equation,

$$P = \frac{V^2 W}{2g}(0.4 + 0.15l - 0.0044\, l^2)$$

where $\frac{V^2 W}{2g}$ is the velocity head and l is the perimeter in meters. This equation holds up to $l = 4$. From what precedes, the maximum total pressure on an indefinitely long rectangle of measurable width is 1.83 times the velocity head pressure. For a very small square, the coefficient may be as small as 1.1. According to the theory usually accepted, this coefficient should be 2.

Air pressures per unit area may be obtained by means of the following formula:

$$p = K\frac{V^2 W}{2g}$$

where

V = velocity of wind per second;
W = weight of air per unit cube;

g = acceleration of gravity in corresponding units.

Table 3, calculated from the above-mentioned article on air pressures, gives the coefficient K for various surfaces.

10. *Wind Pressure on Oblique Surfaces.*—The front-face pressure at an angle of less than 90° can be determined by means of the following formula:

$$p = \frac{V^2 W \sin^{4/3} \alpha}{2g}$$

TABLE 3

Description of Surface	Front	Back	K
10 cm. diameter disc	0.750	0.447	1.197
30 cm. diameter disc	0.754	0.538	1.292
100 cm. diameter disc	0.747	0.813	1.560
10 x 10 cm. square	0.697	0.453	1.150
30 x 30 cm. square	0.696	0.572	1.268
100 x 100 cm. square	0.697	0.893	1.690
10 x 30 rectangle	0.752	0.516	1.268
10 x 100 rectangle	0.771	0.713	1.484
30 x 100 rectangle	0.753	0.757	1.510
4.8 mm. diameter x 25 cm. cylinder	0.222	0.401	0.626
4.8 mm. x 25 cm. rectangle	0.756	0.488	1.244
Long rectangle	0.833	1.000	1.833

Experiments carried on by Drs. Finzi and Soldati proved that the exponent varies between 1.22 and 1.42. The back depression for angles, comprised between 60° and 90°, is fairly uniform over the whole area, and very similar to the values at 90° given in Table 3.

11. *Wind Pressure on Large and Small Surfaces.*—There is a great difference between wind pressure on large and small surfaces. This arises from the fact that the wind pressure acts in gusts or is oscillating, and the oscillations are not synchronous over any considerable extent of space. The English standard for wind pressure is 56 lb. per sq. ft. of plane exposed surface, 60 per cent of this being for cylindrical surfaces (counting diameter into length as exposed area). In this country, 50 lb. per sq. ft. is a frequent standard; but most American engineers are of the opinion that such pressures occur only over small surfaces, and that 30 lb. per sq. ft. of exposed surface is a liberal allowance for areas extending over considerable distances. The New York building code and many other building codes prescribe for high buildings 30 lb. per sq. ft.

The observed wind pressure greatly decreases as one approaches the surface of the ground. This is an additional reason why an assumed

wind pressure of more than 30 lb. per sq. ft. is not warranted for a transmission line, the towers of which are rarely more than 60 ft. high. For the cylindrical parts of a transmission line (poles, wires, etc.), as pressures on cylindrical surfaces have been shown by experiment to be from 50 to 60 per cent of those on plane surfaces, a pressure of 18 lb. per sq. ft. (counting diameter into length as area) is, therefore, a liberal assumption.

Mr. H. W. Buck* has given the results of a series of wind-pressure experiments made at Niagara on a 950-ft. span of 0.58-in. stranded cable, erected so as to be normal to the usual wind. From the data obtained, the following formula was derived:

$$p = 0.0025\ V^2$$

where

$p =$ pressure in lb. per sq. ft. of projected area;
$V =$ wind velocity in miles per hour.

For solid wire, previous experimenters had derived the formula:

$$p = 0.002\ V^2$$

Mr. Buck attributes the 25 per cent increase to the fact that, for a given diameter, cable presents increased surface for wind pressure. Unfortunately he mentions neither the barometer nor the temperature. Fig. 3 was drawn, using Mr. Buck's formula. He points out in his

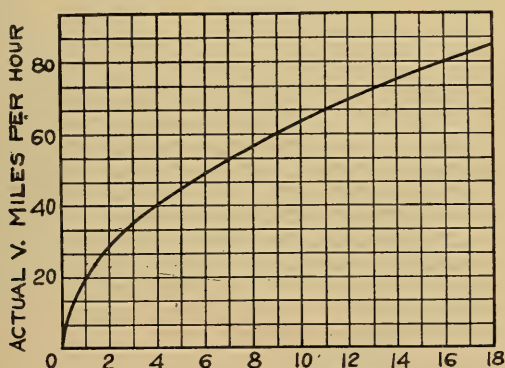

Fig. 3

paper that in the reports of the U. S. Weather Bureau, the wind velocities are higher than the actual velocities on a level with the average transmission line. This, of course, is due to the fact that

*Paper read at the World's Fair in St. Louis, 1904.

the recording instruments are usually located on high, exposed places. Fig. 4 shows graphically his tabular comparison of indicated and actual velocities.

It is highly desirable that proper investigations should be conducted along the line of suitable coefficients for wind-pressure, to see the effect, if any, which the wire diameter exerts on the pressure for a given projected area, and to determine whether the values should not be still greater for cable than for wire.

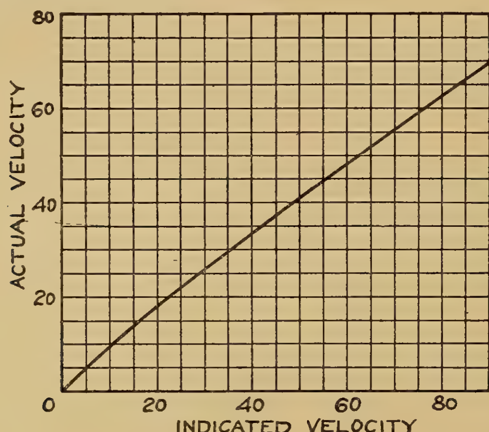

Fig. 4

Based on the fact that the tangent of the angle made by the plane of the wire with the vertical shows the ratio of the wind-pressure to the weight of the wire, A. J. Bowie, Jr., has suggested the following simple and feasible method for keeping record of or measuring the wind pressure on wires. A pivoted lever, provided with a pencil at the upper end, rests constantly against the wire at the center of the span. The pencil traces a diagram on paper from which can be determined the lateral sway of the wire and hence the wind pressure.

IV. Tension in Aerial Lines.*

12. *Analytical Discussion.*—Let ABC represent a wire fastened to insulators A and C, or in other words, imagine ABC to be any span of a transmission line. The internal force or tension in a perfectly flexible string, if hung as shown in Fig 5, would be directed

*The analytical discussion immediately following is substantially that given in text-books on Theoretical Mechanics. It is inserted here for the sake of completeness in the presentation.

at every point along the tangent line to the string; but this is not exactly true of a wire; for, if an attempt be made to bend an imperfectly flexible string, or a conductor, a certain amount of resistance proportional to the degree of rigidity of the string or wire is encountered. However, as the wire under consideration hangs freely, gravity being the only force acting upon it, for all practical purposes, the curve ABC may be considered as a catenary. This curve would be a perfect catenary if the wire were inextensible, perfectly flexible, and of uniform cross-section and density.

The properties of the curve ABC will be the same whether or not the points A and C are at the same level; but, for the sake of simplicity, the equation of this curve, assuming A and C to be in the same horizontal line, will be deduced.

Consider a particle P of the wire (Fig. 5). This particle, as

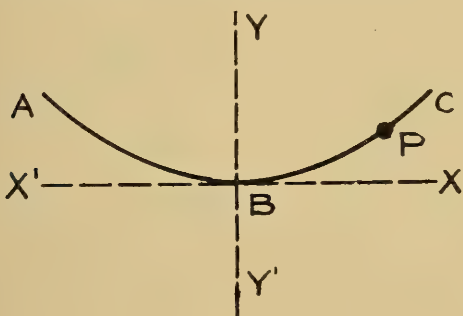

Fig. 5

shown in Fig. 6, is kept in equilibrium by the tensions T_1 and T_2, and by its weight, W acting downwards. Resolving these forces horizontally,

$$T_1 \cos\theta_1 - T_2 \cos\theta_2 = 0$$
$$T_1 \cos\theta_1 = T_2 \cos\theta_2 \quad \text{......................(1)}$$

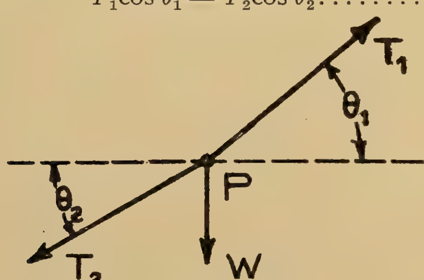

Fig. 6

That is, the horizontal components of the tensions in the different portions of the wire are the same.

The equation of the curve shown in Fig. 5 will now be deduced. The X-axis will be located at B, the lowest point of the catenary and the Y-axis will also pass through this point. Fig. 7 shows as free and in equilibrium, the right-hand half of the catenary ABC (Fig. 5). The tensions H and T, and the weight W, of the wire make this equilibrium stable. For algebraic convenience, make

$$H = cw \quad \text{.........................} (2)$$

That is, the tension H at the lowest point of the curve may be considered replaced by the weight of a length c of wire weighing w pounds

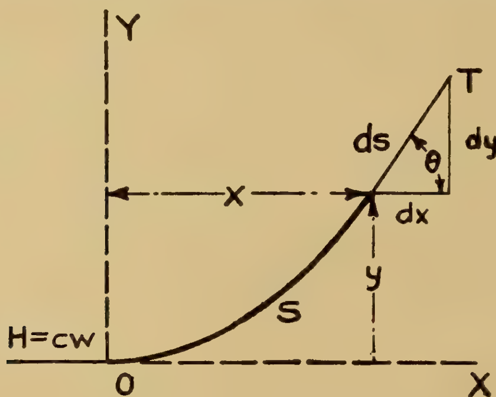

Fig. 7

per unit length. Calling s the length of the curve shown in Fig. 7, resolving vertically,

$$T\frac{dy}{ds} = sw = W \quad \text{.................} (3)$$

Resolving horizontally

$$T\frac{dx}{ds} = cw = H \quad \text{......................} (4)$$

Dividing (3) by (4) and squaring

$$\frac{dy^2}{dx^2} = \frac{s^2}{c^2} \quad \text{.........................} (5)$$

Remembering that

$$dy^2 = ds^2 - dx^2\,,$$

substituting in (5), and solving for dx,

$$dx = \frac{c\,d\,s}{\sqrt{c^2 + s^2}}$$

and

$$x = c\int_0^s \frac{ds}{\sqrt{c^2+s^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Equation (6) can be integrated as follows:

$$\int_0^s \frac{ds}{\sqrt{c^2+s^2}} = \int_0^s \frac{ds}{\sqrt{c^2+s^2}} \times \frac{s+\sqrt{c^2+s^2}}{s+\sqrt{c^2+s^2}} =$$

$$\int_0^s \frac{\sqrt{c^2+s^2}\;ds + s\,d\,s}{\sqrt{c^2+s^2}\;\left[s+\sqrt{c^2+s^2}\;\right]} =$$

$$\int_0^s \frac{ds + \dfrac{s\,d\,s}{\sqrt{c^2+s^2}}}{s+\sqrt{c^2+s^2}} = \Big[\log_e\left(s+\sqrt{c^2+s^2}\right)\Big]_0^s$$

$$= \log_e \frac{s+\sqrt{c^2+s^2}}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Substituting (7) in (6)

$$x = c\,\log_e \frac{s+\sqrt{c^2+s^2}}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Remembering that

$$dx^2 = ds^2 - dy^2,$$

substituting in (5), and solving for y,

$$y = \int_0^s \frac{s\,d\,s}{\sqrt{c^2+s^2}} = \Big[\sqrt{c^2+s^2}\Big]_0^s$$

Therefore

$$y = \sqrt{c^2+s^2} - c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Solving for c,
$$c = \frac{s^2 - y^2}{2y} \quad \text{(10)}$$

Fig. 8 illustrates a principle which will be found of great use

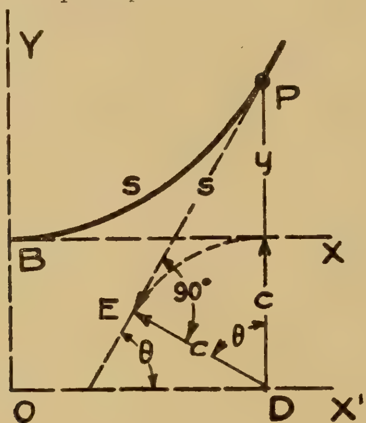

Fig. 8

when it is necessary to draw an accurate catenary.

From equation (3) and (4)

$$\frac{T \sin \theta}{T \cos \theta} = \frac{sw}{cw}$$

$$\tan \theta = \frac{s}{c} \quad \text{(11)}$$

From (9),

$$y + c = \sqrt{c^2 + s^2} \quad \text{(12)}$$

Therefore, (Fig. 8) if PD be parallel to YO and equal to $y + c$, as shown, and EP ($=$ s) be the tangent at P, then ED will be equal to c and perpendicular to tangent EP.

From (12) it may easily be seen that as y is a variable and c a constant, the point D for any other ordinate will necessarily lie on the line OD. This line OD is called the directrix of the catenary.

Equation (8) can be written,

$$e^{\frac{x}{c}} = \frac{s + \sqrt{c^2 + s^2}}{c} \quad \text{(13)}$$

Where e is equal to 2.718, the base of the natural logarithms. Solving for s in (13)

$$s = \frac{c}{2}\left[e^{\frac{x}{c}} - e^{\frac{-x}{c}}\right] \quad \text{(14)}$$

From (9) and (14), the value of y in terms of x can be found. Remembering that in the notation of hyperbolic functions

$$\frac{1}{2}\left[e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right] \equiv \cosh \frac{x}{c}$$

and

$$\frac{1}{2}\left[e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right] \equiv \sinh \frac{x}{c}$$

equation (14) may be written

$$s = c \sinh \frac{x}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

and equation (9)

$$y = \sqrt{s^2 + c^2} - c = c\sqrt{\sinh^2 \frac{x}{c} + 1} - c$$

$$= c \cosh \frac{x}{c} - c$$

$$= \frac{c}{2}\left[e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right] - c \ldots\ldots\ldots (16)$$

We know how to determine the tension at the lowest point of the wire (see equations (2) and (10)). It will now be shown how the tension at the highest point (at the insulator), or at any other point of the wire, can be determined.

Equation (3) may be written,

$$T = \frac{ws}{\sin\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

From Fig 8,

$$\sin\theta = \frac{s}{y + c}$$

Therefore

$$T = \frac{ws}{\sin\theta} = w\,(y + c) \quad \ldots\ldots\ldots\ldots\ldots (18)$$

This last equation shows that the tension at any point is equal to the weight of a portion of the wire whose length is equal to the ordinate of the point plus c.

If we call f the maximum value of y, i. e., the deflection of the wire, then from (2) and (18)

$$T = H + fw \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Remembering that $y_{max} = f$, equation (18) may be written

$$T = fw + cw \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Therefore

$$c = \frac{T - fw}{w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

Equating (10) and 21a)

$$\frac{s^2 - f^2}{2f} = \frac{T - fw}{w}$$

Therefore

$$T = \frac{s^2w + f^2w}{2f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21b)$$

Solving for f

$$f = \frac{T \pm \sqrt{T^2 - s^2 w^2}}{w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Equation (22) may be written

$$T = fw \pm \sqrt{T^2 - s^2w^2} \quad \ldots\ldots\ldots\ldots\ldots (23)$$

Equating (19) and (23)

$$H + fw = fw \pm \sqrt{T^2 - s^2w^2} \quad \ldots\ldots\ldots\ldots (24)$$

and

$$H = \pm \sqrt{T^2 - s^2w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

This last equation shows that the tension T at the insulator, the tension H at the lowest point of the conductor, and the weight W of half the span of wire can be represented as shown in Fig. 5.

(The $\pm$ sign in equation (25) may be taken to show that when considering the right or left-hand half of the span of wire as free and in equilibrium, the horizontal tension H which should be put in

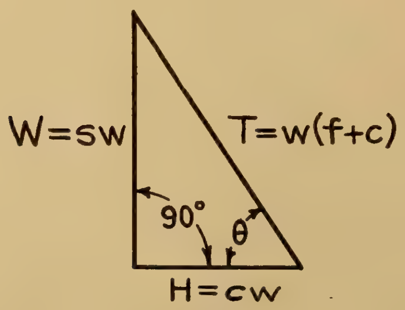

Fig. 9

(as shown in Fig. 9), would point in one or in the other direction.) Equating equations (2) and (25)

$$c\,w = \sqrt{T^2 - s^2 w^2}$$

whence

$$c = \frac{\sqrt{T^2 - s^2 w^2}}{w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

For all ordinary spans, the tension H at the lowest point of the wire is very large compared with the weight w per unit length. Hence from equation (2) the length c is large compared with x, and the ratio $\frac{x}{c}$ is therefore small. This fact permits the use of certain simplifying approximations that may be used if the span is not too long. In the following discussion, we shall first develop the approximate equations and show their application by a numerical example. We shall then take up the case of a very long span and show that for such a span the errors introduced by the approximation are too large to be neglected and that the exact analysis should be employed.

13. *Approximate or Parabola Method.*—Developing the functions $e^{\frac{x}{c}}$ and $e^{-\frac{x}{c}}$ by the aid of Maclaurin's formula,

$$e^{\frac{x}{c}} = 1 + \frac{x}{c} + \frac{x^2}{\lfloor 2\, c^2} + \frac{x^3}{\lfloor 3\, c^3} + \frac{x^4}{\lfloor 4\, c^4} + \text{etc.,}$$

$$\text{and} \quad e^{-\frac{x}{c}} = 1 - \frac{x}{c} + \frac{x^2}{\lfloor 2\, c^2} - \frac{x^3}{\lfloor 3\, c^3} + \frac{x^4}{\lfloor 4\, c^4} - \text{etc.,}$$

(where $\lfloor 2 = 1 \times 2, \lfloor 3 = 1 \times 2 \times 3$, etc).,

Equation (14) therefore can be made to assume the form,

$$s = x + \frac{x^3}{\lfloor 3\, c^2} + \frac{x^5}{\lfloor 5\, c^4} + \frac{x^7}{\lfloor 7\, c^6} + \text{etc.} \quad \ldots\ldots\ldots\ldots (27)$$

In a similar manner, making $y_{\max} = f$, equation (16) may be changed to the form

$$f = \frac{x^2}{\lfloor 2 c} + \frac{x^4}{\lfloor 4 c^3} + \frac{x^6}{\lfloor 6 c^5} + \text{etc.} \quad \ldots\ldots\ldots\ldots (28)$$

Since $\frac{x}{c}$ is small, the higher powers may be neglected for approxi-

mate analysis. If $l = 2x$ and $L = 2s$ (Fig. 10), equations (27) and

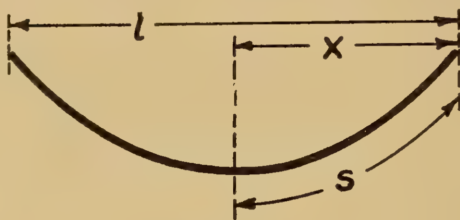

Fig. 10

(28) then become, respectively,

$$L = l + \frac{l^3}{24\,c^2} \quad \text{.........................} (29)$$

$$f = \frac{l^2}{8c} \quad \text{.........................} (30)$$

The effect of this approximation is to substitute a parabola for the true catenary.

Solving for c in equation (30) and substituting (29)

$$L = l + \frac{8f^2}{3l} \quad \text{.........................} (31)$$

Combining (2) and (30)

$$f = \frac{l^2\,w}{8H} \quad \text{.........................} (32)$$

and

$$H = \frac{l^2\,w}{8f} \quad \text{.........................} (33)$$

Equation (32) gives the deflection f at the minimum temperature (say, —20° F.) when w is made equal to the resultant force produced by the weight of wire plus the ice-covering (assumed to be ½in. thick) acting downward, and the maximum wind pressure (18 lb. per sq. ft.) acting normal to the combined weight.

Formula (31) gives the length of the wire at the minimum temperature (—20° F). When the temperature rises to, say, 110° F., the equation for the length, L_1, of the wire assumes the form,

$$L_1 = l + \frac{8f^2}{3l} + \alpha L t - \frac{L(H - H_1)}{AE} \quad \text{.............} (34)$$

where

l = length of span;
f = sag at minimum temperature (—20° F.);
a = coefficient of linear expansion per degree F.;
L = length of wire at minimum temperature;
t = difference between maximum and minimum temperature.
H = tension at lowest point of wire at minimum temperature;
H_1 = tension at lowest point of wire at maximum temperature;
A = cross-sectional area of wire;
E = modulus of elasticity of wire.

In formula (34) the term aLT represents the expansion of the wire due to rise of temperature and the term $L(H-H_1) \div A\ E$ represents the corresponding contraction produced by the decrease in tension due to the higher temperature.

Equation (31) may be written

$$f^2 = \frac{3}{8} l\ (L - l) \ldots\ldots\ldots\ldots\ldots\ldots \quad (35)$$

In a similar manner, if f_1 is the deflection at maximum temperature and L_1 the corresponding length of the conductor

$$f_1^2 = \frac{3}{8} l\ (L_1 - l) \ldots\ldots\ldots\ldots\ldots\ldots \quad (36)$$

Substituting the value of L_1 given in (34), equation (36) becomes

$$f_1^2 = f^2 + \frac{3\ l\ L}{8}\left(a\,t - \frac{H-H_1}{AE}\right) \quad \ldots\ldots (37)$$

Equation (31) may be put in the form,

$$L_1 = l + \frac{8f_1^2}{3l} \ldots\ldots\ldots\ldots\ldots\ldots \quad (38)$$

Dividing (37) by (35) and solving for f_1,

$$f_1 = f\sqrt{\frac{\mathrm{L}\ \left(1 + a\,t - \dfrac{H-H_1}{AE}\right) - l}{L - l}} \ldots\ldots\ldots \quad (39)$$

In (39) there are two unknown quantities, f_1 and H_1. Therefore, the value of H_1 must be found in order to solve for f_1.

From (33)

$$H = \frac{l^2 w}{8f} \ldots\ldots\ldots\ldots\ldots\ldots \quad (40)$$

and $$H_1 = \frac{l^2 w_1}{8f_1} \qquad (41)$$

In (41) w_1 is used instead of w, because for temperatures above freezing point the ice load disappears. If we want to know both the maximum tension and maximum sag at any higher temperature, w_1 is, of course, the resultant, per unit length, of the weight of the conductor and the maximum wind pressure acting normal to the weight,

Combining (41) and (40), and solving for H_1

$$H_1 = \frac{f H w_1}{f_1 w} \qquad (42)$$

Substituting this value of H_1 in equation (37)

$$f_1^2 = f^2 + \frac{3lL}{8}\left(aT - \frac{H}{AE} + \frac{fHw_1}{AEf_1w}\right) \qquad (43)$$

Multiplying through by f_1, collecting terms and transposing

$$f^3 - f_1\left[f^2 + \frac{3lL}{8}\left(aT - \frac{H}{AE}\right)\right] - \frac{3lLfHw_1}{8AEw} = 0 \qquad (44)$$

Equation (44) has only one unknown quantity, f_1, and may be written,

$$f_1^3 + 3Mf_1 + N = 0 \qquad (45)$$

V. Practical Use of Foregoing Formulas.

To show how the preceding formulas can be used in solving practical problems, a numerical example will be given. Several assumptions will have of necessity to be made regarding temperature range, wind pressure, ice coating, etc. The minimum temperature T_2 will be assumed to be —20° F. and the maximum temperature T_1 equal to 110° F. Therefore, the temperature range will be assumed to be 130° F. A ½-in. coat of ice is a conservative figure, making the total diameter 1 in. plus the diameter of the wire or cable. The wind pressure for the ice-covered wire will be assumed to be 18 lb. per sq. ft. (counting total diameter into length as the exposed area). The modulus of elasticity, E*, of hard-drawn copper strands will be taken as 16,000,000 lb. per sq. in. The coefficient of expansion, *a*, per degree Fahrenheit is 0.00000956.

*Mr. F. O. Blackwell has shown that this value of E may be as low as 12 000 000 lb. per sq. in.

As an example, a strand having the following constants will be considered:

Strand = 37 hard-drawn copper wires about No. 10 B & S;
Diameter = 0.679 in.;
Area = 350,000 c.m. or 0.275 sq. in.;
Weight = 1.07 lb. per lin. ft.;
Elastic limit = 9640 lb. or 35,000 lb. per sq. in.

Taking the weight of ice as 0.0332 lb. per cu. in., the weight per ft. of the ½-in. coat of ice on the strand would be

$$0.0332 \times 12 \times \frac{\pi}{4}\left[(1.679)^2 - (0.679)^2\right] = 0.73 \text{ lb.}$$

Therefore, the total weight of and the wind pressure on the strand with its ice covering would be, respectively,

$$1.07 + 0.73 = 1.8 \text{ lb. per foot} \qquad (46)$$

and

$$\frac{1.679 \times 12 \times 18}{144} = 2.52 \text{ lb.} \qquad (47)$$

But (46) and (47) act at right angles to each other, and, therefore, the resultant force acting on the strand is,

$$w = \sqrt{(1.8)^2 + (2.52)^2} = 3.1 \text{ lb. per lin. ft.}$$

The permissible strain in the strand, if taken at two-thirds the elastic limit, would be

$$\frac{2}{3} \times 9640 = 6430 \text{ lb.}$$

Tabulating the data given above, together with the quantities to be determined,

l = length of span = 660 ft.
f = sag at minimum temperature (—20° F.).
f_1 = sag at higher temperature.
H = tension at lowest point of wire at minimum temperature = 6430 lb.
H_1 = tension at lowest point of wire at higher temperature.
w = resultant weight = 3.1 lb. per ft.
L = length of wire in span at minimum temperature.
L_1 = length of wire at higher temperature.
T = temperature range (=130° F.).
a = coefficient of linear expansion for copper = 0.00000956.

From equation (32), we find the sag at minimum temperature to be

$$f = \frac{l^2 w}{8H} = \frac{(660)^2 \times 3.1}{8 \times 6430} = 26.24 \text{ ft.}$$

From (31), the length of the wire at minimum temperature is

$$L = l + \frac{8f^2}{3l} = 660 + \frac{8(26.24)^2}{3 \times 660} = 662.78 \text{ ft.}$$

Before attempting to calculate either the deflection or the length of the cable at the maximum temperature (assumed to be 110°F.) the force acting on this conductor when the ice load disappears must be determined. The weight of the cable is 1.07 lb. per linear ft. The wind pressure acting on it (assuming 18 lb. per sq. ft.), would be

$$\frac{0.679 \times 12 \times 18}{144} = 1.02 \text{ lb. per lin. ft.}$$

Therefore, as these two forces act at 90° to each other, the resultant force is

$$w_1 = \sqrt{(1.07)^2 + (1.02)^2} = 1.48 \text{ lb. per ft.}$$

We are now ready to calculate the deflection of the strand when the thermometer stands at 110° F., and the wind is blowing at such a velocity as to produce on the strand a pressure of 18 lb. per sq. ft.

From (44) and (45), making the required substitutions, the values of M and N are found to be

$$M = -217.5 \qquad (48)$$

and

$$N = -3005 \qquad (49)$$

Substituting the numerical values given in (48) and (49), equation (45) assumes the form

$$f_1^3 - 652.5\, f_1 = 3005 \qquad (50)$$

The above equation solved either by trial or by Horner's method, gives the value of the maximum deflection as

$$f_1 = 27.62 \text{ feet} \qquad (51)$$

From (41), the tension in the strand when both temperature and wind pressure reach a maximum value is

$$H_1 = \frac{l^2 w_1}{8f_1} = \frac{(660)^2 \times 1.48}{8 \times 27.62} = 2920 \text{ lb.} \qquad (52)$$

That is, the decrease in tension due to rise in temperature is

$$H - H_1 = 6430 - 2920 = 3510 \text{ lb.} \qquad (53)$$

As a check on the above value, the length of the strand at maximum temperature will be found following two quite different methods. From equation (38)

$$L_1 = l + \frac{8f^2{}_1}{3l} = 660 + \frac{8(27.62)^2}{3 \times 660} = 663.08 \text{ ft.} \ldots\ldots (54)$$

From (34) and (54)

$$L_1 = l + \frac{8f^2}{3l} + \alpha LT - \frac{L(H - H_1)}{AE}$$

$$= 660 + 2.78 + 0.823 - 0.529.$$

$$= 663.074 \text{ ft.} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

Comparing the lengths as given by (54) and (55), we can readily see that they are practically the same. This close agreement very clearly shows that the value of the maximum deflection given in (52) is the correct one.

VI. Catenary Method

The question has often been asked,—are engineers justified in assuming the curve subtended by a span of wire to be a parabola instead of a true catenary? The writer will endeavor to answer this question, giving a numerical illustration. It is not necessary to mention short spans and small sags because in this case, as it is well known, for all practical purposes, the results obtained by one or the other method are identical. To illustrate this point better, the same 350,000 c.m. copper strand will be discussed. The same constants will be used; but the curve will be regarded as a true catenary.

The tension H at the lowest point of the span can be found using equation (19). Thus

$$T = H + fw = 6430 = H + 26.24 \text{ x } 3.1;$$

Therefore,

$$H = 6430 - 81.34 = 6348.66 \text{ lb.}$$

From equation (2)

$$c = \frac{H}{w} = \frac{6348.66}{3.1} = 2048$$

The approximate value of f (=26.24 ft.; see Table 4) has been used above, because in the present case it can easily be proved that 10

per cent error in the value of the deflection would cause but 0.15 per cent error in the value of the tension constant c. This assumption would be justified even if considering the longest span and the greatest sag in existence. In the present case, the error introduced is only 1.45 per cent (see Table 4) instead of 10 per cent, and, therefore, the error in the value of c will be less than 0.02 per cent.

From (14)

$$s = \frac{c}{2}\left[e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right] \qquad (56)$$

Let, as before, $l = 2x$ and $L = 2s$.

Then

$$\frac{x}{c} = \frac{l}{2c} = \frac{660}{2 \times 2048} = 0.1611.$$

Making the required substitutions, equation (56) assumes the form

$$L = c\left[e^{\frac{l}{2c}} - e^{-\frac{l}{2c}}\right] = 2048\left[e^{0.1611} - e^{-0.1611}\right]$$

$$= 2048\left[1.1748 - 0.8512\right] = 662.74 \text{ ft.}$$

The values of e in the above were found by interpolation, using Table 5.

The deflection of the strand at $-20°$F can be determined as shown below.

$$f = \sqrt{s^2 + c^2} - c = \sqrt{(2048)^2 + (311.37)^2} - 2048$$

$$= 26.62 \text{ ft.}$$

Also

$$f = \frac{c}{2}\left[e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right] - c$$

$$= 1024\left[1.1748 + 0.8512\right] - 2048 = 26.62 \text{ ft.}$$

(For values of e see Table 5.)

Knowing very well that the equations for the catenary are most unwieldly, the writer had almost given up hope of deducing accurate equations for length, sag, etc., which, considering the curve as a catenary, would take into account wind pressure, ice load, temperature changes and corresponding elongation and contraction, etc. After

repeated failure, he succeeded in deducing an equation, the derivation of which will be briefly given below.

Equation (34) may be written,

$$s_1 = s + a\,s\tau - \frac{s\,(T - T_1)}{AE} \quad \text{.............. (57)}$$

where

s_1 = half length of wire at higher temperature.

s = half length of wire at minimum temperature.

τ = temperature range.

T_1 = tension at insulator at higher temperature.

Combining (20) and (16), solving for T_1

$$T_1 = \frac{w_1\ c_1}{2}\left[e^{\frac{x}{c_1}} + e^{-\frac{x}{c_1}}\right] \quad \text{.............. (58)}$$

Substituting this value in (58)

$$s_1 = s + a s\tau - \frac{s\left[T - \frac{w_1\ c_1}{2}\left(e^{\frac{x}{c_1}} + e^{-\frac{x}{c_1}}\right)\right]}{AE} \quad \text{..... (59)}$$

From (14)

$$s_1 = \frac{c_1}{2}\left[e^{\frac{x}{c_1}} - e^{-\frac{x}{c_1}}\right] \quad \text{................. (60)}$$

From (59) and (60)

$$s\left(1 + a\tau - \frac{T}{AE}\right) = \frac{c_1}{2}\left[\left(1 - \frac{sw_1}{AE}\right)e^{\frac{x}{c_1}} - \left(1 + \frac{sw_1}{AE}\right)e^{-\frac{x}{c_1}}\right] \quad \text{.... (61)}$$

All the equations are known except c_1, and by trial the exact value of this constant can be determined.

To show the application of the above formulas for the catenary, a numerical example will be given. As before, the same copper strand, the same span, etc., will be considered.

Substituting the numerical values already given, formula (61) assumes the form

$$331.71 = \frac{c_1}{2}\left[e^{\frac{330}{c_1}} \times 0.9999 - 1.000112\ e^{-\frac{330}{c_1}}\right] \quad \text{........ (62)}$$

Therefore

$$c_1 = 1872 \quad \text{...................... (63)}$$

An almost correct value of c_1 could have been obtained using (27) in the following convenient form

$$s\left(1 + a\tau - \frac{T}{AE}\right) = x + \frac{x^3}{6c_1^2}$$

$$= 331.71 = 330 - \frac{(330)^3}{6c_1^2}$$

Therefore

$$c_1 = 1871.1.$$

As seen, the two values for c_1 are almost identical. This holds even for very long spans.

Subsituting in (60) the value of c_1 given by the exact formula, the length of the cable is found to be

$$L_1 = 2s_1 = c_1\left[e^{\frac{x}{c_1}} - e^{-\frac{x}{c_1}}\right]$$

$$= 1872\ (1.1928 - 0.8384) = 663.44 \text{ ft.}$$

From (12), the deflection is found to be

$$f_1 = \sqrt{c_1^2 + s_1^2} - c_1$$

$$= \sqrt{(1872)^2 + (331.72)^2} - 1872 = 29.17 \text{ ft.}$$

Another way, perhaps more convenient, is to obtain the deflection from (16), as follows:

$$f_1 = \frac{c_1}{2}\left[e^{\frac{x}{c_1}} + e^{-\frac{x}{c_1}}\right] - c_1$$

$$= \frac{1872}{2}(1.1928 + 0.8384) - 1872 = 29.2 \text{ ft.}$$

From an inspection of Table 4 the conclusion is reached that for very long spans and great sags, the deflections and tensions obtained by the parabola method are by no means as accurate as they should be. The present long span and the much longer span of the future require something better. The writer believes that the catenary method suggested is simple and accurate, and could easily be used in practice.

TABLE 4

660-FT. SPAN. SAGS, TENSIONS, AND LENGTHS OF 350,000 C. M. HARD-DRAWN COPPER STRAND.

Temperature,—20°F.			Temperature, 110°F.		
Catenary	*Parabola*	*Per cent Error*	*Catenary*	*Parabola*	*Per cent Error*
		SAG IN FEET			
C	P	(C-P)÷P	C	P	(C-P)÷P
26.62	26.24	1.45	29.20	27.62	5.72
		LENGTH IN FEET			
C	P	(P-C)÷C	C	P	(C-P)÷P
662.74	662.78	0.006	663.44	663.08	0.05
		TENSION IN POUNDS			
C	P	(C-P)÷P	C	P	(P-C)÷C
6430	6430	0	2814	2920	3.77

TABLE 5

HYPERBOLIC FUNCTIONS

$\frac{x}{c}$	$e^{\frac{x}{c}}$	$e^{-\frac{x}{c}}$	Sinh $\frac{x}{c}$	Cosh $\frac{x}{c}$
0.00	1.0000	1.0000	0.0000	1.0000
.01	1.0100	0.9900	.0100	1.0000
.02	1.0202	.9802	.0200	1.0002
.03	1.0305	.9704	.0300	1.0004
.04	1.0408—	.9608	.0400	1.0008
.05	1.0513	.9512	.0500	1.0013
.06	1.0618	.9418	.0600	1.0018
.07	1.0725	.9324	.0701	1.0025
.08	1.0833	.9231	.0801	1.0032
.09	1.0942	.9139	.0901	1.0041
.10	1.1052	.9048	.1002	1.0050
.11	1.1163	.9858	.1102	1.0061
.12	1.1275	.8869	.1203	1.0072
.13	1.1388	.8781	.1304	1.0085
.14	1.1503	.8694	.1405	1.0098
.15	1.1618	.8607	.1506	1.0113
.16	1.1735	.8521	.1607	1.0128
.17	1.1853	.8437	.1708	1.0145
.18	1.1972	.8353	.1810	1.0162
.19	1.2092	.8270	.1911	1.0181
.20	1.2214	.8187	.2013	1.0201
.21	1.2337	.8106	.2115	1.0221
.22	1.2461	.8025	.2218	1.0243
.24	1.2712	.7866	.2423	1.0289

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W. F. M. GOSS,
Director.

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BULLETIN NO. 55

STARTING CURRENTS OF TRANSFORMERS

WITH SPECIAL REFERENCE TO TRANSFORMERS WITH SILICON STEEL CORES

BY

TRYGVE D YENSEN

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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ENGINEERING EXPERIMENT STATION

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STARTING CURRENTS OF TRANSFORMERS

WITH SPECIAL REFERENCE TO TRANSFORMERS WITH SILICON STEEL CORES

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STARTING CURRENTS OF TRANSFORMERS

WITH SPECIAL REFERENCE TO TRANSFORMERS WITH SILICON STEEL CORES

I. Introduction

1. *Preliminary.*—It is generally known that, in closing the primary circuit of a transformer, a transient effect may take place in the form of a momentary rush of current, due to the residual magnetism of the transformer iron. With the introduction of the new silicon steel for transformer cores, with the resulting increase in flux densities, this transient effect has been materially magnified, and may, in some cases, reach dangerous proportions.

It is the object of this bulletin to present some facts with regard to this phenomenon, obtained by means of commercial apparatus, and to show how to protect the system from injury due to this cause.

2. *Acknowledgments.*—Valuable assistance in the preparation of the oscillograms has been rendered by Messrs. C. E. Bennett and A. C. Hobble, of the Electrical Engineering Department.

3. *Theory.*

(*a*) *Inductance without Iron.*—If an alternating e. m. f. $e = E_{max} \sin \theta$, be impressed upon a circuit containing resistance and inductance without iron, in series, this impressed e. m. f. will be consumed by the counter e. m. f. of the inductance, and by the drop through the resistance, and this must be true at every instant, i. e., for every point of the impressed e. m. f. wave. We have, therefore,

$$e = E_{max} \sin \theta = L \frac{di}{dt} + Ri \ldots\ldots\ldots\ldots\ldots (1)$$

where

θ is the phase angle of the impressed e. m. f. $= 2\pi$ ft.

Since

$$\frac{di}{dt} = \frac{di}{d\theta} \frac{d\theta}{dt} = \frac{di}{d\theta} 2\pi f$$

$$L \frac{di}{dt} = 2\pi f L \frac{di}{d\theta} = X_L \frac{di}{d\theta}, \text{ where } X_L = \text{inductive reactance}$$

and

$$E_{max} \sin \theta = X_L \frac{di}{d\theta} + Ri$$

$$E_{max} \sin \theta \, d\theta = X_L \, di + Ri d\theta$$

$$-E_{max} \, d(\cos \theta) = X_L \, di + Ri d\theta$$

$$di = -\frac{E_{\max}}{X_L} d(\cos\theta) - \frac{R}{X_L}\, i\, d\theta \qquad (2)$$

If the circuit be closed at that point of the e. m. f. wave where $e = E_{\max}$, i. e., when $\theta = 90° = \frac{\pi}{2}$, and if the resistance drop be assumed negligible, (2) becomes

$$di = -\frac{E_{\max}}{X_L} d(\cos\theta) \qquad (3)$$

If the circuit be closed at different points of the e. m. f. wave, the current will rise to different values, and these values can now readily be investigated by means of the last equation.

Suppose for instance, that we close the circuit at the 90° point of the e. m. f. wave, i. e., when the e. m. f. is a maximum.
Integrating (3) from $\pi/2$ to π

$$\int_{\frac{\pi}{2}}^{\pi} di = -\frac{E_{\max}}{X_L} \int_{\frac{\pi}{2}}^{\pi} d(\cos\theta) = \frac{E_{\max}}{X_L}$$

which is the maximum current reached, since integrating from $\frac{\pi}{2}$ to $\pi + a$, where a is a constant, less than 2π, results in a value less than

$$\frac{E_{\max}}{X_L}$$

showing that the current decreases from this point.

Suppose, in the next case, that the circuit be closed at the 0° point of the e. m. f. wave, i. e., when $e = 0$ and $\theta = 0$.

Integrating (3) from 0 to $\frac{\pi}{2}$ gives

$$\int_{0}^{\frac{\pi}{2}} di = -\frac{E_{\max}}{X_L} \int_{0}^{\frac{\pi}{2}} d(\cos\theta) = +\frac{E_{\max}}{X_L}$$

the same as before.
Integrating from 0 to π, however,

$$\int_{0}^{\pi} di = -\frac{E_{\max}}{X_L} \int_{0}^{\pi} d(\cos\theta) = +2\frac{E_{\max}}{X_L}$$

i. e., the maximum current obtained in this case is twice that obtained when the circuit is closed at the 90° point of the e. m. f. wave.

In a similar way it can be shown that by closing the circuit at any other point of the e. m. f. wave, the maximum current reached will lie

between

$$\frac{E_{max}}{X_L} \text{ and } 2\frac{E_{max}}{X_L}$$

In general, the current assumes its normal value only when the circuit is closed at that point of the impressed e. m. f. wave, where the permanent value of the current is zero. In the above case, where there is negligible resistance, this is the 90° point of the wave. The effect of the resistance is to move this point towards the zero point.

(*b*) *Inductance with Iron.*—The above calculations assume a constant inductance, i. e., a straight line magnetization curve, obtained by using an inductance without magnetic material as core. If an iron core be employed, such as is the case with the ordinary induction coil or the transformer, the inductance is not constant. As the flux density increases, the inductance decreases, until the iron is perfectly saturated. After this point is reached, the inductance remains constant at a small value, depending only upon the flux passing between the coil and the core through the air or non-magnetic material.

Since the flux is not any longer proportional to the current, the counter e. m. f. due to the inductance must be written

$$A\frac{dB}{d\theta}$$

where $B =$ flux density and $A =$ constant,
instead of

$$L\frac{di}{dt} \text{ or } X_L\frac{di}{d\theta}$$

and equation (1) becomes

$$e = E_{max}\sin\theta = A\,\frac{dB}{d\theta} + Ri \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$-E_{max}\,d\,(\cos\theta) = AdB + Rid\,\theta$$

$$dB = -\frac{E_{max}}{A}\,d\,(\cos\theta) - \frac{R}{A}\,id\,\theta \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Under normal conditions, the resistance drop due to the magnetizing current of a transformer is negligible, and

$$dB = \frac{E_{max}}{A}\,d\,(\cos\theta)$$

The normal maximum value of B is then obtained by integrating dB from $\frac{\pi}{2}$ to π.

$$\int_{\frac{\pi}{2}}^{\pi} dB = B_{max} = -\frac{E_{max}}{A}\Big[\cos\theta\Big]_{\frac{\pi}{2}}^{\pi} = \frac{E_{max}}{A}$$

$$\therefore \frac{E_{\max}}{A} = B_{\max} \text{ and } A = \frac{E_{\max}}{B_{\max}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Substituting (6) in (5)

$$dB = -B_{\max}\, d\,(\cos\theta) - \frac{E_{\max}}{B_{\max}} Rid\,\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Since the relation between the magnetizing current and the resulting flux can not be expressed mathematically in any practical equation, the magnetizing current necessary to produce the required flux according to the above equation can be determined only analytically, as follows:

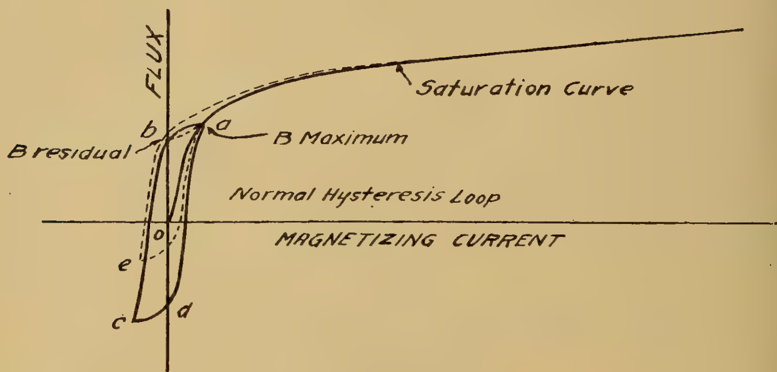


FIG. 1

Suppose Fig. 1 to represent the saturation curve of a transformer and the hysteresis loop for normal voltage and frequency. The hysteresis loop shows what residual magnetism remains in the iron after the current has been removed. *ob* and *od* represent this residual magnetism, depending upon whether the current has died down from a positive or a negative value.

Suppose the circuit is closed when the impressed e. m. f. passes through 0 from negative to positive, i. e., $\theta = 0$, and that the residual magnetism is *ob*. It is evident then that the change of flux to produce the counter e. m. f. must start from *b*. If equation (7) be rewritten in the form

$$\Delta B = -B_{\max}\,\Delta\,(\cos\theta) - \frac{B_{\max}}{E_{\max}} Ri\,\Delta\,\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

and small intervals of θ, say 10°, be taken, the actual conditions can very nearly be approached. Starting from *b*, the flux will follow a curve, such as the dashed curve between *b* and *a*, and will continue on the saturation curve. From equation (8), ΔB can be calculated for

each increment of 10°, starting from 0° in this case, and from Fig. 1 can be obtained the corresponding magnetizing current required to produce the total flux, $B_i + \Delta B$, B_i being the total flux at the beginning of the interval. After having determined the magnetizing current, the resistance drop effect is calculated, equal to

$$\frac{B_{max}}{E_{max}} Ri \Delta \theta \qquad (9)$$

This will, however, reduce the value of ΔB, and a few trials will have to be made before the correct value of ΔB is found.

Proceeding in this manner, the flux and the corresponding magnetizing current may be determined for any number of cycles. For decreasing values of flux, the upper dashed curve in Fig. 1 has to be used. It will be found that the magnetizing current may reach formidable values under unfavorable conditions, particularly for the first cycle. The amplitude of the peaks decreases rapidly, the more so the larger the amplitude of the first peak, on account of the more pronounced effect of the resistance in that case.

4. *Method of Investigation.*—In Part II (a) will be taken up the actual measurements of the magnetizing current of a transformer upon closing the primary circuit at a predetermined point of the e. m. f. wave, and with a known residual magnetism in the iron. These measurements were made by means of oscillograms, showing the impressed e. m. f., the primary magnetizing current, and the secondary induced e. m. f.

Part II (b) takes up the calculations of the flux and magnetizing current for the conditions under which the oscillograms were taken. In order to do this, all the characteristics of the circuit and transformer having any bearing upon the magnetizing current were carefully obtained. The curves plotted show that there is very close agreement between the actual curves, as obtained by means of the oscillograms, and the calculated curves as obtained by means of the circuit and transformer characteristics. This agreement shows that it is possible to make calculations of these phenomena, that can be fully relied upon, and that it is unnecessary to resort to the oscillograph in order to obtain reliable results. It was therefore deemed sufficient for the investigation of the rest of the transformers, covered by this bulletin, to obtain the transformer data necessary to make the calculations as shown in Part III. These calculations cover the most critical condition only, namely, the rush of current upon closing the circuit at the 0° point of the e. m. f. wave with the residual magnetism in the same direction in which the increase of flux will take place upon closing the circuit.

In Part IV, is given the result of placing a resistance or air core inductance in series with the transformer, and it is shown how to calculate a resistance or inductance sufficient to limit the rush of current to safe values.

II. *(a)* Actual Measurements of Phenomena by Means of Oscillograms.

5. *Connections.*—Fig. 2 is a diagram of the connections used for obtaining the oscillograms. *G* is a 10 kw. 440-v. alternator, 60 cycles, with taps, so as to give either 3-phase or 2-phase current, as shown in Fig. 2*a*. Taps 1, 3 and 5 are used for 3-phase, taps 1-4, 2-6, for 2-phase. The closing switch was designed and built specially for the investigation of these phenomena.[1] It was attached to the end of the generator

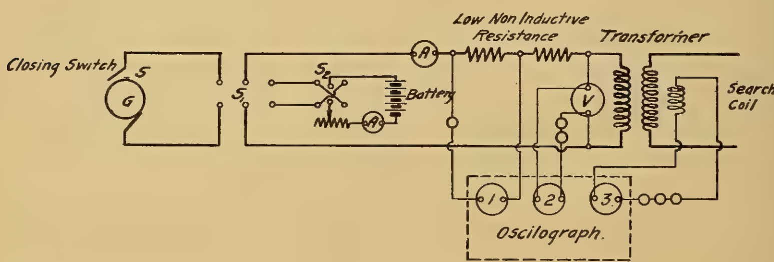


Fig. 2

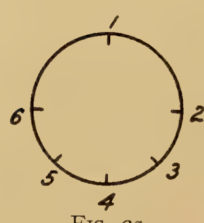


Fig. 2*a*

shaft, and can be set so as to close the circuit at any predetermined point of the e. m. f. wave. However, it would not operate satisfactorily at the normal speed of the generator, 1800 r. p. m. A speed of 650 r. p. m. was finally decided upon, which resulted in a frequency of 22 cycles.

6. *Transformer.*—A 5-kw. 60 cycles 2200, 1100/220, 110-volt transformer of the newest type was used in this test. It was connected for 110 volts primary, i. e., with the low tension coils in parallel. As the normal frequency is 60 cycles, and 22 cycles was used, the voltage

[1]By O. B. Wooten, Research Fellow, Engineering Experiment Station.

had to be reduced in proportion, i. e., the impressed voltage was $110 \times \frac{22}{60} = 40$ volts, to give normal magnetizing current.

As it was desirable to use as stiff a field as possible in the generator, in order to prevent too much of a voltage drop upon closing the transformer circuit, taps 2-3, (Fig. 2*a*), were used, giving 40 volts with about full field and 650 r. p. m. The oscillograms show that the voltage is kept up fairly well at the maximum rush of current.

7. *Residual Magnetism.*—The normal magnetizing current was obtained by impressing 110 volts at 60 cycles upon the transformer. The result is shown in the following table.

TABLE I

Volts	Current	Watts	Freq.
E	I_{ex}	W	F
110	.90	46.5	60

The maximum value of the exciting current $= .90 \times \sqrt{2} = 1.27$ amperes, and this is the current that produces the normal residual magnetism.

A series of experiments was made to ascertain the decrease of the residual magnetism after the removal of the e. m. f. These experiments are described in the Appendix. The following results were obtained:

1. There is no decrease in the residual magnetism of transformers under normal conditions.

2. The decrease of residual magnetism due to vibration or shock is very small, almost negligible.

The oscillograms were taken with a residual magnetism in the iron that would remain after the removal of the normal voltage at normal frequency, which would be the case under normal operating conditions.

This residual magnetism was produced by means of direct current from a storage battery, as shown in Fig. 2. The current used was that corresponding to the normal exciting current of the transformer, the maximum value of which is 1.27 amp. Hence 1.27 amp. D. C. was used.

By means of a reversing switch, S_2, the current could be reversed, producing a residual magnetism in the opposite direction. In order to be sure that the correct residual magnetism was produced, the iron was sent through the regular hysteresis loop a number of times, at least

ten, by reversing the current by means of switch S_2. Let Fig. 3 represent the normal hysteresis loop. Suppose the residual magnetism, at the beginning, is at 2′. Sending $+I_m$ amperes through the transformer increases the magnetism to 1′ along the lower dashed curve. Opening the switch decreases the magnetism to 2′. Reversing the switch brings it near 3. Again opening the switch brings it near 4. Going through the same operations, the loop will approach 1-2-3-4 and, after a few reversals, practically coincide with it, so that when the switch is finally opened, the residual magnetism will be 0-2 or 0-4, according to whether the last current was $+I_m$ or $-I_m$.

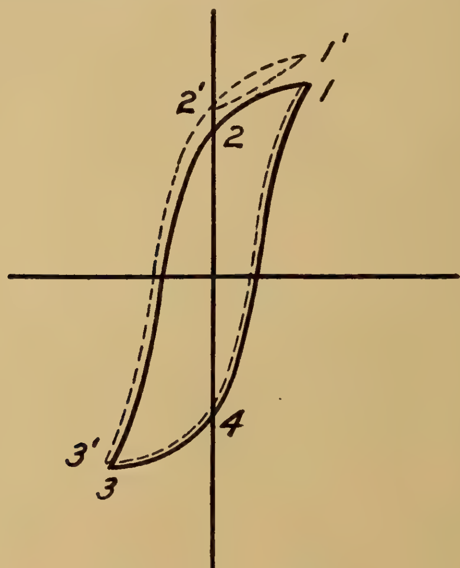


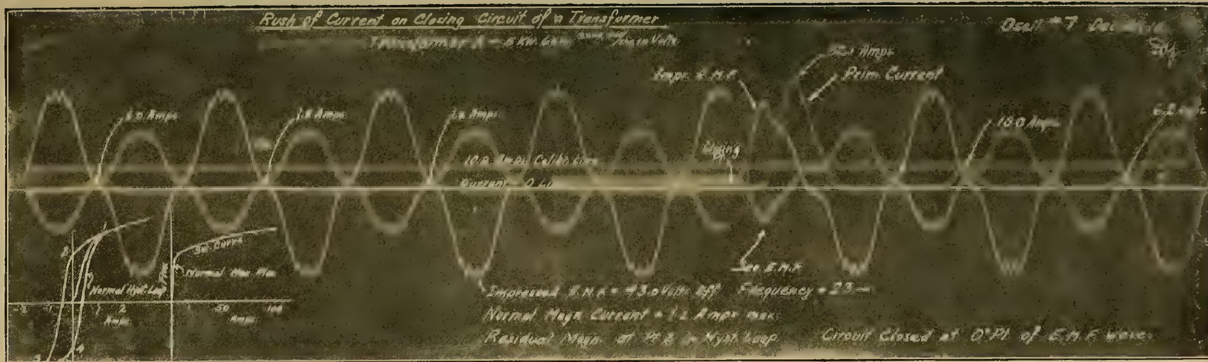
Fig 3

8. *The Oscillograms.*—Out of a total number of eleven oscillograms taken, four are here reproduced, as follows:

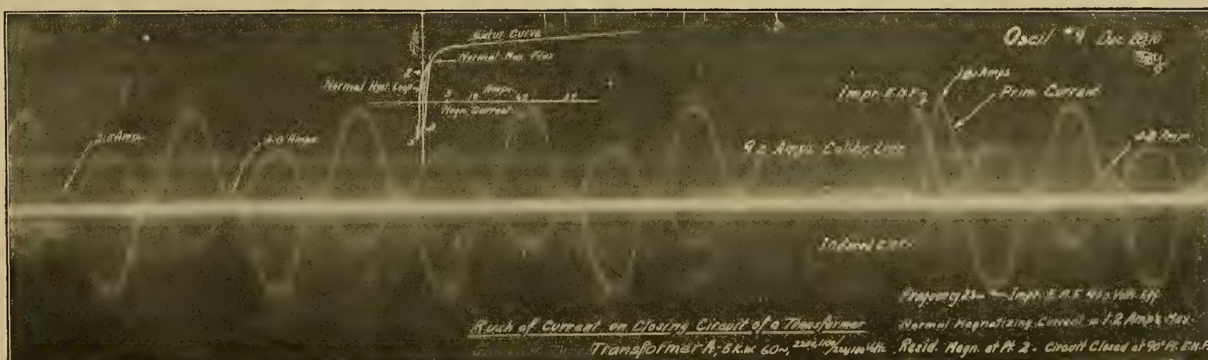
Oscil. 7. Circuit closed at 0° point of e. m. f. wave.
Residual magnetism, positive.
Maximum rush of current = 52.1 amp.

Oscil. 9. Circuit closed at 90° point (more accurately 85° point) of e. m. f. wave.
Residual magnetism, positive.
Maximum rush of current = 18.0 amp.

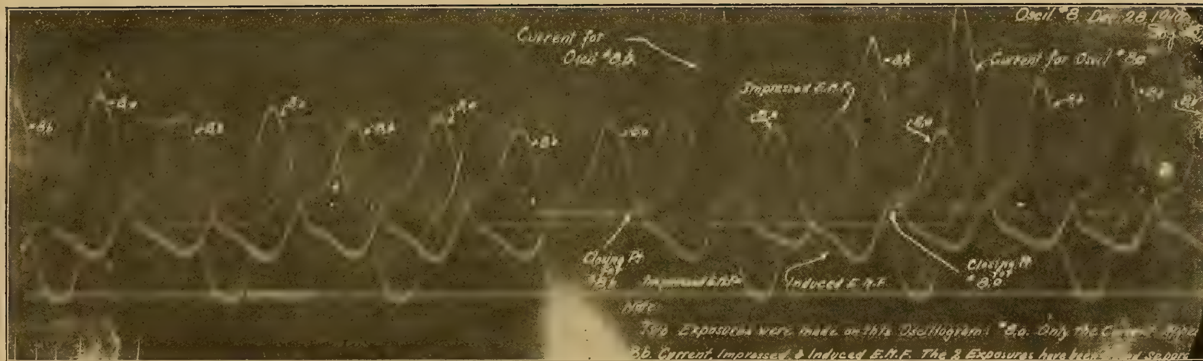
Oscil. 8 (*a*)* Circuit closed at 0° point of e. m. f. wave.
Residual magnetism, negative.
Maximum rush of current = 3.78 amp.



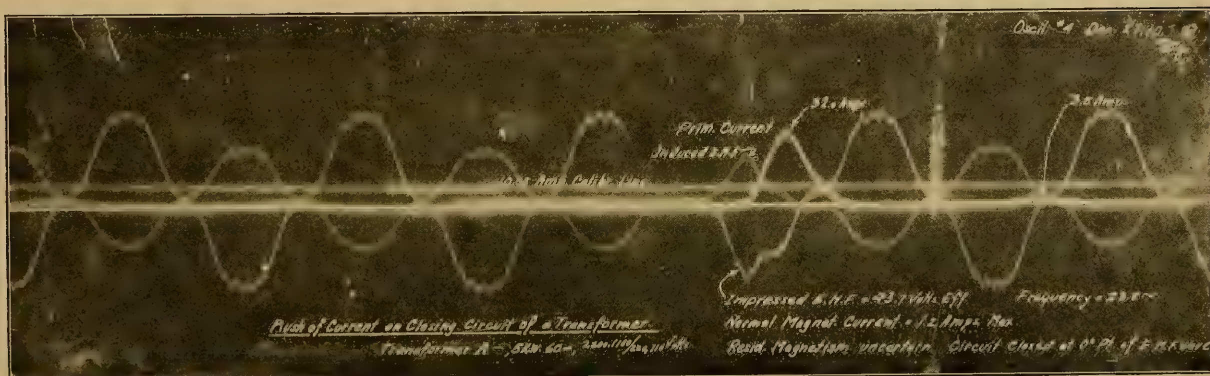
Oscillogram 7



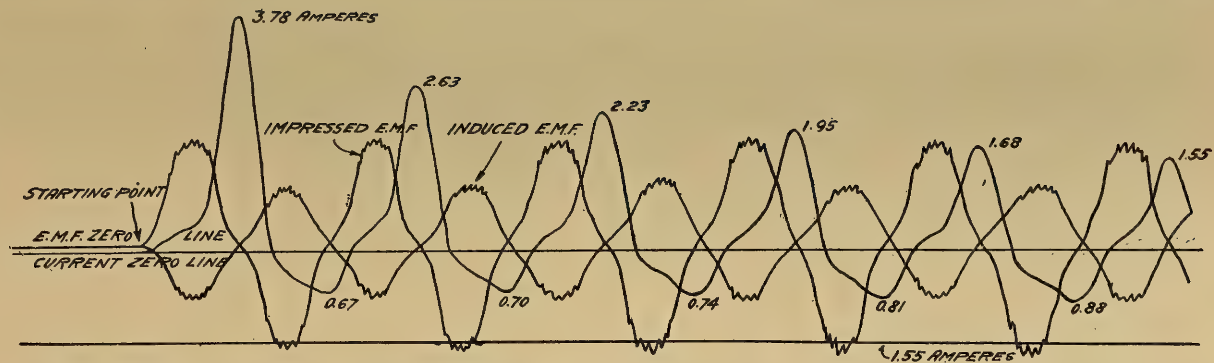
Oscillogram 9



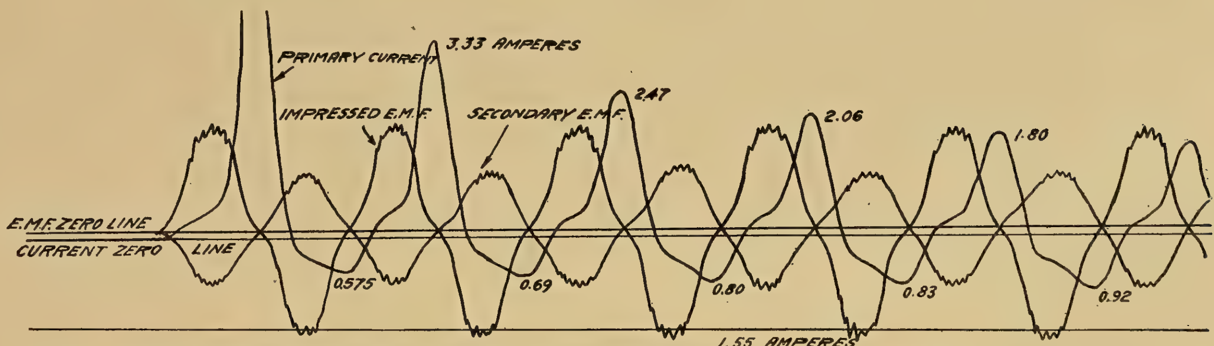
Oscillogram 8



Oscillogram 4



Oscil. 8*a*. Rush of current on closing the primary circuit of Transformer A. Circuit closed at o° point of e. m. f. wave. Residual magnetism equals $-20 \times K$



Oscil 8*b*. Rush of current on closing the primary circuit of Transformer A. Circuit closed at o° point of e. m. f. wave. Residual magnetism equals $-20 \times K$.

8 (*b*)* Same conditions.
Maximum rush of current, unknown.

Oscil. 4. Circuit closed at 0° point of e. m. f. wave.
Residual magnetism, uncertain.
Maximum rush of current = 32.0 amp.

By positive residual magnetism is meant that the magnetism was in the same direction in which the flux would increase upon closing the circuit.

II. (*b*) Theoretical Calculations from Transformer Data.

9. *Magnetization Curves and Hysteresis Loop.*—These were obtained in the following way. The transformer was connected as shown in Fig. 4. Direct current from a storage battery was supplied the high

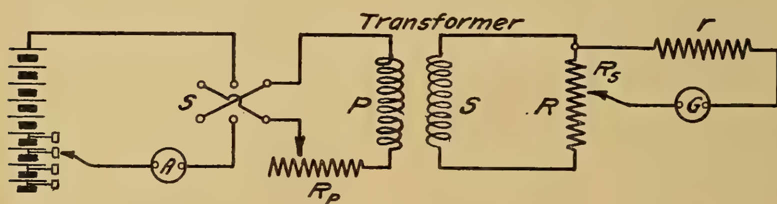


Fig. 4

tension side *P* of the transformer through a reversing switch, *S*, and a resistance R_p. The low tension side was connected to a high resistance *R*, and a D'Arsonval galvanometer, connected across a small part of the resistance, with a very high resistance in series. A change of flux in the transformer would then produce a deflection of the galvanometer coil, proportional to the total change of flux. *R*, R_s and *r* were not changed during the experiment, so that the deflections obtained, multiplied by a constant *K*, gave the flux density in the transformer core. In this investigation, the absolute flux density in gausses was not calculated, as it is only the relative flux values that are needed. The flux density is therefore, throughout this bulletin, expressed as a galvanometer deflection multiplied by a constant, *K*, K_1, K_2, etc. for different transformers.

To obtain the curves, the desired current was sent through the transformer primary, reversed a number of times to be sure that the iron had entered the corresponding loop, and the current left on in the

*Note.—Two exposures were made on Oscil. 8: 8 (*a*) in which the current only appeared; 8 (*b*) containing all quantities. The two exposures have been traced separately.

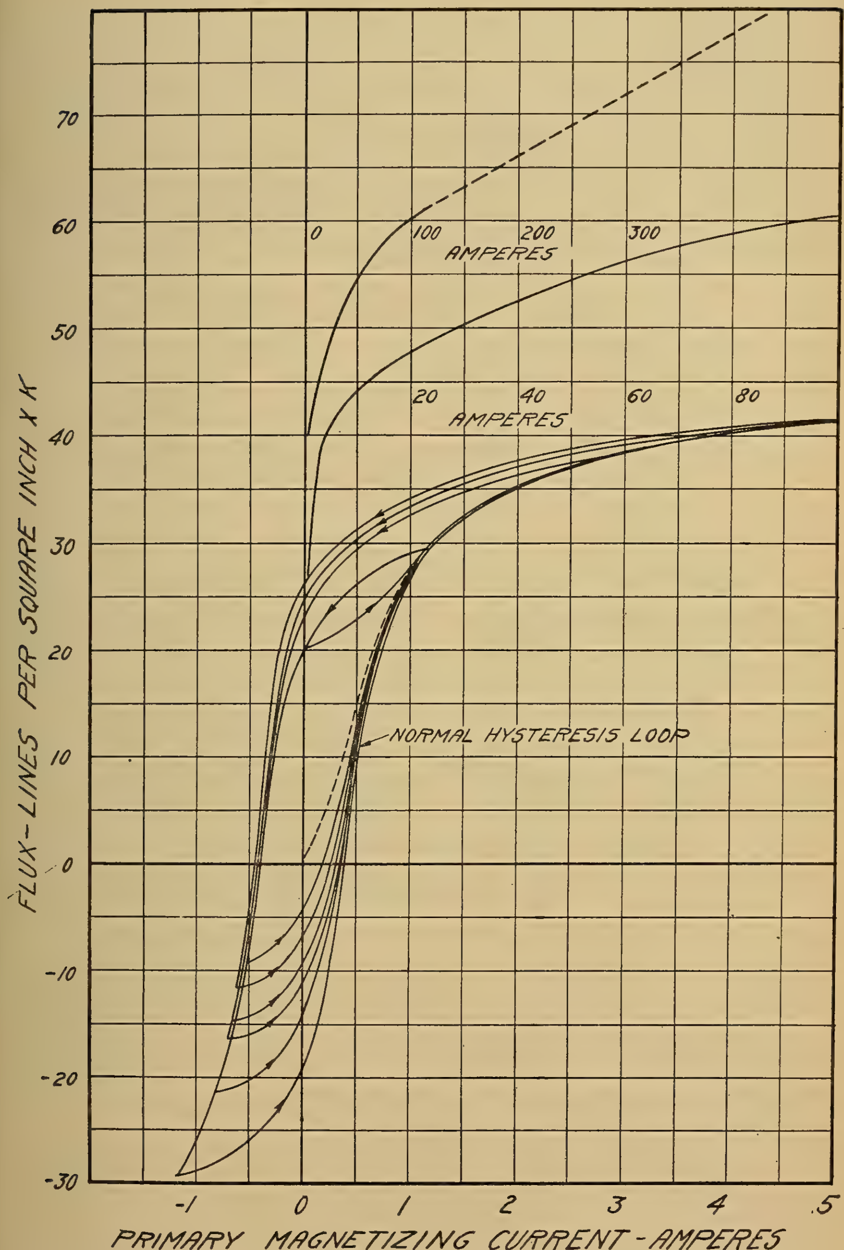


FIG. 5. MAGNETIZATION CURVE AND HYSTERESIS LOOP FOR TRANSFORMER A.

positive direction, corresponding to point 1 in Fig. 3. The galvanometer was then connected. Opening the circuit, the resulting deflection corresponded to a change of flux 1-2, reversing the current produced a change 2-3; opening it produced a change 3-4; again reversing it produced a change 4-1, completing the loop.

Fig. 5 shows the hysteresis loop and magnetization curves for the transformer used for the oscillograms. It was obtained by the method explained above. As an example, is given the galvanometer deflections for 1.22 amp.

TABLE 2

Current	Change	Deflection $= \Delta\phi \times K$	ϕ_{max}	Resid. Mag. ϕ
	1-2	9.5	$\frac{59.5}{2} = 29.75$	$29.75 - 9.5$
1.22	2-3	50.0		
	3-4	9.5	$\frac{59.5}{2} = 29.75$	$= 20.25$
	4-1	50.0		

The saturation curve was carried up to 119 amp. It is seen that at this point the curve has become a straight line, which means that the iron has become saturated, and the increase in flux is taking place only in the non-magnetic space between the iron and coil. Consequently, the curve can be extended indefinitely.

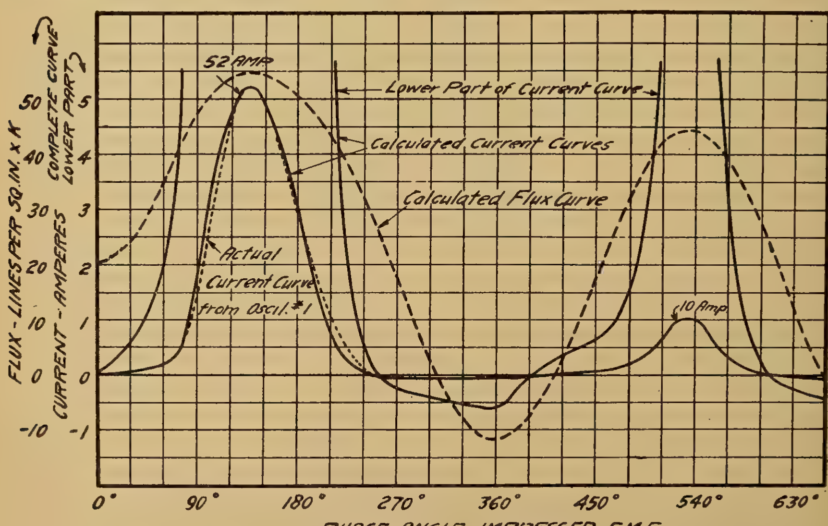


FIG. 6. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A. Circuit closed at 0° point of e. m. f. wave. Residual magnetism $= +20 \times K$.

9. *Calculations.*—The data needed for the calculations of the magnetizing current are as follows:

1. Normal hysteresis loop;
2. Magnetization curve up to straight line relation;
3. Total effective voltage impressed upon the transformer circuit;
4. Total resistance of circuit;
5. Total inductance of circuit.

Sine wave e. m. f. is assumed in these calculations. Equation (8), gives the relation:

$$\Delta B = -B_{max} \Delta (\cos \theta) - \frac{B_{max}}{E_{max}} Ri \Delta \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

assuming the circuit to have negligible inductance outside the transformer.

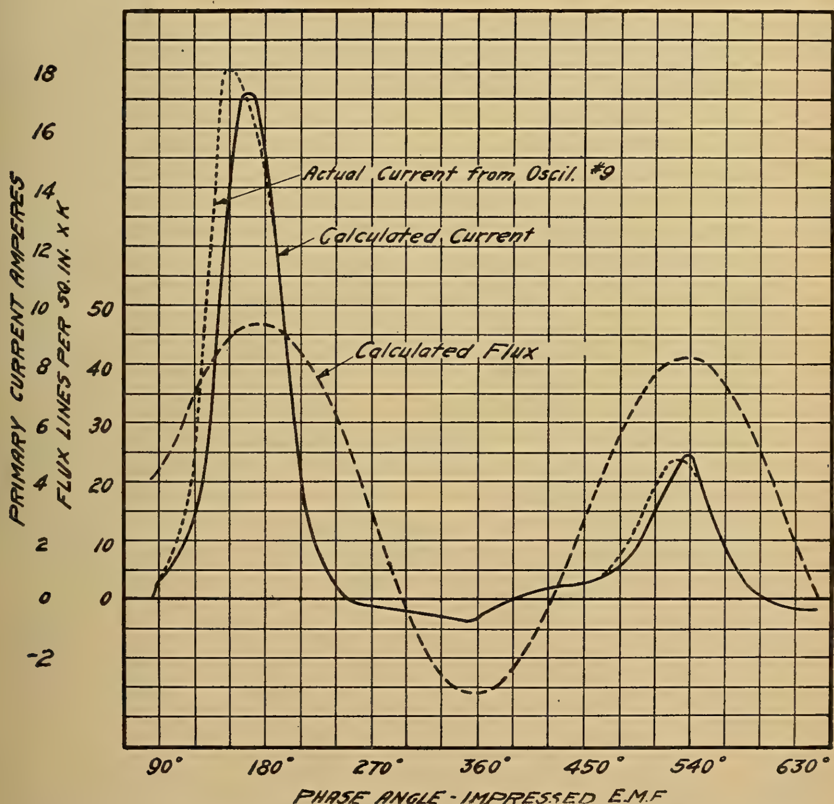


FIG. 7. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A. Circuit closed at 85° of e. m. f. wave. Residual magnetism equals $+20 \times K$.

B = flux, B_{max} = max. flux of normal hysteresis loop;

E_{max} = max. impressed e. m. f. = $\sqrt{2}\,E_{eff}$;

R = total resistance of circuit;

i = instantaneous value of magnetizing current.

In the present case,

B_{max} (from Fig. 5) $= 29.5 \times K$, where K = const.

$E_{eff} = 40$ volts. $E_{max} = \sqrt{2} \times 40 = 56.5$ volts.

$R = .745$ ohms.

Substituting in (8), for increments of θ of 10°, i. e. $\Delta\theta = 10° = .175$ radians,

$$\Delta B = -29.5\,K\,\Delta(\cos\theta) - .0685\,K\,i.$$

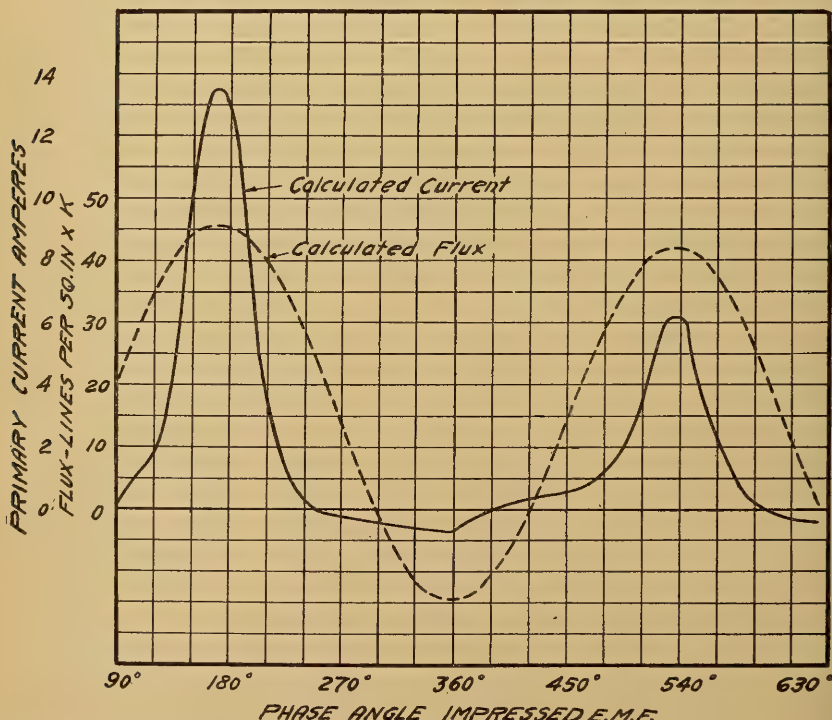


Fig. 8. Rush of Current on Closing the Primary Circuit of Transformer A. Circuit closed at 90° point of e. m. f. wave. Residual magnetism equals $+20 \times K$

In Table 3 are given the calculations for a number of different conditions, viz.,

CONDITIONS

	Closing Point on e. m. f. wave	Residual Magnetism	Impressed e. m. f. $_{eff}$	Frequency Cycles/Sec.	Resist. of Circuit
Columns 4 to 7	0°	+ 20 K	40 volts	22	.745
Columns 8 to 11	85°	+ 20 K	40 volts	22	.745
Columns 12 to 15	90°	+ 20 K	40 volts	22	.745
Columns 16 to 19	0°	— 20 K	40 volts	22	.745
Columns 20 to 23	90°	— 20 K	40 volts	22	.745

These conditions correspond to those under which the oscillograms were taken.

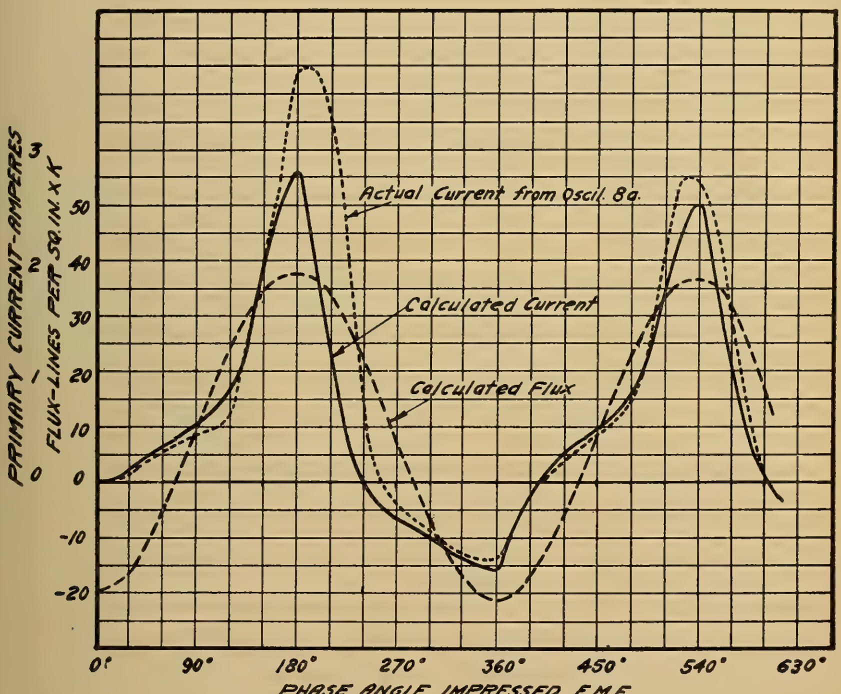


FIG. 9. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A. Circuit closed at 0° point of the e. m. f. wave. Residual magnetism equals $-20 \times K$.

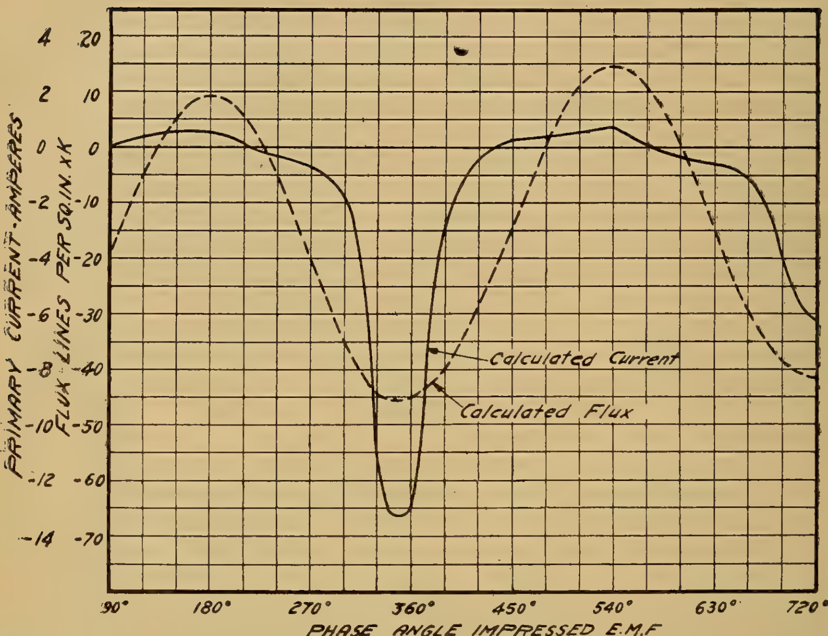


FIG. 10. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A. Circuit closed at 90° point of the e. m. f. wave. Residual magnetism = — 20 × *K*.

In Fig. 11 are plotted the calculated values of current, flux and impressed e. m. f. for the various conditions, to the same scale, in order to compare readily the effect of the closing point and the residual magnetism. Fig. 11 is a summary of Fig. 6 to 10 inclusive and Table 3.

11*a* corresponds to Table 3, Columns 4-7, and Fig. 6.
11*b* corresponds to Table 3, Columns 12-15, and Fig. 8.
11*c* corresponds to Table 3, Columns 16-19, and Fig. 9.
11*d* corresponds to Table 3, Columns 20-23, and Fig. 10.

11*e* represents the condition in which the circuit is closed at the 90° point of the e. m. f. wave with no residual magnetism. This is the condition for normal closing, since the flux and current then will enter at once upon their normal path. The same result would be obtained under conditions of Fig. 11*a* if the initial magnetism were negative maximum; under conditions in Fig. 11*b*, if the initial magnetism were 0; under conditions of Fig. 11*c*, if the initial magnetism were negative maximum, and under conditions of Fig. 11*d* if the initial magnetism were 0. It is seen that the closer the conditions come to these normal

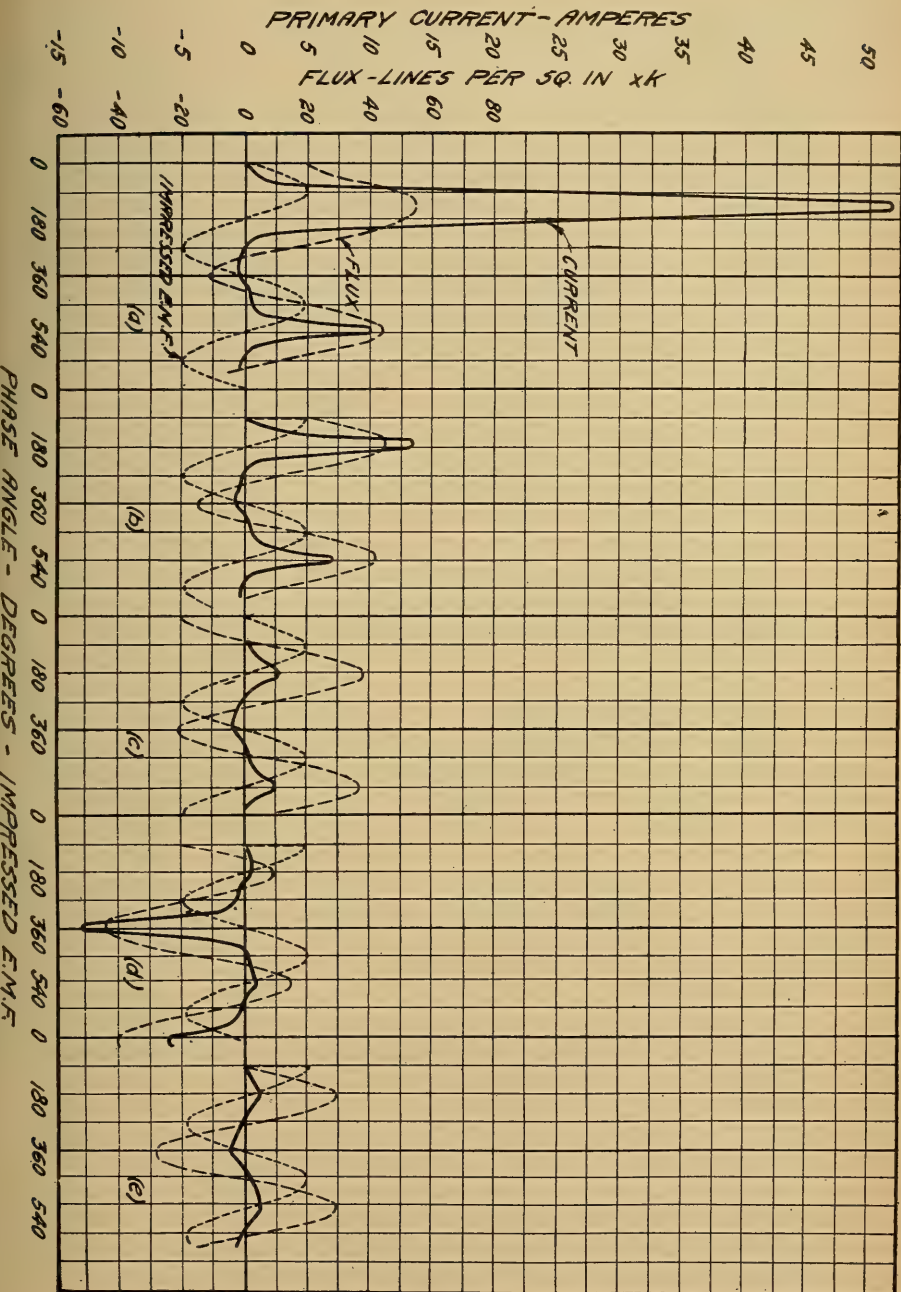


FIG. 11. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A.

TABLE 3—DETERMINATION OF FLUX AND CURRENT CURVES FOR TRANSFORMER A

IMPRESSED E. M. F. = 40 VOLTS EFFECTIVE AT 22 CYCLES RESISTANCE OF CIRCUIT = .745 OHMS.

1	2	3	4	5	6	7
			Circuit Closed at 0° Point of E. M. F. Wave Residual Magnetism = 20.0 K			
θ	$\cos\theta$	$-29.5\,K\,\Delta(\cos\theta)$	$\Delta B = -29.5\,K\,\Delta(\cos\theta) - .0685\,Ki_m$	Flux B	Mag. Current i_m	$.0685\,Ki_m$ b
0	+ 1.00	....	....	+ 20.0 K	+ .00	.00
10	.98	+ .59 K	+ .58 K	20.6	.20	.01
20	.94	1.18	1.15	21.8	.40	.03
30	.87	2.06	2.02	23.8	.60	.04
40	.77	2.95	2.88	26.7	.90	.06
50	.64	3.83	3.75	30.45	1.25	.085
60	.50	4.13	4.00	34.45	1.90	.13
70	.34	4.72	4.50	38.95	3.25	.22
80	+ .17	5.00	4.43	43.38	8.40	.575
90	.00	5.00	3.77	47.15	18.00	1.23
100	— .17	5.00	3.00	50.15	29.50	2.02
110	.34	5.00	2.30	52.45	39.50	2.70
120	.50	4.72	1.50	53.95	47.00	3.22
130	.64	4.13	.63	54.55	51.00	3.50
140	.77	3.83	+ .27	54.82	52.00	3.56
150	.87	2.95	— .45	54.37	49.50	3.40
160	.94	2.06	1.00	53.37	44.40	3.04
170	.98	1.18	1.37	52.00	37.60	2.57
180	1.00	+ .59	1.53	50.47	31.00	2.12
190	.98	— .59	2.11	48.36	22.20	1.52
200	.94	1.18	2.20	46.16	15.00	1.03
210	.87	2.06	2.65	43.51	8.60	0.59
220	.77	2.95	3.20	40.31	3.50	.25
230	.64	3.83	3.95	36.36	1.60	.12
240	.50	4.13	4.18	32.18	0.65	.045
250	.34	4.72	4.73	27.45	+ .10	+ .010
260	— .17	5.00	4.99	22.46	— .15	— .010
270	.00	5.00	4.98	17.48	.25	.020
280	+ .17	5.00	4.98	12.50	.32	.022
290	.34	5.00	4.97	7.53	.37	.025
300	.50	4.72	4.69	+ 2.84	.42	.029
310	.64	4.13	4.10	— 1.26	.45	.031
320	.77	3.83	3.80	5.06	.50	.034
330	.87	2.95	2.95	7.97	.53	.036
340	.94	2.06	2.02	9.99	.57	.039
350	.98	1.18	1.14	11.13	.60	.04
360	1.00	— .59	— .55	11.68	.60	.04
370	.98	+ .59	+ .62	11.06	.50	.03
380	.94	1.18	1.20	9.86	.25	.017
390	.87	2.06	2.06	7.80	— .07	— .0048
400	+ .77	+ 2.95	+ 2.94	— 4.86	+ .10	+ .007

TABLE 3—(Continued)—DETERMINATION OF FLUX AND CURRENT CURVES FOR TRANSFORMER A
IMPRESSED E. M. F. = 40 VOLTS EFFECTIVE AT 22 CYCLES RESISTANCE OF CIRCUIT = .745 OHMS.

	Circuit Closed at 85° Point of E. M. F. Wave Residual Magnetism = + 20.0 K				Circuit Closed at 90° Point of E. M. F. Wave Residual Magnetism = + 20.0 K			
1	8	9	10	11	12	13	14	15
θ	$\Delta B = -29.5 K \Delta(\cos\theta) - .0685 K i_m$	Flux B	Mag. Current i_m	$.0685 K i_m$ b	$\Delta B = -29.5 K \Delta(\cos\theta) - .0685 K i_m$	Flux B	Mag. Current i_m	$.0685 K i_m$ b
0								
10								
20								
30								
40								
50								
60								
70								
80	........*	+ 20.0 K	.00	.00				
90	+ 2.47	+ 22.47	.45	.031	....	20.0 K	.00	.00
100	4.93	27.40	.98	.067	+ 4.95 K	24.95	+ .74	.05 K
110	4.90	32.30	1.50	.100	4.92	29.87	1.17	.08
120	4.55	36.85	2.45	.168	4.60	34.47	1.82	.125
130	3.83	40.68	4.45	.305	3.92	38.39	3.00	.21
140	3.18	43.86	9.50	.650	3.43	41.82	6.00	.41
150	1.97	45.83	14.20	.98	2.30	44.12	10.00	.685
160	.90	46.73	17.00	1.165	1.18	45.30	12.75	.875
170	.015	46.75	17.10	1.17	+ .26	45.56	13.50	.92
180	− .48	46.26	15.60	1.07	− .30	45.29	13.00	.89
190	1.39	44.87	11.70	.80	1.27	44.02	10.00	.68
200	1.74	43.13	8.20	.56	1.64	42.38	6.70	.46
210	2.35	40.78	4.20	.29	2.30	40.08	3.55	.24
220	3.11	37.69	2.30	.16	3.08	37.00	2.00	.137
230	3.91	33.78	1.20	.08	3.90	33.10	1.00	.0685
240	4.16	29.62	+ .40	+ .03	4.15	28.95	+ .35	.024
250	4.72	22.90	− .06	.0	4.72	24.23	.00	.00
260	4.99	17.91	.20	− .01	4.99	19.24	− .15	.01
270	4.98	12.93	.26	.02	4.98	14.26	.25	.017
280	4.98	7.95	.33	.02	4.98	9.28	.30	.0206
290	4.97	+ 2.92	.38	.026	4.97	+ 4.31	.35	.024
300	4.69	− 1.77	.43	.03	4.69	− 0.38	.40	.027
310	4.10	5.87	.49	.034	4.10	4.48	.475	.0325
320	3.79	9.66	.55	.038	3.80	8.28	.525	.036
330	2.91	12.75	.60	.041	2.91	11.19	.575	.0394
340	2.02	14.77	.67	.046	2.02	13.21	.625	.043
350	1.13	15.90	.70	.048	− 1.13	14.34	.650	.045
360	− .54	16.44	.71	.048	− .54	14.88	.670	.046
370	+ .62	15.82	.40	.03	+ .62	14.25	.45	.035
380	1.20	14.62	.25	.02	1.20	13.06	.26	.018
390	2.06	12.54	− .06	− .00	2.06	11.00	− .08	.005
400	+ 2.95	9.59	+ .09	+ .006	+ 2.95	− 8.05	+ .08	.005

* 85° = θ

TABLE 3—DETERMINATION OF FLUX AND CURRENT CURVES FOR TRANSFORMER A

IMPRESSED E. M. F. = 40 VOLTS EFFECTIVE AT 22 CYCLES RESISTANCE OF CIRCUIT = .745 OHMS.

			Circuit Closed at 0° Point of E. M. F. Wave Residual Magnetism = 20.0 K			
1	2	3	4	5	6	7
θ	$\cos\theta$	$-29.5\,K\,\Delta(\cos\theta)$	$\Delta B = -29.5\,K\,\Delta(\cos\theta) - .0685\,K i_m$	Flux B	Mag. Current i_m	$.0685\,K i_m$ b
0	+ 1.00	....	....	+ 20.0 K	+ .00	.00
10	.98	+ .59 K	+ .58 K	20.6	.20	.01
20	.94	1.18	1.15	21.8	.40	.03
30	.87	2.06	2.02	23.8	.60	.04
40	.77	2.95	2.88	26.7	.90	.06
50	.64	3.83	3.75	30.45	1.25	.085
60	.50	4.13	4.00	34.45	1.90	.13
70	.34	4.72	4.50	38.95	3.25	.22
80	+ .17	5.00	4.43	43.38	8.40	.575
90	.00	5.00	3.77	47.15	18.00	1.23
100	− .17	5.00	3.00	50.15	29.50	2.02
110	.34	5.00	2.30	52.45	39.50	2.70
120	.50	4.72	1.50	53.95	47.00	3.22
130	.64	4.13	.63	54.55	51.00	3.50
140	.77	3.83	+ .27	54.82	52.00	3.56
150	.87	2.95	− .45	54.37	49.50	3.40
160	.94	2.06	1.00	53.37	44.40	3.04
170	.98	1.18	1.37	52.00	37.60	2.57
180	1.00	+ .59	1.53	50.47	31.00	2.12
190	.98	− .59	2.11	48.36	22.20	1.52
200	.94	1.18	2.20	46.16	15.00	1.03
210	.87	2.06	2.65	43.51	8.60	0.59
220	.77	2.95	3.20	40.31	3.50	.25
230	.64	3.83	3.95	36.36	1.60	.12
240	.50	4.13	4.18	32.18	0.65	.045
250	.34	4.72	4.73	27.45	+ .10	+ .010
260	− .17	5.00	4.99	22.46	− .15	− .010
270	.00	5.00	4.98	17.48	.25	.020
280	+ .17	5.00	4.98	12.50	.32	.022
290	.34	5.00	4.97	7.53	.37	.025
300	.50	4.72	4.69	+ 2.84	.42	.029
310	.64	4.13	4.10	− 1.26	.45	.031
320	.77	3.83	3.80	5.06	.50	.034
330	.87	2.95	2.95	7.97	.53	.036
340	.94	2.06	2.02	9.99	.57	.039
350	.98	1.18	1.14	11.13	.60	.04
360	1.00	− .59	− .55	11.68	.60	.04
370	.98	+ .59	+ .62	11.06	.50	.03
380	.94	1.18	1.20	9.86	.25	.017
390	[illegible]	[illegible]	+ 2.94	− 4.86	+ .10	+ .007

TABLE 3—(Continued)—DETERMINATION OF FLUX AND CURRENT CURVES FOR TRANSFORMER A

IMPRESSED E. M. F. = 40 VOLTS EFFECTIVE AT 22 CYCLES RESISTANCE OF CIRCUIT = .745 OHMS.

	Circuit Closed at 85° Point of E. M. F. Wave Residual Magnetism = + 20.0 K				Circuit Closed at 90° Point of E. M. F. Wave Residual Magnetism = + 20.0 K			
1	8	9	10	11	12	13	14	15
θ	$\Delta B =$ $-29.5\,K$ $\Delta(\cos\theta)$ $-.0685\,Ki_m$	Flux B	Mag. Current i_m	$.0685\,Ki_m$ b	$\Delta B =$ $-29.5\,K$ $\Delta(\cos\theta)$ $-.0685\,Ki_m$	Flux B	Mag. Current i_m	$.0685\,Ki_m$ b
0								
10								
20								
30								
40								
50								
60								
70								
80	............*	+ 20.0 K	.00	.00				
90	+ 2.47	+ 22.47	.45	.031	....	20.0 K	+ .00	.00
100	4.93	27.40	.98	.067	+ 4.95 K	24.95	+ .74	.05 K
110	4.90	32.30	1.50	.100	4.92	29.87	1.17	.08
120	4.55	36.85	2.45	.168	4.60	34.47	1.82	.125
130	3.83	40.68	4.45	.305	3.92	38.39	3.00	.21
140	3.18	43.86	9.50	.650	3.43	41.82	6.00	.41
150	1.97	45.83	14.20	.98	2.30	44.12	10.00	.685
160	.90	46.73	17.00	1.165	1.18	45.30	12.75	.875
170	.015	46.75	17.10	1.17	+ .26	45.56	13.50	.92
180	— .48	46.26	15.60	1.07	— .30	45.29	13.00	.89
190	1.39	44.87	11.70	.80	1.27	44.02	10.00	.68
200	1.74	43.13	8.20	.56	1.64	42.38	6.70	.46
210	2.35	40.78	4.20	.29	2.30	40.08	3.55	.24
220	3.11	37.69	2.30	.16	3.08	37.00	2.00	.137
230	3.91	33.78	1.20	.08	3.90	33.10	1.00	.0685
240	4.16	29.62	+ .40	+ .03	4.15	28.95	+ .35	.024
250	4.72	22.90	— .06	.0	4.72	24.23	.00	.00
260	4.99	17.91	.20	— .01	4.99	19.24	— .15	.01
270	4.98	12.93	.26	.02	4.98	14.26	.25	.017
280	4.98	7.95	.33	.02	4.98	9.28	.30	.0206
290	4.97	+ 2.92	.38	.026	4.97	+ 4.31	.35	.024
300	4.69	— 1.77	.43	.03	4.69	— 0.38	.40	.027
310	4.10	5.87	.49	.034	4.10	4.48	.475	.0325
320	3.79	9.66	.55	.038	3.80	8.28	.525	.036
330	2.91	12.75	.60	.041	2.91	11.19	.575	.0394
340	2.02	14.77	.67	.046	2.02	13.21	.625	.043
350	1.13	15.90	.70	.048	— 1.13	14.34	.650	.045
360	— .54	16.44	.71	.048	— .54	14.88	.670	.046
370	+ .62	15.82	.40	.03	+ .62	14.25	.45	.035
380	1.20	14.62	.25	.02	1.20	13.06	.26	.018
390	2.06	12.54	— .06	— .00	2.06	11.00	— .08	.005
400	+ 2.95	9.59	+ .09	+ .006	+ 2.95	— 8.05	+ .08	.005

* 85° = θ

TABLE 3—(Concluded)—DETERMINATION OF FLUX AND CURRENT CURVES FOR TRANSFORMER A

IMPRESSED E. M. F. = 40 VOLTS EFFECTIVE AT 22 CYCLES RESISTANCE OF CIRCUIT = .745 OHMS.

	Circuit Closed at 0° Point of E. M. F. Wave Residual Magnetism = — 20.0 K				Circuit Closed at 90° Point of E. M. F. Wave Residual Magnetism = — 20.0 K			
1	16	17	18	19	20	21	22	23
θ	$\Delta B =$ — 29.5 K $\Delta(\cos\theta)$ — .0685 Ki_m	Flux B	Mag. Current i_m	.0685 Ki_m b	$\Delta B =$ — 29.5 K $\Delta(\cos\theta)$ — .0685 Ki_m	Flux B	Mag. Current i_m	.0685 Ki_m b
0	....	— 20.00 K	.00	.00				
10	+ 0.59	19.41	+ .01	.00				
20	1.18	18.23	.02	.00				
30	2.05	16.18	.12	+ .008				
40	2.94	13.24	.19	.01				
50	3.81	9.43	.26	.018				
60	4.11	5.32	.31	.021				
70	4.69	— 0.63	.38	.026				
80	4.97	+ 4.34	.45	.031				
90	4.97	9.31	.50	.034	....	— 20.0 K	.00	+ .00
100	4.96	14.27	.60	.041	+ 4.99	15.01	+ .15	.01
110	4.95	19.22	.72	.05	4.98	10.03	.25	.02
120	4.66	23.88	.88	.06	4.70	5.33	.30	.02
130	4.05	27.93	1.03	.08	4.10	— 1.43	.37	.03
140	3.73	31.66	1.42	.10	3.80	+ 2.57	.41	.03
150	2.82	34.48	1.90	.13	2.92	5.49	.45	.03
160	1.90	36.38	2.37	.16	2.03	7.52	.48	.03
170	1.00	37.38	2.65	.18	1.15	8.67	.50	.03
180	+ .40	37.78	2.80	.19	+ .56	9.23	.51	.03
190	— .75	37.03	2.30	.16	— .62	8.61	.40	.03
200	1.30	35.73	1.80	.12	1.20	7.41	.30	+ .02
210	2.15	33.58	1.25	.09	2.06	5.35	+ .05	.00
220	3.00	30.58	+ .65	.045	2.94	+ 2.41	— .10	— .01
230	3.85	26.73	+ .25	+ .02	3.82	— 1.41	.20	.01
240	4.13	22.60	.00	.00	4.11	5.52	.34	.02
250	4.71	17.89	— .15	— .01	4.69	10.21	.44	.03
260	4.98	12.91	.25	.02	4.96	15.17	.55	.04
270	4.98	7.93	.34	.02	4.95	20.12	.70	.05
280	4.97	+ 2.96	.37	.03	4.94	25.06	.90	.06
290	4.97	— 2.01	.43	.03	4.92	29.98	1.20	.08
300	4.68	6.69	.50	.035	4.60	34.58	1.90	.12
310	4.09	10.78	.56	.038	3.92	38.50	3.05	.21
320	3.79	14.57	.66	.044	3.42	41.92	6.00	.41
330	2.90	17.47	.69	.047	2.23	44.15	10.50	.72
340	2.01	19.48	.75	.051	1.16	45.31	13.00	.90
350	1.13	20.61	.78	.053	— .27	45.58	13.30	.91
360	— .53	21.14	.80	.055	+ .30	45.28	12.90	.89
370	+ .62	20.58	.50	.034	1.28	44.00	10.00	.69
380	1.20	19.38	.35	.024	1.53	42.47	5.10	.35
390	2.07	17.31	.15	— .01	2.30	40.17	3.50	.24
400	2.95	— 14.36	— .00	.00	+ 3.09	— 37.08	— 2.00	— .14

conditions, the less rush of current takes place. Fig. 11*c* comes very close to these normal conditions, while Fig. 11*a* is farthest away. Fig. 11*b* and *d* are practically identical, with the exception that in Fig. 11*b* the rush of current is positive, while in Fig. 11*d* it is negative.

10. *Agreement between Oscillograms and Calculated Curves.*—The values of flux and current from Table 3 have been plotted in Fig. 6 to 10 inclusive, together with the actual currents, as given by the oscillograms. The full lines give the calculated currents. The dashed lines give the calculated flux. The dotted lines give the actual currents.

From these plates it may be seen that the agreement between the actual curves and the calculated curves is very close. Indeed, for the first case, corresponding to Oscil. 7 and Table 3, Columns 4-7, the two current curves practically coincide. For the second case, corresponding to Oscil. 9 and Table 3, Columns 12-15, the maximum disagreement is only 4.5 per cent, while in the third case, corresponding to Oscil. 8*a* and Table 3, Columns 16-19, the disagreement is 25 per cent.

The closer agreement in the first case was to be expected, considering that a small variation in the residual magnetism in Oscil. 8 would have a greater effect than in Oscil. 7, on account of the dampening effect of the resistance in 7, while the resistance has practically no effect in 8. While the attempt was made to have the residual magnetism constant in all cases, it is possible that it may have varied a small amount. Assume, for instance, that the residual magnetism for Oscil. 8 was $-18.0\ K$ instead of $-20\ K$, the maximum positive flux would be approximately $37.8 + 2.0 = 39.8\ K$ corresponding to a current of 3.75 amp. (instead of 2.8 amp.) which is the current shown by the oscillogram.

However, the agreement between the oscillograms and the calculations is such as to warrant the conclusion that reliable results of the starting current of transformers can be obtained by calculations, if the complete data of the transformer and circuits are at hand, as tabulated on p. 15.

III. Calculation of Maximum Starting Current of Transformers of Various Types and Makes.

In the preceding section, have been given the starting currents of a 110-volt 60-cycles transformer by impressing upon it 40 volts at 22 cycles. While this resulted in normal magnetizing currents under normal operating conditions, the percentage of resistance drop in

terms of total impressed e. m. f. is much greater for the same current than if 110 volts were impressed.

For 110 volts, 60 cycles, equation (8) takes the following form: (the resistance of the circuit remaining .745 ohms)

$$\Delta B = -29.5\, K \Delta (\cos \theta) - \frac{29.5\, K}{110 \sqrt{2}}\, Ri \Delta \theta$$

$$\Delta B = -29.5\, K \Delta (\cos \theta) - .025\, Ki$$

which shows that the effect of the resistance in decreasing ΔB, and consequently the current, is decreased by 110/40, or in proportion to the voltage.

In this section, calculations are given for the case in which the transformers are connected directly to constant potential busbars with sufficient power behind to maintain the voltage constant in spite of large starting currents. The potential in this case is the normal voltage of the transformers, and the resistance of the leads is assumed negligible.

The following transformers have been treated:

Designation	Capacity K.V.A.	VOLTS Primary	VOLTS Secondary	Freq.	Make	Year of Mfg.	Remarks
Transformer A	5	2200/1100	220/110	60	X	1910	Same transformer as was used in obtaining oscillograms.
Transformer B	5	2080/1040	460/230	60	X	Old Type	
Transformer C	50	2200/1100	440/220	60	Y	1910	
Transformer D	7½	440	110	60	Y	Old Type	
Transformer E	15	440/220	220/110	60	Z	1911	

The transformers will now be taken up in order, and the current calculated for the case where the circuit is closed at the 0° point of the e. m. f. wave, with the residual magnetism positive, i. e., for the conditions of Oscil. 7, which give the maximum rush of current.

Transformer A

5-kw., 2200, 1100/220, 110 volts, 60 cycles, new type, 110-volt winding used as primary.

Data

Hysteresis loop and magnetization curve are given in Fig. 5.
Normal effective voltage = 110 volts.
Resistance of circuit = resistance of transformer = .0253 ohms.
Maximum value of normal exciting current = 1.27 amp.

Hence equation (8) becomes

$$\Delta B = -29.5 K \Delta (\cos \theta) - .00084 Ki.$$

Table 4 gives the calculations for this and the following cases from 0 to 200°. For transformer A, it gives a maximum current of 390 amp., while the maximum value of the full load current is only $\sqrt{2} \times 45 = 64.3$ amp., i. e., the maximum rush of current is 6.1 times normal full load.

Transformer B

5-kw. 2080, 1040/460, 230 V., 60 cycles, old type, 2080-volt winding used as primary.

Data

Hysteresis loop and magnetization curve are given in Fig. 12.
Normal eff. e. m. f. = 2080 volts.
Resistance of transf. (2080-volt winding) = 9.35 ohms.
Maximum value of normal exciting current = 0.1 amp.

From Fig. 12

$B_{max} = 26.25 \times K_4$
Normal residual magnetism = $20.0 \times K_4$.
$E_{max} = \sqrt{2} \times 2080 = 2940.$

Equation (8) becomes

$$\Delta B = -26.25 \times K_4 \Delta (\cos \theta) - .0146 K_4 i.$$

From Table 4, Columns 8-11, the maximum current is 13.5 amp. or about 4 times the maximum value of the full load current, viz., $\sqrt{2} \times 2.4 = 3.4$ amp.

Transformer C

50 kw. 2200, 1100/440, 220 volts, 60 cycles, new type, 2200-volt side used as primary.

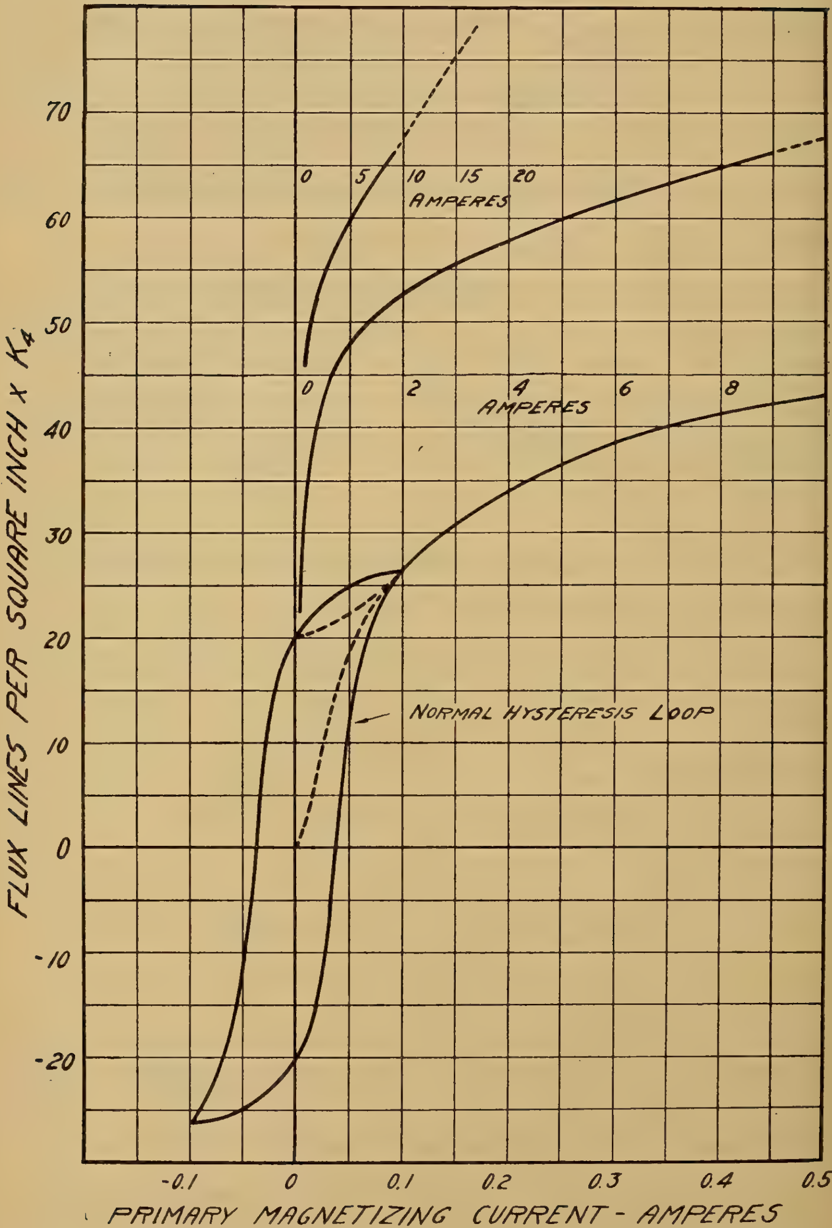


FIG 12. MAGNETIZATION CURVE AND HYSTERESIS LOOP OF TRANSFORMER B.

Data

Hysteresis loop and magnetization curve are given in Fig. 13.

Normal eff. e. m. f. = 2200 volts.

Resistance of transformer (2200-volt winding) = .446 ohms.

Maximum value of normal exciting current = .5 amp.

From Fig. 13

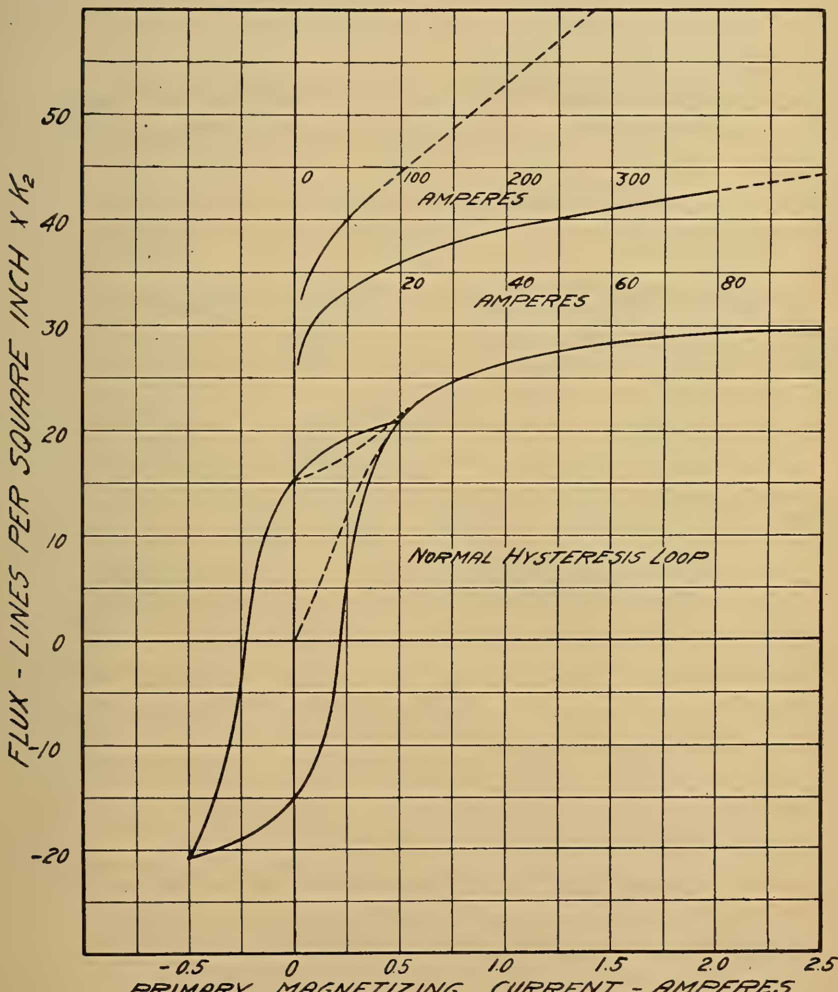


FIG. 13. MAGNETIZATION CURVE AND HYSTERESIS LOOP FOR TRANSFORMER C.

$$B_{max} = 20.75\, K_2$$

Normal residual magnetism $= 15.25\, K_2$

$$E_{max} = \sqrt{2} \times 2200 = 3110 \text{ volts}$$

Equation (8) becomes

$$\Delta B = -20.75\, K_2\, \Delta\,(\cos\theta) - .00052\, K_2\, i$$

From Table 4, Columns 12-15, the maximum current is 235 amp. or about 7.3 times the maximum value of the normal full load current, viz., $\sqrt{2} \times 22.7 = 32.1$ amp.

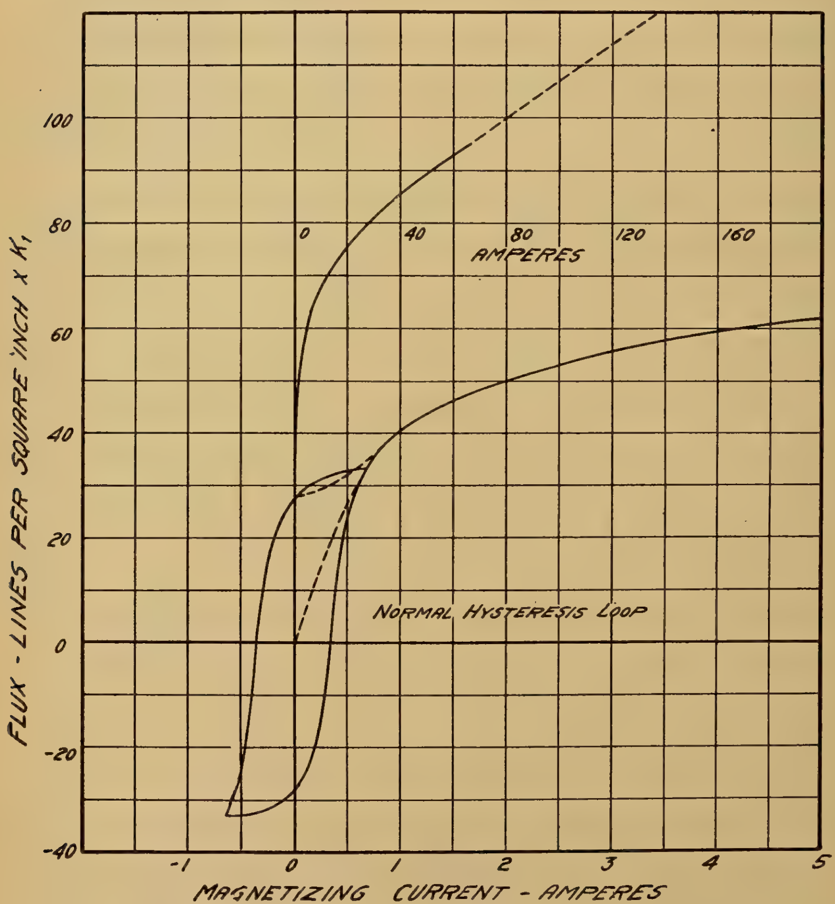


FIG. 14. MAGNETIZATION CURVE AND HYSTERESIS LOOP FOR TRANSFORMER D.

TABLE 4

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

NEGLIGIBLE RESISTANCE AND INDUCTANCE IN PRIMARY LEADS

		TRANSFORMER A 5 Kw. 110 Volts, Impressed at 60 Cycles Residual Magnetism = + 20.0 K Circuit Closed at 0° Point of E. M. F. Wave Total Res. = .0253ω				
1	2	3	4	5	6	7
θ	$\cos\theta$	$-29.5\,K$ $\Delta\cos\theta$	Δ B = $-29.5\,K$ $\Delta\cos\theta$ $-.00084\,Ki_m$	Flux B	Mag. Current i_m	$.00084\,Ki_m$
0	+ 1.00	....	....	20.0 K	.00	.00
10	.98	+ .59 K	+ .6	20.6	.20	.00
20	.94	1.18	1.18	21.8	.40	.00
30	.87	2.06	2.06	23.9	.60	.00
40	.77	2.95	2.95	26.9	.90	.00
50	.64	3.83	3.83	30.7	1.30	.00
60	.50	4.13	4.13	34.8	1.90	.00
70	.34	4.72	4.72	39.5	3.60	.00
80	+ .17	5.00	5.00	44.5	11.00	.00
90	.00	5.00	4.98	49.5	27.00	+ .02
100	− .17	5.00	4.96	54.5	50.00	.04
110	.34	5.00	4.93	59.4	82.50	.07
120	.50	4.72	4.58	64.0	165.00	.14
130	.64	4.13	3.94	67.9	230.00	.19
140	.77	3.83	3.60	71.5	290.00	.24
150	.87	2.95	2.66	74.2	340.00	.29
160	.94	2.06	1.75	76.0	370.00	.31
170	.98	1.18	.86	76.9	385.00	.32
180	− 1.00	+ .59	+ .26	77.2	390.00	.33
190	.98	− .59	− .91	76.3	380.00	.32
200	− .94	− 1.18	− 1.47	74.8	345.00	.29

TABLE 4—(Continued)

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

NEGLIGIBLE RESISTANCE AND INDUCTANCE IN PRIMARY LEADS

TRANSFORMER B 5 KW. 2080 VOLTS IMPRESSED AT 60 CYCLES RESIDUAL MAGNETISM = 20.0 K					TRANSFORMER C 50 KW. 2200 VOLTS IMPRESSED AT 60 CYCLES RESIDUAL MAGNETISM = + 15.25 K_2				
Circuit Closed at 0° Point of E. M. F. Wave Res. of Winding on 2080 Volt Side = 9.35ω					Circuit Closed at 0° Point of E. M. F. Wave Res. of Winding on 2200 Volt Side = .446ω				
8	9	10	11	12	13	14	15	16	17
$-26.25\,K_4$ $\Delta \cos\theta$	$\Delta B =$ $-26.25\,K_4$ $\Delta \cos\theta$ $-.0146\,K_4 i_m$	Flux B	Mag. Current i_m	$-.0146\,K_4 i_m$ b	$-20.75\,K_2$ $\Delta \cos\theta$	$\Delta B =$ $-20.75\,K_2$ $\Delta \cos\theta$ $-.00052$ $K_2 i_m$	Flux B	Mag. Current i_m	.00052 $K_2 i_m$
....	....	20.0 K_4	.00	.00	....	....	+ 15.25 K_2	.00	.00
+ .53 K_4	+ .53	20.5	.015	.00	+ .42 K_2	+ .42 K_2	15.70	.05	.00
1.06	1.06	21.6	.04	.00	.84	.84	16.5	.15	.00
1.84	1.84	23.4	.065	.00	1.45	1.45	18.00	.30	.00
2.63	2.63	26.0	.095	.00	2.08	2.08	20.00	.40	.00
3.42	3.42	29.4	.135	.00	2.70	2.70	22.70	.55	.00
3.68	3.68	33.1	.185	.00	2.90	2.90	25.60	.90	.00
4.22	4.22	37.3	.27	.00	3.32	3.32	28.90	1.70	.00
4.48	4.48	42.8	.50	.00	3.53	3.53	32.40	7.50	.00
4.48	4.48	47.3	.90	.00	3.53	3.52	35.90	20.00	.01 K_2
4.48	4.48	51.8	1.80	.00	3.53	3.51	39.40	41.00	.02
4.48	4.43	56.2	3.30	+ .05	3.53	3.49	42.90	80.00	.04
4.22	4.14	60.3	5.30	.08	3.32	3.26	46.20	120.00	.06
3.68	3.57	63.9	7.70	.11	2.90	2.82	49.00	150.00	.08
3.42	3.28	67.2	9.80	.14	2.70	2.60	51.60	185.00	.10
2.63	2.46	69.7	11.50	.17	2.08	1.97	53.60	205.00	.11
1.84	1.66	71.4	12.50	.18	1.45	1.33	54.90	225.00	.12
1.06	.85	72.3	13.25	.19	.84	.72	55.60	230.00	.12
+ .53	+ .33	72.6	13.50	.20	+ .42	+ .30	55.90	235.00	.12
− .53	− .72	71.9	12.90	.19	− .42	− .54	55.40	228.00	.12
− 1.06	− 1.24	70.7	12.00	.18	− .84	− .95	54.40	215.00	.11

TABLE 4—(Concluded)

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

NEGLIGIBLE RESISTANCE AND INDUCTANCE IN PRIMARY LEADS

TRANSFORMER D 7.5 Kw. 440 Volts Impressed at 60 Cycles Residual Magnetism $= +28\,K_1$					TRANSFORMER E 15 Kw. 440 Volts Impressed at 60 Cycles Res. Magnetism $= +25.0\,K_3$				
Circuit Closed at 0° Point of E. M. F. Wave Res. of Winding on 440 Volt Side $= .26\omega$					Circuit Closed at 10° Point of E. M. F. Wave Res. of Winding on 440 Volt Side $= .195\omega$				
18	19	20	21	22	23	24	25	26	27
$-33\,K_1$ $\Delta\cos\theta$	$\Delta B = -33\,K_1$ $\Delta\cos\theta$ $-.0024\,K i_m$	Flux B	Mag. Current i_m	$.0024\,K_1 i_m$ b	$-38\,K_3$ $\Delta\cos\theta$	$\Delta B = -38\,K_3$ $\Delta\cos\theta$ $-.0021\,K_3 i_m$	Flux B	Mag. Current i_m	$.0021\,K_3 i_m$ b
....	....	$28.0\,K_1$	.00	.00	....	....	$25.0\,K_3$	.. .00	.00
$+ .66\,K_1$	$.66\,K_1$	28.7	.20	.00	$+ .76\,K_3$	$+ .76\,K_3$	25.8	.05	.00
1.32	1.32	30.0	.30	.00	1.52	1.52	26.3	.10	.00
2.30	2.30	32.3	.50	.00	2.65	2.65	29.0	.30	.00
3.30	3.30	35.6	.75	.00	3.80	3.80	32.8	.50	.00
4.30	4.30	39.9	.97	.00	4.95	4.95	37.8	.75	.00
4.60	4.60	44.5	1.30	.00	5.30	5.30	43.1	1.20	.00
5.30	5.30	49.8	2.00	.00	6.10	6.10	49.2	2.00	.00
5.60	5.60	55.4	3.00	.00	6.50	6.50	55.7	3.50	.00
5.60	5.60	61.0	5.00	.00	6.50	6.50	62.2	10.00	.00
5.60	5.60	66.6	9.00	.00	6.50	6.50	68.7	20.00	.00
5.60	5.60	72.2	16.00	.00	6.50	6.40	75.1	42.00	+ .088
5.30	5.20	77.4	24.00	$.06\,K_1$	6.10	5.90	81.0	90.00	.19
4.60	4.50	81.9	32.00	.08	5.30	5.00	86.0	140.00	29
4.30	4.20	86.1	42.00	.10	4.95	4.60	90.6	190.00	.40
3.30	3.20	89.3	51.00	.12	3.80	3.30	93.9	220.00	.46
2.30	2.20	91.5	57.00	.14	2.65	2.10	96.0	240.00	.50
1.32	1.20	92.7	60.50	.14	+ 1.52	1.00	97.0	250.00	.53
+ .66	+ .5	93.2	62.00	.15	+ .76	+ .26	97.3	255.00	.54
− .66	− .8	92.4	60.50	+ .14	− .76	− 1.26	96.0	240.00	.50
− 1.32	.00	.00	.00	.00	− 1.52	− 2.00	94.0	220.00	+ .46

Transformer D

7½-kw. 440/110 volts, 60 cycles, old type, 440-volt side used as primary.

Data

Hysteresis loop and magnetization curve are given in Fig. 14.

Normal eff. e. m. f. = 440 volts.

Resistance of transformer (440 volt-winding) = .26 ohms.

Maximum value of normal exciting current = .655 amp.

From Fig. 14

$B_{max} = 33.0 \times K_1$

Normal residual magnetism = 28.0 K_1

$E_{max} = \sqrt{2} \times 440 = 622.5$ volts.

Equation (8) becomes

$$\Delta B = -33\, K_1\, \Delta\, (\cos\theta) - .0024\, K_1\, i$$

From Table 4, Columns 16-19, the maximum current is 62 amp. or about 2.6 times normal full load current, viz., $\sqrt{2} \times 17 = 24.1$ amp.

Transformer E

440, 220/220, 110 volts, 60 cycles, new type, 440-volt side used as primary.

Data

Hysteresis loop and magnetization curve are given in Fig. 15.

Normal eff. e. m. f. = 440 volts.

Resistance of transformer (440-volt winding) = .195 ohms.

Maximum value of normal exciting current = .87 amp.

From Fig. 15

$B_{max} = 38.0\, K_3$

Normal residual magnetism = 25.0 K_3

$E_{max} = \sqrt{2} \times 440 = 622.5$ volts

Equation (8) becomes

$$\Delta B = -38.0\, K_3\, \Delta\, (\cos\theta) - .0021\, K_3 i$$

From Table 4, Columns 20-23, the maximum current is 255 amp. or 5.3 times the maximum value of the normal full load current, viz., $\sqrt{2} \times 34.1 = 48.2$ amp.

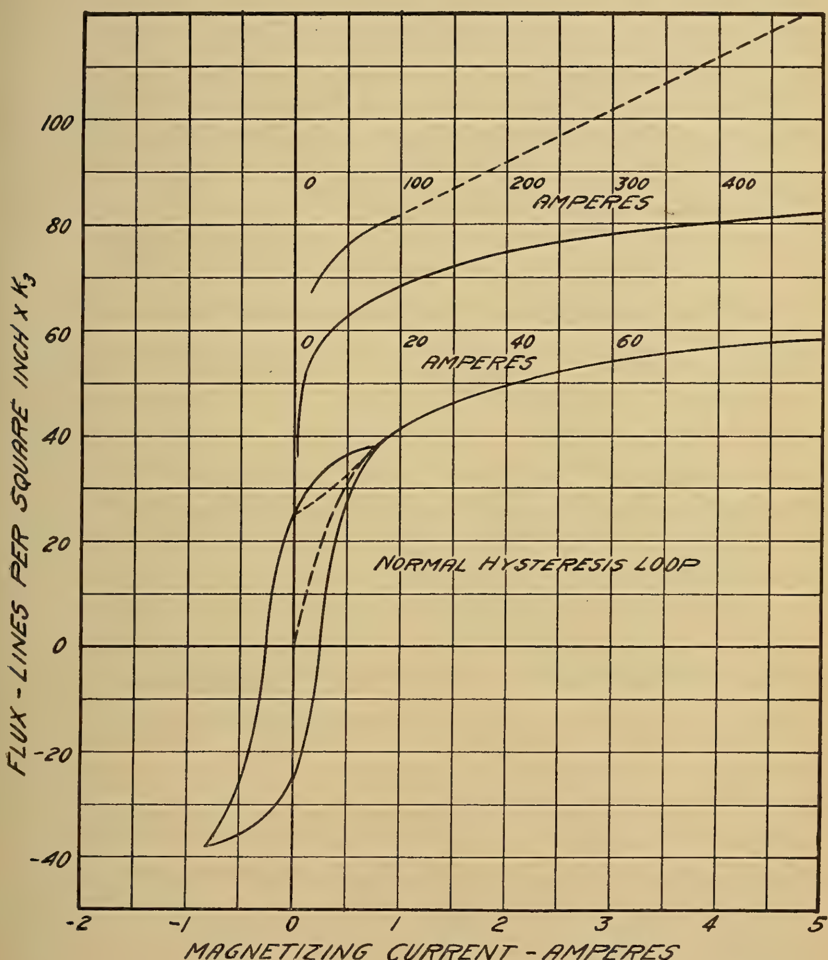


FIG. 15. MAGNETIZATION CURVE AND HYSTERESIS LOOP FOR TRANSFORMER E.

IV. Effect of Resistance and Inductance in Series with Transformer Primary.

In the preceding section, have been presented the results of connecting some transformers directly to the busbars with negligible resistance in the leads. It was shown that with the old type of transformers, the initial rush of current may amount to two to four times normal full load current, while with the new type, with silicon steel cores, the initial rush may exceed seven times full load current.

The only remedy for reducing these abnormal currents, where the transformer is to be connected to constant potential busbars, is the introduction of resistance or inductance in series with the primary winding, i. e., that side of the transformer which is to be connected to

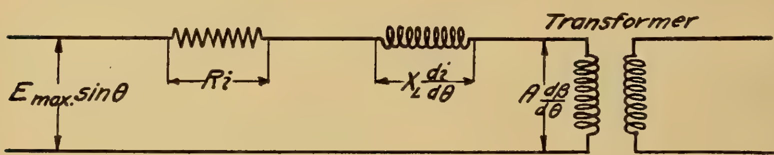


Fig. 16.

the power station. This inductance or resistance will take care of a part of the impressed e. m. f., leaving only a fraction of it to be taken care of by the counter e. m. f. of the transformer. This is shown diagrammatically in Fig. 16. The general equation then is

$$E_{\text{max}} \sin \theta = A \frac{dB}{d\theta} + X_{\text{L}} \frac{di}{d\theta} + Ri \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

which, if solved for ΔB in the same way as in Part I, gives

$$\Delta B = - B_{\text{max}} \Delta (\cos \theta) - \frac{B_{\text{max}}}{E_{\text{max}}} X_{\text{L}} (\Delta i) - \frac{B_{\text{max}}}{E_{\text{max}}} Ri (\Delta \theta) \quad \ldots\ldots \quad (11)$$

This reduces to (8) if X_{L} is negligible.

R is the total resistance of the circuit, including the transformer primary, and X_{L} is the inductive reactance outside the transformer.

In the following, will be calculated the maximum rush of current with either resistance or inductance in series with the transformer primary for two of the silicon steel transformers, transformer A and transformer C.

Transformer A.

Case 1.—For negligible inductance outside the transformer equation (11) becomes

$$\Delta B = -B_{max} \Delta (\cos \theta) - \frac{B_{max}}{E_{max}} Ri (\Delta \theta)$$

Suppose now that $R = 1.21$[1] ohms, i. e., that the outside resistance is $1.21 - .025 = 1.185$ ohms, since the resistance of the transformer primary is .025 ohms. For 10° intervals of $\theta, \Delta \theta = .175$.

$$B_{max} = 29.5 \text{ K}$$
$$E_{max} = 155 \text{ volts}$$
$$R = 1.21 \text{ ohms}$$
$$\Delta B = -29.5 K \Delta (\cos \theta) - .0403 K i.$$

Case 2.—For negligible resistance in the leads, the total resistance of the circuit may be neglected, and (11) becomes

$$\Delta B = -B_{max} \Delta (\cos \theta) - \frac{B_{max}}{E_{max}} X_L (\Delta i)$$

Assume $X_L = 1.21$ ohms

$$\Delta B = -29.5 K \Delta (\cos \theta) - .23 K (\Delta i)$$

Table 5 gives the calculations for these and the following cases. It is seen from this table that the maximum rush of current for Case 1 is 78.0 amp., and for the second case 86.0 amp. or less than twice full load current.

Transformer C.

Case 1—Negligible inductance outside transformer.

$$\text{For } R = \frac{\frac{1}{2} \text{ normal voltage}}{\text{full load current}} = \frac{1100}{24} = 43.5 \text{ ohms}$$

or $43.5 - .5 = 43$ ohms in series with the 2200-volt winding, equation (11) reduces to

$$\Delta B = -20.75 K_2 \Delta (\cos \theta) - .051 K_2 i.$$

Case 2.—Negligible resistance outside transformer.

For $X_L = 43.5$ ohms in series with the 2200 volt-winding, equation (11) becomes, neglecting resistance:

$$\Delta B = -20.75 K_2 \Delta (\cos \theta) - .29 K_2 (\Delta i).$$

From Table 5, Columns 12-14, it may be seen that the maximum rush of current for Case 1 is 50.0 amp., and for Case 2, 55 amp., i. e., in both cases less than twice full load current.

From the above calculations, it may be seen that the initial rush of current upon closing the primary circuit of a transformer can be limited to safe values by inserting either a resistance or an air core inductance in series with the primary circuit. In the particular cases above, the current was limited to less than twice full load current by

[1]This resistance multiplied by full load current, 45.5, gives a drop equal to half normal voltage: $1.21 \times 45.5 = 55$ volts.

TABLE 5

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

INDUCTANCE OR RESISTANCE IN SERIES WITH TRANSFORMERS

		TRANSFORMER A. 5 Kw. 2200, 1100/220, 110 Volts 60 Cycles. Residual Magnetism $= +20.0\,K$								
		Circuit Closed at 0° Point of E. M. F. Wave. Total Resistance $= 1.21\omega$ Inductance $= 0$					Circuit Closed at 0° Point of E. M. F. Wave. Total Resistance $= 0$. Ind. $= 1.21\omega$			
1	2	3	4	5	6	7	8	9	10	11
θ	$\cos\theta$	$-29.5\,K_2\,\Delta\cos\theta$	$\Delta B = -29.5\,K\,\Delta\cos\theta - .0403\,K i_m$	Flux B	Mag. Current i_m	$.0403\,K i_m$ b	$\Delta B = -29.5\,K\,\Delta\cos\theta - .23\,K\,\Delta i_m$	Flux B	Mag. Current i_m	$.23\,K\,\Delta i_m$
0	+ 1.00	....	....	+ 20.0 K	.00	.00	.	+ 20.0 K	.00	.00
10	.98	+ .59 K	+ .60	20.6	.20	.00	+ .6 K	20.6	.00	.00
20	.94	1.18	1.20	21.8	.40	.00	1.20	21.8	.40	.00
30	.87	2.06	2.10	23.9	.60	.00	2.10	23.9	.60	.00
40	.77	2.95	2.95	26.9	.90	.00	2.90	26.8	.90	+ .07 K
50	.64	3.83	3.80	30.7	1.25	.05	3.70	30.5	1.25	.08
60	.50	4.13	4.00	34.7	1.90	.08	4.00	34.5	1.85	.14
70	.34	4.72	4.60	39.3	3.50	.14	4.40	38.9	3.25	.32
80	+ .17	5.00	4.60	43.9	10.00	.40	4.00	42.9	7.5	.975
90	.00	5.00	4.10	48.0	21.00	.85	3.30	46.2	15.0	1.72
100	− .17	5.00	3.60	51.6	36.00	1.40	2.80	49.0	24.7	2.20
110	.34	5.00	3.00	54.6	50.00	2.00	2.50	51.5	35.5	2.50
120	.50	4.72	2.10	56.7	63.50	2.60	2.30	53.8	46.5	2.50
130	.64	4.13	1.30	58.0	72.00	2.90	1.78	55.58	56.75	2.35
140	.77	3.83	+ .70	58.7	78.00	3.10	1.55	57.13	66.5	2.23
150	.87	2.95	− .20	58.5	76.00	3.10	1.15	58.25	74.5	1.85
160	.94	2.06	− .70	57.8	70.00	2.80	.71	58.96	80.5	1.38
170	.98	1.18	.00	.00	.00	.00	.34	59.30	84.0	.80
180	1.00	+ .59	.00	.00	.00	.00	+ .20	59.50	86.0	+ .46
190	.98	− .59	.00	.00	.00	.00	− .20	59.30	84.0	− .46
200	.94	− 1.18	.00	.00	.00	.00	− .34	58.96	80.5	− .80

TABLE 5—(Concluded)

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

INDUCTANCE OR RESISTANCE IN SERIES WITH TRANSFORMERS

TRANSFORMER C

50 Kw. 2200, 1100/440, 220 VOLTS 60 CYCLES

RESIDUAL MAGNETISM = 15.25 K_2

	Circuit Closed at 0° Point of E. M. F. Wave Total Resistance = 43.5ω. Ind. = 0				Circuit Closed at 0° Point of E. M. F. Wave Total Res. = 0. Ind. = 43.5ω			
12	13	14	15	16	17	18	19	20
$-20.75 K_2$ $\Delta \cos\theta$	$\Delta B =$ $-20.75 K_2$ $\Delta \cos\theta$ $-.051 K_2 i_m$	Flux B	Mag. Current i_m	$.051 K_2 i_m$	$\Delta B =$ $-20.75 K_2$ $\Delta \cos\theta$ $-.29 K_2 \Delta i_m$	Flux B	Mag. Current i_m	$.29 K_2 \Delta i_m$ b
....	....	15.25 K_2	.00	.00	....	15.25 K_2	.00	.00
+ .42 K_2	+ .42 K_2	15.70	.05	.00	+ .42 K_2	15.7	.05	.00
.84	.84	16.50	.15	.00	.84	16.5	.15	+ .03 K_2
1.45	1.45	18.00	.30	.00	1.45	18.0	.30	.045
2.08	2.08	20.00	.40	.00	2.08	20.0	.40	.03
2.70	2.70	22.70	.55	.00	2.70	22.7	.55	.045
2.90	2.90	25.60	.90	.00	2.80	25.5	.85	.09
3.32	3.20	28.80	1.65	.08 K_2	3.00	28.5	1.50	.19
3.53	3.17	32.00	7.00	.36	2.55	31.05	4.00	1.00
3.53	2.81	34.80	15.00	.72	2.20	33.25	9.50	1.34
3.53	2.25	37.05	25.00	1.28	1.70	34.05	15.75	1.80
3.53	1.70	38.75	36.00	1.85	1.55	36.5	22.50	1.96
3.32	1.00	39.75	45.00	2.30	1.35	37.85	29.25	1.96
2.90	.35	40.10	49.00	2.50	.00	38.75	36.00	1.96
2.70	+ .15	40.15	50.00	2.55	.80	39.55	42.50	1.80
2.08	− .3	39.85	45.50	2.32	.50	40.05	48.00	1.00
1.45	− .00	39.25	40.5	2.06	.35	40.40	51.70	1.07
.84	.00	.00	.00	.00	.15	40.55	54.00	.07
+ .42	.00	.00	.00	.00	+ .13	40.68	55.00	+ .29
− .42	.00	.00	.00	.00	− .13	40.55	54.0	− .29
− .84	.00	.00	.00	.00	− .15	40.40	51.7	− 1.07

inserting a resistance or inductive reactance equal to

$$R = X_L = \frac{\frac{1}{2}\text{ normal voltage}}{\text{full load current}}$$

in series with the primary.[1]

V. Summary and Conclusions

Table 6 gives the results of the calculations of the previous sections. Columns 9 and 10 give the results for the case in which the transformers are connected to the busbars with sufficient power behind to keep the voltage constant and with negligible resistance and inductance in the leads. Columns 11 and 12 give the results for the same conditions, but with a resistance of

$$R = \frac{\frac{1}{2} E}{I_{\text{full load}}}$$

in series with the primary.

Columns 13 and 14 give the results for the same conditions but with a reactance

$$X_L = 2\pi fL = \frac{\frac{1}{2} E}{I_{\text{full load}}}$$

in series with the primary.

Transformers A, C, and E are of recent manufacture with silicon steel cores, while B and D are of an old type.

In the preceding sections, it has been shown that transient currents amounting to several times the maximum value of full load current may occur upon closing the primary circuit of a transformer. While this transient current for the old type transformers may amount to two to four times full load current, it may rise above seven times the maximum value of full load current for transformers with cores made from silicon steels with high flux densities.

This transient current becomes a serious problem only for stations containing step-up transformers, that are connected directly to the station busbars through leads of negligible resistance and inductance. If the generators are belted and of only moderate capacity, the system may be flexible enough to stand the shock due to the enormous current that may follow upon closing the transformer switches. If, however, the generators are of large capacity and direct-connected, the shock may be sufficient to cause a rupture between the generator and the prime mover. It will readily be seen that this current will be of the same order as a direct short-circuit current of the generator.

[1]For calculations of inductance coils for this purpose reference is made to University of Illinois Bulletin 53, by Prof. Morgan Brooks and Mr. H. M. Turner, entitled, "Inductance of Coils".

TABLE 6

SUMMARY OF RESULTS

1	2	3	4	5	6	7	8		9	10	11	12	13	14
		Volts							Transformer Connected Directly to Busbars		Connected to Busbars through Res. $= \frac{.5\,E^1}{I_{\text{full load}}}$		Connected to Busbars through React. $= \frac{.5\,E^1}{I_{\text{full load}}}$	
Transformers	Capacity K.V.A	Pri-mary	Secon-dary	Freq. f	Make	Type	Winding Used	Eff. Value of Full Load Cur'nt Amp.	Max. Value of Max. Rush of Current amperes	Ratio to Max. Value of Full Load Current	Max. Value of Max. Rush of Current amperes	Ratio to Max. Value of Full Load Current	Max. Value of Max. Rush of Current amperes	Ratio to Max. Value of Full Load Current
A	5	2200	220	60	X	New	110	45.50	390	6.1	78.0	1.21	86.0	1.34
		1100	110											
B	5	2080	460	60	X	Old	2018	2.40	13.5	4.0				
		1040	230											
C	50	2200	440	60	Y	New	2200	22.70	235	7.3	50.0	1.56	55.0	1.72
		1100	220											
D	7.5	440	110	60	Y	Old	440	17.05	62	2.6				
E	15	440	220	60	Z	New	440	34.10	255	5.3				
		220	110											

[1]E = Normal Voltage of Primary Winding.

If each set of transformers be connected to one generator, the problem will consist simply in bringing the voltage up slowly with the transformers connected, but in cases where it becomes necessary to connect transformers to busbars of full potential, it becomes necessary, for safe operation, to insert in the primary circuit a resistance or inductance to limit the transient current to safe values. It has been shown that a resistance or inductive reactance amounting to

$$\frac{\frac{1}{2}\text{ normal voltage}}{\text{full load current}}$$

will limit the current to less than twice full load current under the most critical conditions.

This resistance or inductance needs to be in the circuit for only a very short time, since the current will fall down to below full load

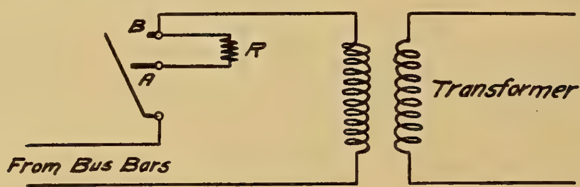


Fig. 17

current after a few cycles. The resistance or inductance may be connected as shown diagramatically in Fig. 17, where an extra contact, A, is provided on the switch, in such a way that, in closing the switch the contact A is reached before the main contact B is reached. It might be possible to provide such a contact even on oil switches. As the interval between the time the switch touches A until it touches B, need be only a fraction of one second, no change in the operating mechanism of the switch would be necessary. Usually, it takes an oil switch about 0.5 second to close, i. e., from the time it starts until the switch is closed. If the contact A is located 1/3 of the way from the closed position, it may take the contact 0.1 second to travel from A to B, and this time will be sufficient even for 25 cycle systems.

Appendix

Residual Magnetism.—It has been generally believed that residual magnetism is of a transient nature, i. e., that the magnetism that remains in the iron, after removing the magnetizing force, gradually decreases.

As this point is very important in connection with the starting current of transformers, an experiment was undertaken to ascertain whether the residual magnetism is a permanent quantity or not. Connections were made as shown in Fig. 18.

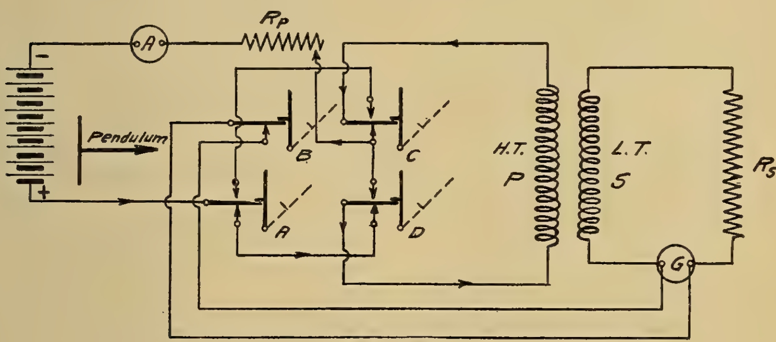


Fig. 18

Referring to the hysteresis loop of Fig. 3, it may be seen that, starting at 1, opening the circuit produces a change of flux corresponding to 1-2; reversing the current produces a change 2-3, etc. However, if the residual magnetism at 2 decreases before the current is reversed, the change of flux will be less than 2-3. Consequently, the problem consists in determining whether this change varies with the time elapsing between opening the circuit and closing it again in the opposite direction.

The time interval was controlled by means of a pendulum and four contacts, A, B, C and D. By tracing out the connections in Fig. 18, it will be seen that with all the levers in the upright position, the current flows through the circuit in the direction of the arrows, while the galvanometer G is shortcircuited by means of contact B. If now the pendulum is started from the position shown, lever A is first knocked down opening the primary circuit. The change of flux 1-2 does not produce any deflection of the galvanometer, because it is shortcircuited until lever B is knocked down. Finally, C and D are knocked down,

reversing the current, producing a change of flux 2-3, and since the galvanometer is no longer shortcircuited, this change produces a deflection of the galvanometer, proportional to the change of flux.

The shortest distance between A and D corresponded to about .1 second, and it could be increased to about ½ second.

The transformer used was the 5-kw. 2080, 1040/460, 230 volt, 60-cycle transformer, designated as transformer B. A resistance R_p, of 400 ohms, was inserted in the primary to decrease the time constant, and the current was maintained at the maximum value of the normal magnetizing current, namely, .1 amp.

1. The deflection corresponding to change 2-3 and .1 second interval between A and D was 48.0 cm. The interval was then in turn increased to ½ second, and by hand operation to 1 min., 5 min., 90 min., 12 hrs., and 24 hrs.

The deflection corresponding to the change 2-3 was in every case the same.

2. To ascertain whether there was any decrease of flux during the .1 second period, the resistance R_g amounting to more than 1 megohm was cut out. The sensitiveness of the galvanometer is such that 10^{-8} amp. corresponds to 1 mm. deflection. The transformer core has a cross-section of about 4 sq. in. and the number of turns of the 230-volt winding is 80. The normal flux density is about 50 000 lines per sq. in., and the resistance of the galvanometer circuit was less than 1000 ohms. Hence to produce a deflection of one cm. required 10^{-7} amp, or $10^{-7} \times 10^{3} = 10^{-4}$ volts.

$$E = 10^{-4} = \frac{80 \times \phi}{10^{8}}$$

$$\phi = \frac{10^{-4}\ 10^{8}}{80} = \frac{10^{4}}{80} = 125 \text{ lines per sec.}$$

That is, it requires a change of flux of 125 lines per sec. to produce a large deflection of the galvanometer. As the total normal flux is $50\,000 \times 4 = 200\,000$ lines, this is less than 1/10 per cent, so that any material change of flux occurring within the first 1/10 sec. after opening A would be recorded. The time elapsing between A and B was less than .01 sec. The pendulum was stopped after knocking down B, so that any deflection occurring after the change 1-2 would be due to the decrease of the residual magnetism.

As a result of the several trials, not the slightest deflection could be detected.

3. It was finally attempted to determine the effect of vibration and blows upon the permanency of the residual magnetism. With the same connections as in 2, A and B were knocked down. The trans-

former core was then given a series of blows with a hammer. The first blow produced a deflection of about 50 cm., and the successive blows produced deflections decreasing very rapidly. This deflection corresponds to about $50/1000 = .05$ cm. with the resistance R_g in circuit.

Change 1-2 produced a deflection of 8.5 cm.
Change 2-3 produced a deflection of 48.0 cm.
Total change = 56.5 cm.

This means that the maximum value of the normal flux corresponds to 28.3 centimeters deflection, and the residual magnetism to $28.3 - 8.5 = 19.8$ cm. deflection. Consequently, a deflection of .05 cm. corresponds to a decrease of residual magnetism due to severe blows of 5/20 per cent = 1/4 per cent.

The transformer was finally given continuous hard blows for 5 minutes (one blow every other second) after point 2 had been reached. With the resistance R_g cut out, the effect of the last blows could hardly be noticed. R_g was then replaced in the circuit and the deflection corresponding to the change 2-3 was observed. The result showed that the effect of the above severe treatment was to decrease the residual magnetism by 4 per cent.

Conclusion.—From the above, the conclusion seems justified that there is no decrease in the residual magnetism of a transformer under normal conditions, and that the decrease due to vibration and ordinary shocks is negligible.

LIST OF PUBLICATIONS

ALL BULLETINS, THE TITLES OF WHICH ARE NOT STARRED, WILL BE SENT FREE UPON APPLICATION.

**Bulletin No.* 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

**Circular No.* 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

**Bulletin No.* 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

**Circular No.* 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

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**Bulletin No.* 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

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W. F. M. GOSS,
Director.

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BULLETIN NO. 56

TESTS OF COLUMNS: AN INVESTIGATION OF THE VALUE OF CONCRETE AS REINFORCEMENT FOR STRUCTURAL STEEL COLUMNS

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 56 MARCH 1912

TESTS OF COLUMNS: AN INVESTIGATION OF THE VALUE OF CONCRETE AS REINFORCEMENT FOR STRUCTURAL STEEL COLUMNS

BY ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING AND IN CHARGE OF THEORETICAL AND APPLIED MECHANICS, AND ARTHUR R. LORD, RESEARCH FELLOW IN THEORETICAL AND APPLIED MECHANICS

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I. INTRODUCTION

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TESTS OF COLUMNS: AN INVESTIGATION OF THE VALUE OF CONCRETE AS REINFORCEMENT FOR STRUCTURAL STEEL COLUMNS.

I. Introduction.

1. *Preliminary.*—In reinforced concrete building construction columns form an important element, and in the case of very high or very heavily loaded buildings, the size of the columns and the space they occupy become important considerations. Various types of reinforced columns are in use. Columns with longitudinal reinforcement and hooped columns are common. During the past few years designers have used structural steel columns encased in concrete. Sometimes the structural steel shapes form a relatively small proportion of the column section and are considered as reinforcement for the concrete. In other designs the amount of steel is much larger and the structural shapes will carry a large proportion of the load so that the column instead of being a reinforced concrete column is really a steel column reinforced with the concrete in which it is embedded. Such columns may occupy less space than the reinforced concrete column as usually designed.

Two points of view seem to exist with reference to columns having a large percentage of structural steel: (a) that the concrete surrounding the steel simply affords protection from fire and corrosion and that the additional strength afforded by the concrete is not considerable in amount and is not available for design; and (b) that if the concrete be present it must act in unison with the steel and that its strengthening effect and its effect upon the permissible deformation of the column should be taken into account. The present building codes either directly or through the relation of stresses allowed virtually occupy the first position when the steel column forms more than 8% of the column section.

The series of tests described in this bulletin was planned to throw light upon the action of columns formed of structural steel shapes by filling the space between the shapes with concrete or encasing them in concrete as exemplified in a form of column section which has been used in reinforced concrete building construction. It is hoped that the results will be helpful in discussing fundamental principles underlying the design of such columns.

2. *Scope of Bulletin.*—The investigation was planned with the view of securing information on the following principal points for the section and type of column tested: (1) the effect of length and slenderness on the strength of the plain steel columns; (2) the effect of length upon the strength of similar columns made up with a core of concrete; (3) the effect of richness of concrete in the core filling upon the strength of the column; (4) the effect of adding an exterior coat around the steel section upon the strength of the column, and the action of this coat under load; (5) the effect of spiral hooping upon the strength and stress-deformations of the column.

The tests were planned in an effort to obtain the most information on these points with the 32 columns available for the

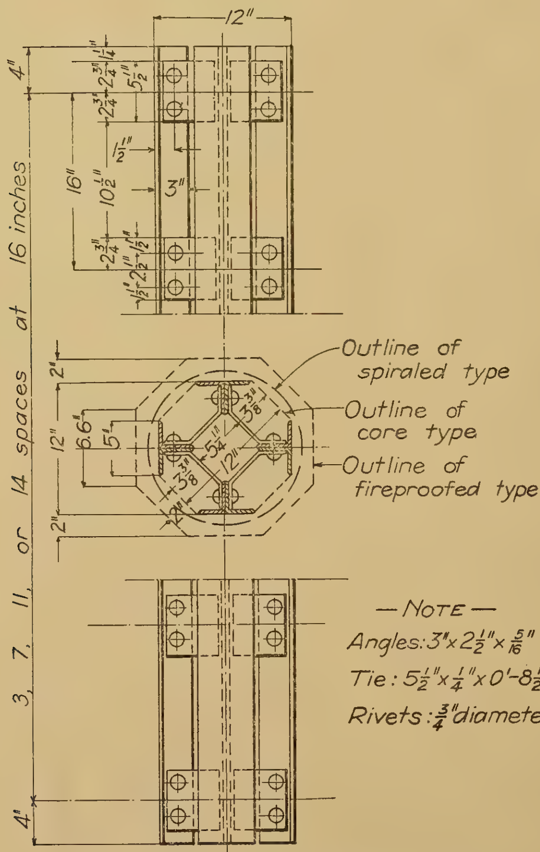

FIG. 1. DETAILS OF TEST COLUMNS.

tests. The results may not be applicable to sections or types of columns which differ from the form used.

3. *Acknowledgment.*—The steel columns used in the tests were furnished through the courtesy of the Illinois Steel Company, Mr. E. J. Buffington, President, and were made at the North Works of the Company in Chicago. The spirals for the six spiraled columns were furnished by the American System of Concrete Reinforcing, of Chicago. The work of concreting the columns was directly supervised by Mr. D. A. Abrams, Associate in the Engineering Experiment Station of the University of Illinois. The tests were made by Mr. Lord with the assistance of Mr. R. K. Steward and members of the staff of the Engineering Experiment Station. Mr. Lord is responsible largely for outlining the scope of the tests and for working up the results. Mr. W. A. Slater, First Assistant in the Engineering Experiment Station, has given helpful assistance in putting the bulletin into final form. Especial acknowledgment is made to Professor Frank P. McKibben and the Department of Civil Engineering of Lehigh University for the work of testing the four spiraled columns sent to the Fritz Engineering Laboratory and for the great interest and care so freely given.

II. Materials, Test Pieces, and Methods of Testing.

4. *The Steel Columns.*—The structural steel columns used were of the Gray type and were composed of eight angles, $3 \times 2\frac{1}{2} \times \frac{5}{16}$-in., tied at intervals of 16 inches by $5\frac{1}{2} \times \frac{1}{4}$-in. plates as shown in Fig. 1. This type of column has been used in the construction of reinforced concrete buildings and has proved very satisfactory. The size of angles, radii of gyration, dimensions of tie plates, slenderness ratio of the flanges between tie plates, radius of gyration of column and slenderness ratio of columns give relations which are in many ways comparable to column sections which have been used in reinforced concrete building construction.

The method of fabricating these columns was in no sense unusual. The ends, although milled, did not present a true bearing as they were received in the laboratory. The steel was open hearth structural steel. Tension tests of $1\frac{1}{2} \times \frac{5}{16} \times 18$-in. specimens cut from an untested flange gave an ultimate strength of 62 000 lb. per sq. in. and a yield point of 39 800 lb. per sq. in.

The ultimate compressive strength of flanges composed of two $3 \times 2\frac{1}{2} \times \frac{5}{16}$-in. angles riveted back to back averaged 39 700 lb. per sq. in. for two specimens 32 inches long, and 33 200 lb. per sq. in. for two specimens 7 ft. 10 in. long.

5. *Sections of Columns.*—Table 5, page 14, gives the schedule of the columns tested. Ten steel columns were tested without concrete reinforcement; these columns are called "plain steel columns" in the schedule and in the discussion. For studying the effect of concrete in connection with the steel, the space inclosed within the outline of the structural shapes, as shown in Fig. 1, was filled with concrete. This combination of structural shapes and concrete will be termed the "core type of column". It was adopted as the principal form of test piece because it was believed wisest to obtain the larger part of the data with the section which is considered to be the effective section in design. For purposes of comparison three columns were made with 2 inches of concrete outside of the steel (see Fig. 1), and the action of this outer shell under load was studied. This combination of steel and concrete is here termed the "fireproofed type." The effect of richness of concrete upon the strength of the core type was sought by the testing of columns with 1–1–2 and 1–3–6 mixtures in addition to the 1–2–4 mixture. In six columns the steel was inclosed within a wire spiral and the space filled with concrete to the outer face of the spiral. The spirals were 14 in. in diameter, were of $\frac{1}{4}$-in. steel wire with a pitch of 2 in. and $1\frac{1}{2}$ in., respectively, the percentages of spiral reinforcement used being 0.75 and 1.0. The percentage of the structural steel section in terms of the whole column area varied, being 10.8 % for the core type, 6.1 % for the fireproofed type and 8.5 % for the spiraled columns.

TABLE 1.

TENSILE STRENGTH OF CEMENT.

Ref. No.	Age 7 Days			Age 28 Days		
	Neat	1:3 Standard Sand	1:3 Sand Used in Columns	Neat	1:3 Standard Sand	1:3 Sand Used in Columns
1	589	198	265	674	278	323
2	684	227		709	283	
3	653	240		731	319	

6. *Cement and Aggregates.*—The cement used was furnished by the Universal Portland Cement Company. Tests of samples taken at times through the season and made by B. L. Bowling, Assistant in charge of the Cement Laboratory, are given in Table 1. Sample No. 1 was taken October 14, No. 2 November 22, and No. 3 January 15. In the fineness test 98.5 per cent passed No. 50 sieve, 96.5 per cent passed No. 100 sieve, and 82.5 per cent passed No. 200 sieve.

The sand used was torpedo sand from Attica, Indiana. It was of good quality, fairly sharp, clean and well-graded. It combined with the cement used in a very satisfactory manner and gave a higher briquette test than did the same cement with standard Ottawa sand. It was from the same locality and of the same quality as the sand used in making reinforced concrete test specimens for several years at the University of Illinois.

A good quality of rather hard limestone from Kankakee, Illinois, specified to pass through a 1-in. screen and over a $\frac{1}{4}$-in. screen, was used. It is representative of the stone most used in building construction of reinforced concrete in Illinois and it was of the grade which has been used in the previous experimental work of the Laboratory. In the columns tested the failure did not appear to result from the crushing or breaking of the stone in any way.

7. *Concrete.*—Table 2 gives the proportions of the materials used in the different batches of concrete from which the columns were made. The sand and stone were first measured by loose volume. A bag of cement (95 lb.) was considered as 1 cubic foot of cement. The resulting proportions by weight are given in the table.

Men skilled in mixing concrete and making test pieces were employed in the work. The foreman and the other workmen are experienced concrete workmen; they have made the specimens for the laboratory for six seasons. The mixing was done with shovels by hand. The sand and cement were first mixed dry; the stone, which had previously been thoroughly moistened, was added and the mix then turned until of a uniform appearance. Usually the first operation included five or six turnings and the second three or more. Water was added in sufficient quantity to produce a distinctly wet mixture which would run rapidly from the shovel. The whole was then turned until thoroughly mixed.

8. *Making of Columns.*—The forms for the core type were extremely simple inasmuch as the concrete was confined to the octagon determined by the edges of the steel flanges. Four planks of the correct length were placed in a vertical position directly against the flanges and were held in position by means of yokes. For the fireproofed type octagonal forms of wood were built to give the required 2 inches of clearance over all faces. For the spiraled type circular metal forms were placed directly against the wire spiral and held in place by bands as shown in Fig. 1 of Bulletin No. 20 of the University of Illinois Engineering Experiment Station.

TABLE 2.

PROPORTIONS OF CONCRETE INGREDIENTS.

Column No.	By Volume			By Weight		
	Cement	Sand	Stone	Cement	Sand	Stone
8907	1	2	4	1	2.16	3.55
8908	1	2	4	1	2.05	3.55
8912	1	2	4	1	2.16	3.55
8913	1	2	4	1	2.05	3.55
8917	1	2	4	1	2.11	3.35
8918	1	2	4	1	1.88	3.62
8922	1	2	4	1	2.09	3.42
8923	1	2	4	1	2.04	3.54
8925	1	1	2	1	1.06	1.81
8926	1	1	2	1	1.04	1.76
8927	1	3	6	1	3.14	5.43
8928	1	3	6	1	3.02	5.30
8929	1	2	4	1	2.04	3.52
8930	1	2	4	1	2.13	3.60
8931	1	2	4	1	2.07	3.60
8933	1	2	4	1	2.08	3.61
8934	1	2	4	1	2.02	3.51
8935	1	2	4	1		
8936	1	2	4	1	2.02	3.56
8937	1	2	4	1	2.04	3.52
8938	1	2	4	1	2.08	3.67

The concrete was placed by pouring it into the forms at the top of the column a bucketful at a time. It was worked around the sides and in the center by means of a pole. As an aid in securing uniform concrete successive bucketfuls were taken from different portions of the pile. The forms were filled practically level with the top of the steel.

Previous to pouring the concrete, the steel column was placed in a vertical position on a 14 x 14 x 1-in. cast-iron bearing plate (upon which it was later tested) and the forms were placed in position. The day after pouring, after the concrete in the column had had time to shrink, the top of the column was pre-

pared for testing by setting another bearing plate upon it, using a neat cement mortar as a bed. Care was taken to have this plate rest equally upon the four flanges and also to bear on the mortar. The layer of mortar between the plate and the steel was kept as thin as possible. A film of cement over the end of the steel was necessarily present; but in most cases it was extremely thin and in no case did this thickness reach $\frac{1}{8}$ inch. In pouring the spiraled type it was found that the concrete did not flow into the small space at the back of the flanges between the wires of the spiral, and it was necessary to grout into this space with a sand mortar. In columns of the fireproofed type the concrete outside the flanges was found to be less dense and in some cases the surface over the flanges was more or less pitted. As has always been the practice in the preparation of concrete specimens in the laboratory, the fabrication was done seriatim, one specimen of a kind well through the series, followed later with the fabrication of the second specimen seriatim. By this means any accidental variation of cement or aggregate or of temperature conditions would be likely to have the same effect on one type of specimen as on another, whereas if all the specimens of the same kind were made at the same time the presence of such variations might be mistakenly considered to be caused by variations in type. This practice is explained at some length here, because considerable variation was found in the concrete of specimens of the same type made at different times. The dates of making specimens were as follows: On October 2, 1910, No. 8907 and 8912 were made from one batch of concrete and No. 8917 from another batch. On October 29, No. 8922 was made from one batch and No. 8925 from another batch. On November 4, No. 8927 was made. On November 8, No. 8908 and 8913 were made from the same batch, No. 8918 from a batch, and No. 8923 from another batch. November 12, No. 8926 was made and November 19, No. 8928 was made.

9. *Auxiliary Test Specimens.*—From each batch of concrete used three 6-in. cubes and one 8 x 16-in. cylinder were made from which to determine the properties of the concrete in the columns. These were stored in damp sand until a few days before the corresponding column was to be tested when they were removed to the testing laboratory and two faces prepared for the test by the addition of a thin coat of plaster of paris. It was originally intended to test all these specimens at the same date as the corresponding column, but an unexpectedly low strength was found

in the cube tests of the first specimens made. One cube from each of the later batches was then reserved for test at 90 days in order to know whether the concrete was poor or whether it merely lacked curing. It was found that these later specimens gave what may be considered to be normal strength, and hence no light was thrown on the cause of the low strength of the concrete from the first batches. The results of the cube and cylinder tests are given in Table 3. These auxiliary tests show that the strength of the concrete at the time of testing was far from uniform. The possible causes of this are discussed in a later paragraph.

TABLE 3.

COMPRESSION TESTS OF CUBES AND CYLINDERS.

Loads are given in pounds per square inch.

Corresponding Column No.	6-in. Cubes		8 x 16-in. Cylinders 60 days
	60 days	90 days	
8907	1790		1350
8908	2430	2650	1490
8912	1790		1350
8913	2430	2650	1490
8917	1420		1140
8918	2150	2600	1260
8922	1320		970
8923	1970	2640	1150
8825	2970		2420
8926	3280	4000	2520
8927	1320	1800	700
8928	1440	1740	660
8929	1760	2580	1370
8930	2020	2740	1330
8931	1670	2760	1280
8933	1440	2000	1160
8934	2070	2350	1340
8935	2270	2720	1330
8936	1780	2180	1110
8937	1620	2900	1460
8938	1980	2580	1120

10. *Storage and Handling.*—The columns were stored in the room where they were made. Forms were removed at the end of a week and from that time on the columns were occasionally sprinkled. The records show that the room temperature varied from 60° to 70° Fahrenheit, but it seems probable that a larger variation may have occurred in different parts of the building. It is also probable that the later columns dried out much more and attained a higher percentage of their final concrete strength than did the cubes at the same age.

Before removing the columns to the testing laboratory the bearing plates at top and bottom were connected by rods to prevent displacement, and these rods were not removed until the column was in its final position in the testing machine.

11. *Method of Measuring Deformations.*—The extensometer used and the method of attachment are shown in Fig. 2. For the determination of deformations in the steel, holes were tapped in the flanges at the lower point to receive the shaft of a wire-wound dial extensometer and at the upper point to receive a bolt; from this bolt a wire was suspended, wrapped once around the drum of the extensometer and weighted with a nut at the end. The deformation occurring in the gauge length between the bolt and the

TABLE 4.

GAUGE LENGTHS AND PRECISION OF MEASUREMENTS.

Length of Column	Gauge Length	Least Unit-deformation Measurable
2 ft.-0 in.	10 in.	.00002
4 ft.-8 in.	40 in.*	.000005
10 ft.-0 in.	100 in.	.000002
15 ft.-4 in.	150 in.	.000001
19 ft.-4 in.	200 in.	.000001

*Average.

lower shaft was registered by the movement of the pointer of the dial. Measurements were made on each of the four flanges, four instruments being used for the purpose. Where measurements were desired on concrete faces, specially prepared plugs were inserted in the column during the pouring and these were tapped to receive the bolts and the shafts of additional extensometers. The wire suspended upon the upper bolts was in general 1 inch from the face of the column, and the accuracy of the observations depends upon the conservation of the plain section in the column as a whole. The results obtained indicated that this condition was not fully satisfied in the case of plain steel columns. An arrangement of instruments which gave the deformations at the center of gravity of the flanges direct was used in the test of flanges and on column No. 8914. The result indicated that the error in the first arrangement of instruments was not great. The gauge lengths used varied with the lengths of the specimens. The dials could be read to an indicated movement of 0.0002 inch. The gauge

length and the least unit-deformation obtainable for the different lengths of the columns are given in Table 4.

In the case of the plain steel columns 10 feet or more in length, in addition to the four measurements of deformation taken over the gauge lengths noted in Table 4, six other measurements of deformation were taken over gauge lengths about one-third as long. These shorter gauge lengths were located at different portions of the columns in an effort to detect local bending. The measurements did not show that material bending occurred at loads below the maximum load, and if such bending occurred, it was confined to shorter distances.

12. *Method of Testing.*—The columns were tested in the Riehle vertical 600 000-lb. screw-power testing machine in the Laboratory of Applied Mechanics of the University of Illinois. It was appreciated in advance that the full strength of the spiraled columns could not be developed in the 600 000-lb. testing machine, but it was thought that there would be a sufficient indication of their action to determine the critical yield-point. However, after testing them it was concluded that a determination of the action of such columns at higher loads would be of value, and further tests of four of the columns were made in the Riehle 800 000-lb. testing machine at Lehigh University. In the case of the plain steel columns a specimen was placed with its lower end bearing directly on the weighing table of the testing machine and centered with respect to the screws. The compression head of the machine was then lowered until the suspended spherical bearing block rested on top of the column. This block was then centered on the column. Although the ends of the columns were milled in the shops, in some cases it was necessary to file the ends to secure a satisfactory bearing and in some cases to use carefully prepared steel shims. In the case of the concreted columns the bearing plates of the columns rested directly on the weighing table. When the column was in its final position in the machine, the rods connecting the end bearing plates, used while transporting the column to the testing room, were removed and the spherical bearing block was lowered into position as noted for the plain steel columns. An initial reading of the extensometer was taken with no load on the column except the weight of the spherical bearing block. The compression head of the testing machine was then brought to bear on the bearing block and was run down at the slowest speed (0.05 inch per minute)

until a load of 25 000 lb. was registered. The machine was stopped at this load and after an interval of 30 to 45 seconds the extensometers were read. In like manner the load was increased by increments of 25 000 lb. at the speed of 0.05 inch per minute, with readings of the extensometers between applications of load, until the maximum strength of the column was passed or the capacity of the testing machine was reached. With the first five columns tested (No. 8905, 8906, 8910, 8915, 8920) no effort was made to restrain any movement of the spherical bearing block. In the remaining tests, when a load of 50 000 lb. had been applied, special wedges were inserted and adjusted in the bearing block to restrain it from a rolling movement. In the tests of many of the columns the operation of the machine was continued after the maximum load was passed, to determine the effect of a further application of load and a further shortening of the column. This was done in order that the critical section might be definitely determined and the nature of the final failure observed.

Three conditions mentioned incidentally above merit more complete discussion:

(a) It is evident that the speed at which the testing machine is run is of importance and that a column will carry a higher maximum load if the load be applied rapidly than it will under a slow application. In actual use the load is not momentarily applied but it is a dead weight and follows at once any shortening or other movement of the column. In testing, the load should be held a sufficient time to ensure that the material has attained its full deformation. In the tests herein described the slowest speed of the machine (0.05 in. per min.) was used in all cases. The instruments were read starting about 30 seconds after the indicated load had been momentarily applied.

(b) The condition of the bearing surfaces at the ends of the column has an important effect upon the load which the column will carry. In a building a column receives its load from story to story and especial attention is given to the bearing arrangement at its base, but in a testing machine the load is applied directly upon the end section. If the load is to be uniformly distributed, the bearings of the ends must be perfect. In these tests care was taken to get a fairly good bearing, but it should be noted that the load was not always uniformly distributed over the section and that some of the flanges were more highly stressed than others. The records of observed deformations of the various gauge lengths in the different flanges show this effect. When

TABLE 5.

DATA OF TEST COLUMNS.

Column No.	Description	Length ft. in.	$\frac{l}{r}$	Area of Gross Section sq. in.	Concrete Mixture	Age of Test days
8902	Plain Steel	2-0	6.1	13		
8905	Plain Steel	4-8	14.4	13		
8906	Plain Steel	4-8	14.4	13		
8907	Core Type	4-8		120	1-2-4	60
8908	Core Type	4-8		120	1-2-4	59
8910	Plain Steel	10-0	30.8	13		
8911	Plain Steel	10-0	30.8	13		
8912	Core Type	10-0		120	1-2-4	60
8913	Core Type	10-0		120	1-2-4	62
8914	Plain Steel	10-0	30.8	13		
8915	Plain Steel	15-4	47.2	13		
8916	Plain Steel	15-4	47.2	13		
8917	Core Type	15-4		120	1-2-4	61
8918	Core Type	15-4		120	1-2-4	59
8920	Plain Steel	19-4	59.5	13		
8921	Plain Steel	19-4	59.5	13		
8922	Core Type	19-4		120	1-2-4	60
8923	Core Type	19-4		120	1-2-4	60
8925	Core Type	10-0		120	1-1-2	61
8926	Core Type	10-0		120	1-1-2	60
8927	Core Type	10-0		120	1-3-6	59
8928	Core Type	10-0		120	1-3-6	60
8929	Fireproofed	10-0		213	1-2-4	60
8930	Fireproofed	10-0		213	1-2-4	60
8931	Fireproofed	10-0		213	1-2-4	60
8933	Spiraled*	10-0		153	1-2-4	60
8934	Spiraled*	10-0		153	1-2-4	59
8935	Spiraled*	10-0		153	1-2-4	59
8936	Spiraled*	10-0		153	1-2-4	60
8937	Spiraled†	10-0		153	1-2-4	60
8938	Spiraled†	10-0		153	1-2-4	59

*0.75 % of spiral reinforcement.
†1.0 % of spiral reinforcement.

the load is unevenly distributed over the section the tendency towards bending is increased. The effect of poor end conditions is more serious in short columns than in long ones; in the 19 ft. 4-in. columns its effect is probably negligible. In studying the effect of length on column strength the effect of end conditions must be borne in mind. Columns No. 8902 and 8914 had the best end conditions of any tested. In the 2-ft. column (No. 8902) a test was found to be impracticable without carefully turning the ends in a lathe.

TABLE 6.

Maximum Loads Carried by Columns and by Steel and Concrete.

Column No.	Total Load in Pounds			Pounds per square inch of Section	
	Column Load	Load Considered as Carried by Steel	Load Considered as Carried by Concrete	Steel	Concrete
8902	487 300				
8905	440 200			33 700	
8906	449 000			34 700	
8907	577 000	444 600	132 400	34 200	1240
8908	602 000	444 600	157 400	34 200	1470
8910	410 400			31 600	
8911	425 600			32 700	
8912	510 000	418 000	92 000	32 150	860
8913	584 700	418 000	166 700	32 150	1560
8914	424 000			32 600	
8915	368 000			28 300	
8916	376 000			28 900	
8917	468 500	372 000	96 500	28 600	900
8918	532 200	372 000	160 200	28 600	1500
8920	374 200			28 800	
8921	345 000			26 500	
8922	491 400	359 600	131 800	27 650	1230
8923	495 500	359 600	135 900	27 650	1270
8925	636 000	418 000	218 000	32 150	2040
8926	655 000	418 000	237 000	32 150	2210
8927	516 000	418 000	98 000	32 150	920
8928	530 500	418 000	112 500	32 150	1050
8929	600 000*				
8930	630 700	418 000	212 700	32 150	1060
8931	635 700	418 000	217 700	32 150	1090
8933	600 000	Maximum load applied five times; not broken.			
8934	600 000	Not broken.			
8935	600 000	Maximum load applied three times; not broken.			
8936	625 000	Not broken.			
8937	600 000	Maximum load applied three times; not broken.			
8938	600 000	Not broken.			
8934	856 000	Second test; near ultimate.			
8935	830 000	Second test; near ultimate.			
8936	600 000	Second test; not broken.			
8937	830 000	Second test; not broken.			
8938	827 000	Second test; not broken.			
8937	714 000	Third test with spiral and outside concrete removed.			

*Not broken.

(c) The conditions of the end restraint will affect the curve taken by a column. The lower end, bearing on the unyielding weighing table of the testing machine, is quite firmly restrained until serious bending occurs in the column. The upper end is loaded through a spherical bearing block in order to secure adjustment

with the compression head of the testing machine. With this arrangement, this adjustable bearing block may finally move on itself as the tendency to bend in the column overcomes the friction between the surfaces of the spherical bearing block, and thus only partial restraint will exist. In the first five plain steel columns tested (No. 8905, 8906, 8910, 8915, 8920) no effort was made to prevent this movement, and the column was in the condition of one end fixed and one end partly restrained. Neither the fixedness nor the freedom of the ends can be considered as in any sense absolute; they must be taken as relative terms. In all the column tests after the five noted above, special wedges and angle blocks were driven under the upper head (at a load of 50 000 lb.), the

TABLE 7.

LOADS CARRIED AT VARIOUS UNIT-DEFORMATIONS.

Loads are given in thousands of pounds for the unit-deformation given in the column caption and in the last column for the maximum load applied.

Column No.	.0002	.0004	.0006	.0008	.0010	.0012	.0014	.0016	.0018	.0020	.0030	.0040	Max. Load
8905	83	160	229	290	341	379	405	422					440
8906	83	161	235	300	355	396	421						449
8907	115	206	286	360	419	467	515	557					577
8908	120	216	311	390	465	529	565	592					602
8910	82	158	226	286	338	372	393						410
8911	82	155	221	281	335	377	402						426
8912	121	207	286	355	414	455							510
8913	123	224	319	390	461	516	548						585
8914	80	160	233	300	355	390	410	419	423				424
8915	84	154	216	271	319	352							368
8916	86	159	221	276	321	353	373						376
8617	109	193	268	336	393	432							468
8918	118	208	288	363	429	482	509						532
8920	80	150	211	262	308	349							374
8921	84	156	214	265	310								345
8922	115	199	269	331	387								491
8923	114	206	285	358	427	477							495
8925	132	240	336	426	505	568	604	624					636
8926	119	226	320	405	486	553	594	619	635	647			655
8927	107	190	265	334	397	447	480	500					516
8928	105	187	264	335	399	452	486	500					530
8929	141	257	357	452	534	594							600‡
8930	117	218	310	398	474	536	581	611					631
8931	144	254	347	424	492	546	585	610	626				636
8933	126	231	322	409	489	550	588						600‡
8934	134	235	321	394	459	511	545	568	588				600‡
8935	136	242	338	430	516	578	600	600					600‡
8936	129	234	324	406	483	541	579						625‡
8937	135	242	338	427	508	567	600						600‡
8938	124	230	326	416	491	543	581						600‡
8934*	112	220	325	430	525	625	685	715	737	755	808	835	856‡
8935*	100	200	300	410	525	640	690	720	743	761	815		830‡
8936*	110	200	290	375	460	540	592						600‡
8937*	96	194	295	404	520	628	680	710	733	753	815		830‡
8938*	110	218	322	430	540	653	705	730	750	768	820		827‡
8937†	111	225	340	453	567	656	700	712	714				714

*Second test; all second tests made at Lehigh University except column No. 8936.
†Third test; made at Lehigh University.
‡The column did not fail under maximum load applied.

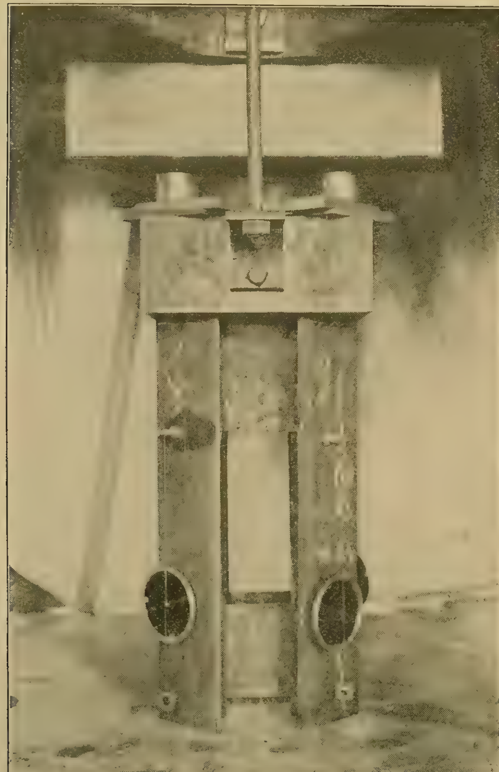

Fig. 2. View of Bearing Block and Attachment of Instruments.

Fig. 3. View Showing Core Type of Column beyond the Maximum Load.

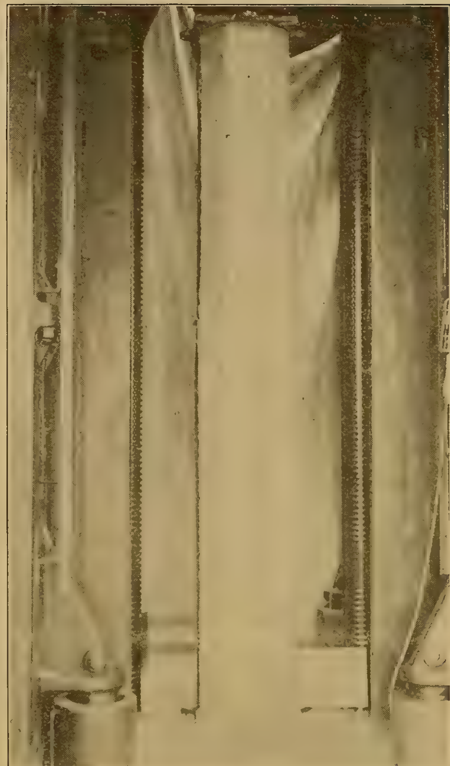

Fig. 4. View Showing Fireproofed Column beyond the Maximum Load.

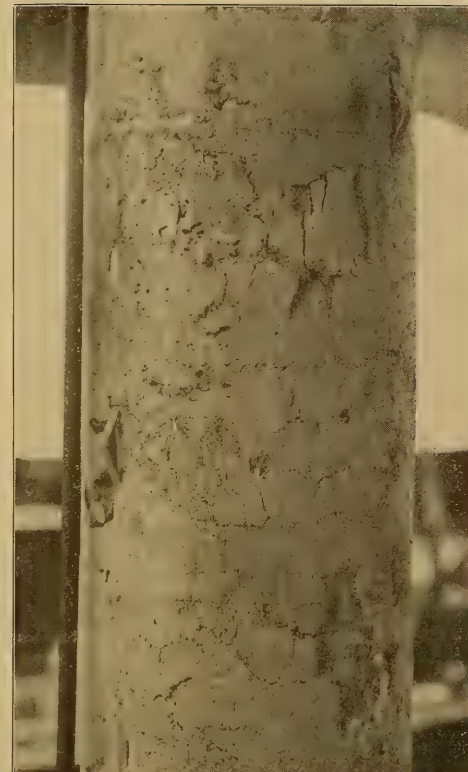

Fig. 5. View Showing Spiraled Column after Test.

spherical bearing block having had opportunity to adjust the bearing satisfactorily by this time and no appreciable tendency to bend having yet developed in the column. In this manner the spherical bearing block was restrained from further motion and became in effect as rigid a loading block as the base. Fig. 2, facing p. 16, shows the arrangement used in the later tests. That the apparatus accomplished the desired purpose is evidenced by the fact that failure in bending occurred almost invariably at or near the center of the column. It may be noted in this connection that the conditions of restraint in the testing machine are probably not as near fixed-end conditions as are those of a reinforced concrete building.

III. Experimental Data and Discussion.

13. *Data.*—In Table 6, page 15, are given the maximum loads carried by the test columns. The loads carried at various unit-deformations (interpolated between readings) are given in Table 7. The load-deformation diagrams are given in Fig. 6, 7, 11, 15, 18, 19, and 20. Complete data for all tests are on file in the Laboratory of Applied Mechanics of the University of Illinois, but as there were two hundred or more readings of deformations for most of the tests, only the summarized data are given in this bulletin.

The discussion of the tests will be made under the following heads: A. Plain Steel Columns—Effect of Length; B. Core Type—Effect of Length; C. Core Type—Value of Concrete; D. Fireproofed Columns; E. Spiraled Columns; F. Summary.

A. Plain Steel Columns—Effect of Length.

14. *Phenomena of the Tests.*—The plain steel columns showed test phenomena which were consistently uniform. At loads of from 225 000 to 250 000 pounds cracking sounds were heard and these continued intermittently throughout the remainder of the test. The time required to add the 25 000-lb. increment of load gradually became longer with the testing machine running at a uniform speed, and at the maximum load the weighing beam floated within a range of 1000 lb. for a period of 10 or 15 minutes. At maximum load no bending was visible to the eye except in the longer columns and very little was shown by the deformation readings. After the maximum load was reached and the machine head was run down to complete the failure of the column

(generally at a faster speed), bending developed very gradually. In general this bending was fairly symmetrical about the middle of the length.

15. *Stress-deformation Relations.*—The load-deformation diagrams for the plain steel columns are given in Fig. 6. It is seen that the load-deformation curves bend at low loads. This may be due partly to imperfect end bearings of the steel shapes and to the arrangement of instruments which presupposed the conservation of a plane section in the column during the test. Column No. 8914 whose ends were dressed quite carefully to a fairly true surface and for which the extensometer was arranged to avoid any effect of bending shows a straight line up to 175 000

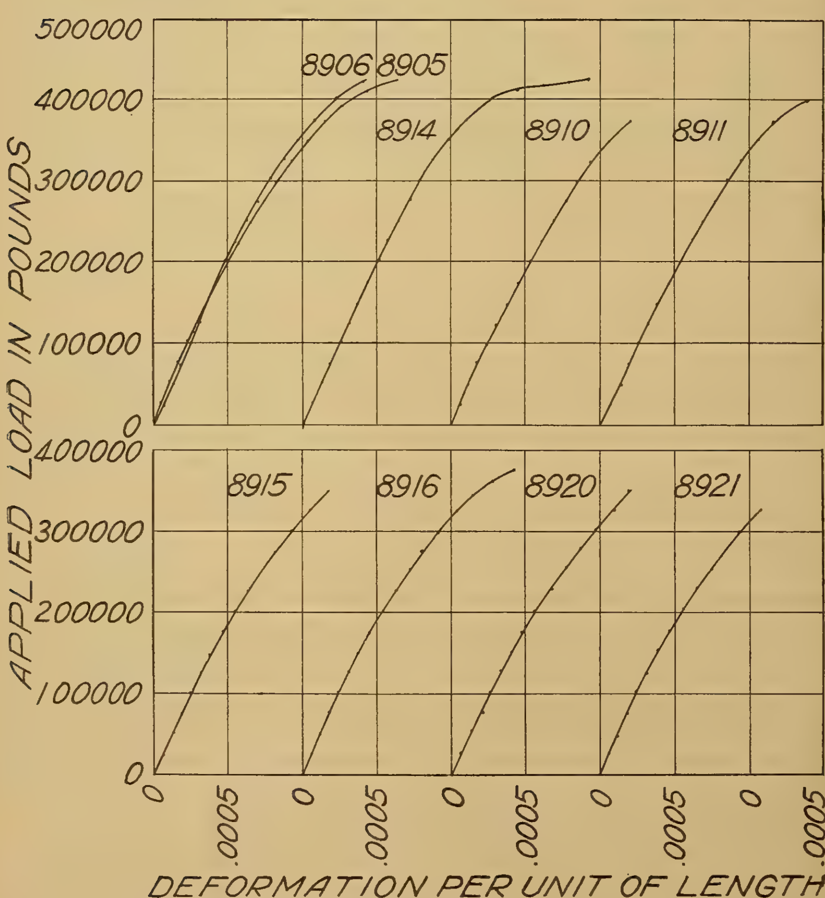

FIG. 6. LOAD-DEFORMATION DIAGRAMS FOR PLAIN STEEL COLUMNS.

lb. load and also an increase in stiffness over the other two columns of the same length. Even in this case a marked bending is present in the diagram well below the maximum load.

In Table 8, page 22, are given the secant moduli of elasticity, calculated from the curves, for unit-deformations of .0004, .0007, .0010, and for a point near the ultimate load. From these computations, it would appear that the modulus of elasticity for low deformations may be considered as lying between 30 000 000 and 31 500 000 lb. per sq. in. In Fig. 7 the average load-deformation curves for the several lengths of column are also

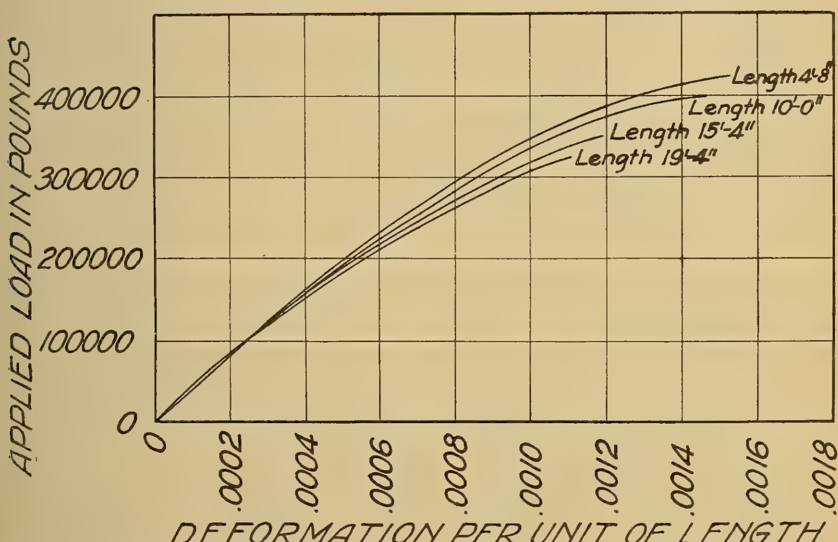

FIG. 7. AVERAGE DEFORMATION DIAGRAMS FOR PLAIN STEEL COLUMNS OF VARIOUS LENGTHS.

shown. It is apparent that the modulus of elasticity of the column as a whole decreases as the length of the column increases. It is also seen that the unit-deformation in the column as a whole at the maximum load becomes less as the length of the column increases. This is an indication of localized high stresses in the column.

16. *Relation of Strength to Length.*—In Table 6, page 15, are given the maximum loads carried by the plain steel columns of the several lengths.

Tests which have been made in the past on relatively small columns indicate that for columns having a ratio of length (l) to radius of gyration (r) less than, say, 100, the results may be ex-

pected to fall along a line slightly inclined to the $\frac{l}{r}$ axis. In arriving at an expression for the effect of length upon strength it may add to our clearness of perception to trace the development of the effect of length upon strength at various stages of the test. In

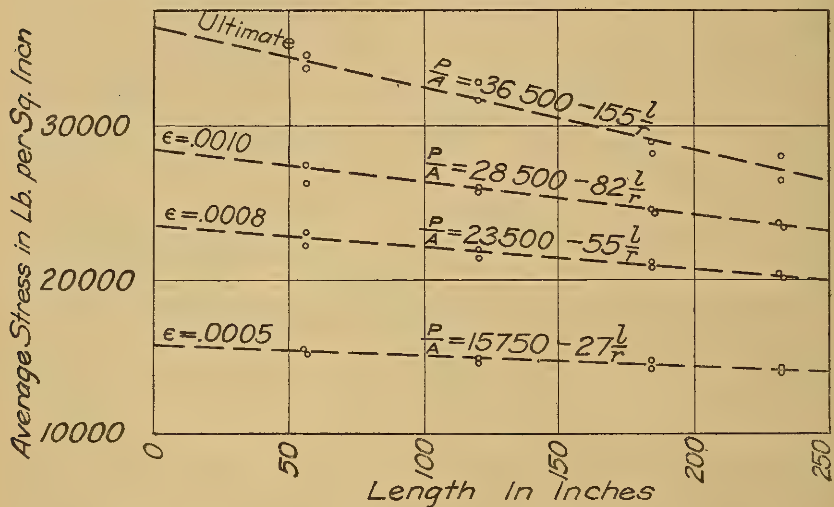

FIG. 8. LOAD-LENGTH DIAGRAMS FOR PLAIN STEEL COLUMNS.

Fig. 8 the results for the four lengths of column are plotted for unit-deformations of .0005, .0008, .0010, and for ultimate load. The four equations give the relation between $\frac{l}{r}$ and unit-stress at these deformations. The equations show that the effect of length upon stress is small at low deformations and is relatively much higher at the higher deformations. In a similar manner results were plotted for other unit-deformations from .0003 to .0012, and from these data Fig. 9 has been prepared. In this figure values of f and k for the column formula $\frac{P}{A}=f-k\frac{l}{r}$ are given for a range of values of the unit-deformation. The lower curve also shows the increasing effect of the slenderness ratio on the strength as the test progresses. If we should produce the tangent back to the horizontal axis the intersection is at .0004, and it is not far from the facts to say that up to a unit-deformation of .0004 the slenderness ratio has no effect and that beyond

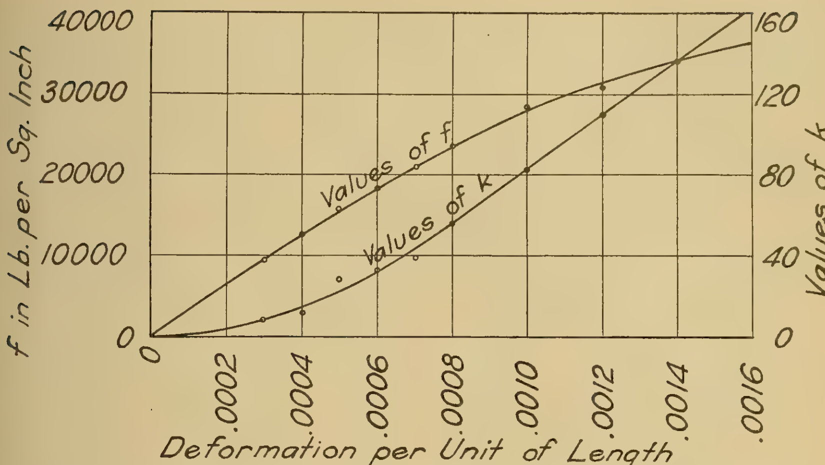

FIG. 9. VALUE OF TERMS IN STRAIGHT-LINE FORMULA FOR PLAIN STEEL COLUMNS. Column formula: $\frac{P}{A} = f - k\frac{l}{r}$

this deformation it increases in a constant ratio to the increase in deformation. From these diagrams it appears that a straight line will represent the results very satisfactorily for any given unit-deformation.

For the ultimate load, the nature of the strength-length relation is somewhat affected by the fact that the ultimate general unit-deformation of a column at its maximum load is smaller for the greater lengths of column, as is shown in Fig. 7. While for deformations less than the ultimate the straight-line equation

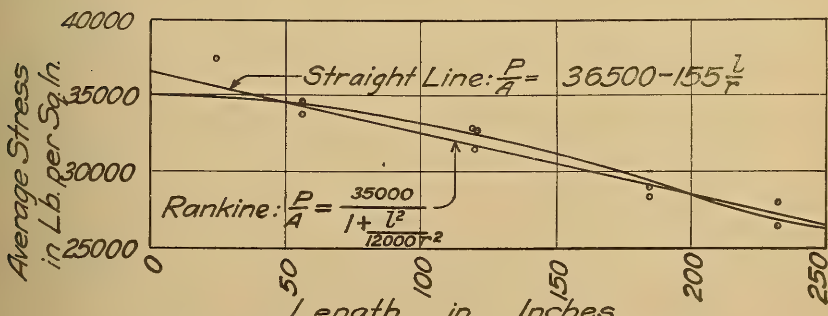

FIG. 10. COLUMN FORMULAS FOR MAXIMUM LOAD.

TABLE 8.

Moduli of Elasticity.

The values of the moduli are given in millions of pounds per square inch.

Column No.	Secant Modulus at Unit-deformation or .0004			Secant Modulus at Unit-deformation of .0007			Secant Modulus at Unit-deformation of .0010			Secant Modulus near Ultimate Strength			Initial Modulus		Last Unit-deformation*
	Steel	Concrete	Ratio n	Steel	Concrete	Ratio n	Steel	Concrete	Ratio n	Steel	Concrete	Ratio n	Column Concrete	Cylinder	
8902	34.2			32.5			30.6			26.5					.0017
8905	31.3			28.7			26.4			22.8					.0016
8906	31.1			29.6			27.3			25.1					.0014
8907	31.2	1.07	29	29.1	0.80	35	26.8	0.70	38	23.8	0.64	37	2.00	2.09	.0018
8908	31.2	1.33	23	29.1	1.20	24	26.8	1.11	24	23.8	1.03	23	1.67	2.08	.0015
8910	30.6			28.3			26.0			20.3					.0015
8911	29.8			27.8			25.7			22.3					.0014
8912	30.2	1.21	25	28.0	0.92	30	25.9	0.73	35	21.3	0.56	37	2.00	2.00	.0017
8913	30.2	1.46	21	28.0	1.32	21	25.9	1.19	22	21.3	1.03	21	2.30	2.08	.0014
8914	31.3			29.4			27.4			24.2					.0013
8915	29.8			26.9			24.4			23.0					.0012
8916	30.8			27.5			24.6			20.5					.0014
8917	30.3	0.86	35	27.2	0.75	36	24.5	0.67	36	21.8	0.63	34	1.46	1.85	.0012
8918	30.3	0.98	31	27.2	1.07	25	24.5	1.03	24	21.8	0.90	24	1.79	1.55	.0013
8920	28.9			26.0			23.5			22.4					.0012
8921	30.0			26.5			23.7			23.1					.0011
8922	29.4	0.94	31	26.3	0.84	31	23.6	0.74	32	22.8	0.74	31	1.81	1.30	.0010
8923	29.4	1.23	24	26.3	1.13	23	23.6	1.11	21	22.8	1.05	22	1.87	1.33	.0012
8925	30.2	1.98	15	28.0	1.72	16	25.9	1.61	16	21.3	1.40	15	2.33	2.84	.0016
8926	30.2	1.63	18	28.0	1.47	19	25.9	1.43	18	21.3	1.36	16	1.85	3.15	.0020
8927	30.2	0.79	38	28.0	0.63	44	25.9	0.52	45	21.3	0.58	37	1.30	1.25	.0016
8928	30.2	0.74	40	28.0	0.63	44	25.9	0.60	43	21.3	0.60	35	1.06	1.25	.0015
8929	30.2	1.27	24	28.0	1.09	26	25.9	1.00	26	21.3	0.93	23	1.60	2.00	.0012
8930	30.2	0.76	40	28.0	0.73	38	25.9	0.69	37	21.3	0.66	32	1.00	1.67	.0018
8931	30.2	1.22	25	28.0	0.95	29	25.9	0.78	33	21.3	0.67	32	1.85	2.00	.0018
8933	30.2	1.34	23	28.0	1.16	24	25.9	1.11	23				1.67	1.37	
8934	30.2	1.41	21	28.0	1.06	26	25.9	0.88	29				2.00	1.25	
8935	30.2	1.52	20	28.0	1.34	21	25.9	1.29	20				2.00	2.00	
8936	30.2	1.36	22	28.0	1.14	25	25.9	1.06	24				1.70	2.00	
8937	30.2	1.55	19	28.0	1.33	21	25.9	1.22	21				2.00	2.00	
8938	30.2	1.30	23	28.0	1.18	24	25.9	1.11	23				1.80	2.00	

* This column gives the unit-deformation observed immediately before the removal of the extensometers from the test columns, which was at loads varying from 80 % to 100 % of maximum load.

alone appears to apply, at the ultimate it is possible to construct an equation of the Rankine type which will represent the result quite closely. In Fig. 10 such an equation and a straight-line equation are plotted. The equations are as follows:

$$\text{Straight-line: } \frac{P}{A} = 36\,500 - 155\frac{l}{r}$$

$$\text{Rankine type: } \frac{P}{A} = \frac{35\,000}{1 + \dfrac{l^2}{12000\, r^2}}$$

It appears that the straight-line equation represents the test results as closely as the Rankine equation. The ultimate load for $\frac{l}{r} = 0$ is higher for the straight-line equation than by the Rankine equation, and both are lower than the stress of 37 500 lb. per sq. in. carried by the test column two feet long and the stress of 39 700 lb. per sq. in. carried by the compression test pieces taken from the flanges. After making a study of the data it is believed that the straight-line equation given above may be considered best to represent the effect of length upon ultimate load for the plain steel columns tested.

B. Core Type—Effect of Length.

17. *Phenomena of the Tests.*—The columns in which the core only was filled with concrete acted in much the same way as the plain steel columns. The concrete was somewhat restrained by the structural shapes, but the capacity for carrying an increasing load accompanied by the development of very high deformations which has been found in hooped concrete columns was not present. The columns exhibited much toughness and gave slow failures.

All the columns were tested with the upper bearing block restrained from motion, and in all cases the bending was symmetrical about the center or it occurred below the center. Very little bending was apparent at the maximum load. The bending shown in the view in Fig. 3, facing p. 16, occurred some time after the maximum strength of the column had been developed. In the final failure of the column, at deformations well beyond the maximum load, the crushing of the concrete was frequently more marked at the top of the column than elsewhere, due probably to the smaller density of the concrete at this place. In the columns 4 ft. 8 in. long practically no bending occurred, and the columns failed by general crushing which was more marked over the upper half. The columns 10 feet long, in the continuation of the

test beyond the maximum load, failed finally in bending, No 8912 bending about the center and No. 8913 bending sharply near the base. The columns 15 feet 4 in. long and 19 feet 4 in. long, after passing the maximum load, bent symmetrically about the center. In the latter part of the test, the concrete crushed on one side of

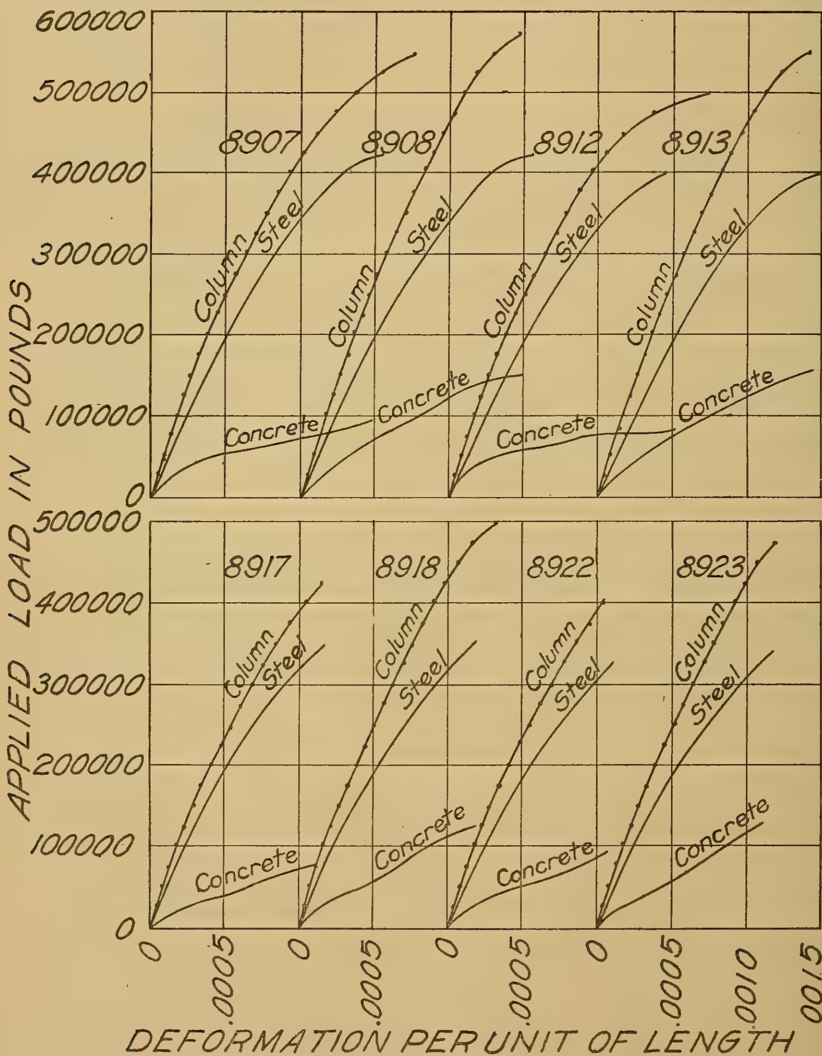

Fig. 11. Load-deformation Diagrams for Core Type of Column.

the column at the center and on the opposite side at the top and bottom. The varying density of the concrete caused by the obstruction to settlement and shrinkage in setting offered by the steel was shown in the final crushing. In all cases the crushing

was more marked at points at or immediately above the tie plates in the steel columns than at points between these plates. Apparently the concrete was less dense at the first-named places. As noted above, and as was to be expected, the concrete at the top of the column was weaker and less dense than that lower down.

18. *Stress-deformation Relations.*—Fig. 11, page 24, gives load-deformation diagrams for these columns. It was evident from the test results that in these columns the conservation of plane section was well maintained during the test, and this condition has been assumed in the interpretation of the data. The load-deformation curves are similar to those of the plain steel columns. In the diagrams in Fig. 11 the average load-deformation curve for the two plain steel columns of the same length has also been drawn.

19. *Effect of Test Conditions.*—The thickness of the mortar cushion between the end of the steel column at the top and the bearing plate may have exerted an influence on the strength of the columns, but it is difficult to arrive at any estimate of the amount of this. Where the joint was extremely thin, as was generally the case, its effect was undoubtedly negligible, but in one case where this joint was nearly $\frac{1}{8}$ inch thick it may have exerted an appreciable influence on the strength developed by the column.

20. *Effect of Length.*—The maximum loads carried by the columns are given in Table 6, page 15.

In the discussion of the effect of length of column the slenderness function may be expressed in terms of $\frac{l}{d}$ where l is the length of the column and d is the short diameter of the column section. In a following paragraph the effect of the length will be expressed in terms of $\frac{l}{r}$, where r is the radius of gyration of the steel section.

In Fig. 12 are plotted the loads for eight columns for unit-deformations of .0005, .0008, .0010 and also for ultimate load. Owing to variations in the concrete strength these points can not be expected to show as close agreement as did the plain steel columns. It may be noted that for each length of column the specimen showing greater strength was made on Nov. 8 and the weaker one on an earlier date, the variation in the strength agreeing with variation of strength of cubes and cylinders, as discussed elsewhere. The straight lines on this diagram express fairly well

the relation between $\frac{l}{d}$ and the average unit-stress over the cross-section of the column for several unit-deformations and for the maximum loads. P represents the load on the column and A the area of the cross-section of both steel and concrete. The equation for the maximum load is seen to be $\frac{P}{A} = 5150 - 52\frac{l}{d}$.

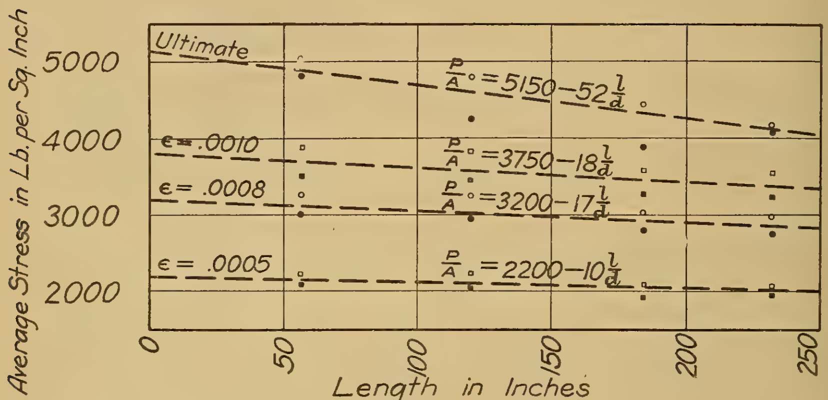

FIG. 12. LOAD-LENGTH DIAGRAM FOR CORE TYPE OF COLUMN.

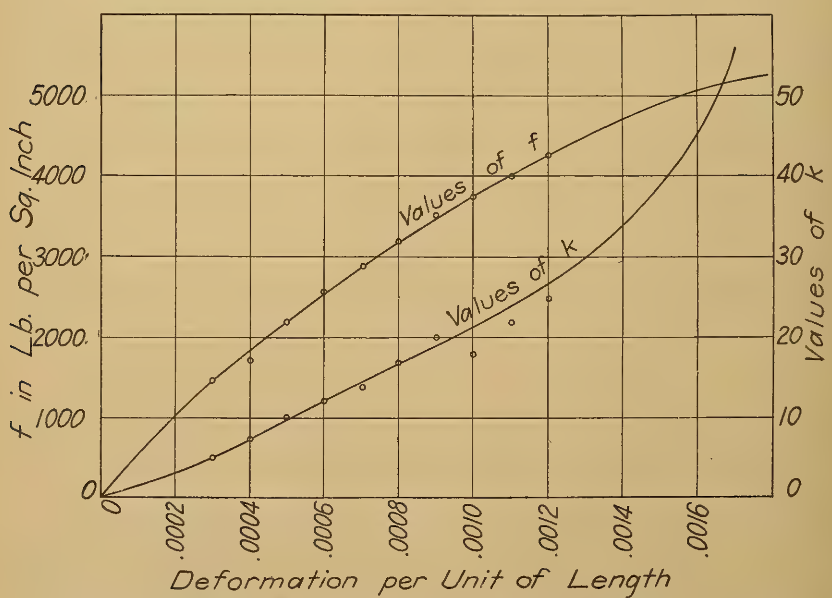

FIG. 13. VALUE OF TERMS IN STRAIGHT-LINE FORMULA FOR CORE TYPE OF COLUMN. Column formula : $\frac{P}{A} = f - k\frac{l}{d}$

Fig. 13 gives values of f, the first term of the second member in the straight-line column formula, and of k, the coefficient of $\frac{l}{d}$, for the range of unit-deformations.

21. *Comparison with Plain Steel Columns.*—In order to obtain a comparison of the effect of slenderness in columns of the core type and in plain steel columns, Fig. 14 has been prepared. In this diagram, for the purpose of comparison, the total load has

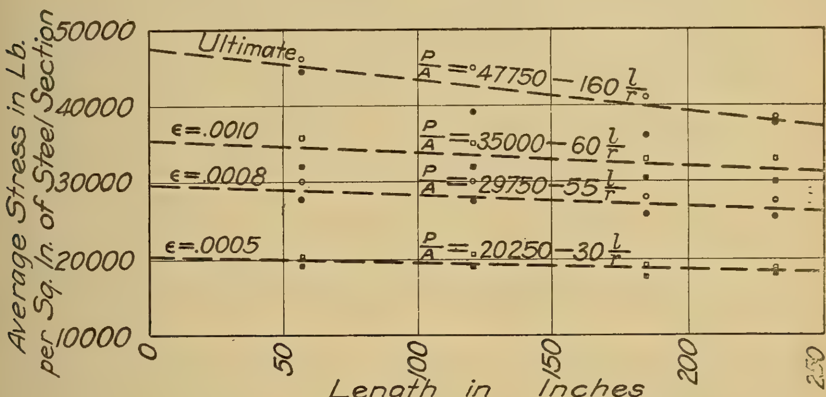

FIG. 14. LOAD-LENGTH DIAGRAM IN TERMS OF THE STEEL SECTION FOR CORE TYPE OF COLUMN.

been considered to be carried by the steel alone. A is the area of the cross-section of the steel and r is its radius of gyration. By comparing with Fig. 8 it is seen that the coefficients of $\frac{l}{r}$ agree very closely with the coefficients of $\frac{l}{r}$ found in the tests of plain steel columns at the same compression deformations. Thus at ultimate load the coefficient of $\frac{l}{r}$ is 155 for the plain steel columns and 160 for the reinforced steel columns. This would indicate that in the core type of column within the limits of length tested, the effect of length upon strength of column is a function of the slenderness ratio of the steel itself and is almost independent of the slenderness ratio of the concrete. This conclusion is in accord with results of tests of long concrete columns made in the Laboratory of Applied Mechanics, plain concrete columns 20 diameters long giving nearly as great strength as columns 10 diameters long. The discussion of the amount of stress taken by the concrete is given in a later paragraph.

C. CORE TYPE—VALUE OF CONCRETE.

22. *Test Phenomena and Stress-deformation Relations for Various Mixtures.*—All the columns of the core type showed substantially the same test phenomena for the different mixtures of concrete, a fact which may be accounted for by the presence of sufficient steel to make the concrete effect the smaller element. As in none of these columns did the total load taken by the concrete exceed one-half of that taken by the steel, it would be expected that the steel would govern the general behavior of the column under test. The 1–1–2 columns were found to sustain a greater ultimate unit-deformation than the columns of leaner mixtures. The load-deformation diagrams for the columns of 1–1–2 mix and 1–3–6 mix are given in Fig. 15. The close similarity of the load-deformation diagrams and those of the 1–2–4 mix of the same length, shown in Fig. 11 and 15, may be noted.

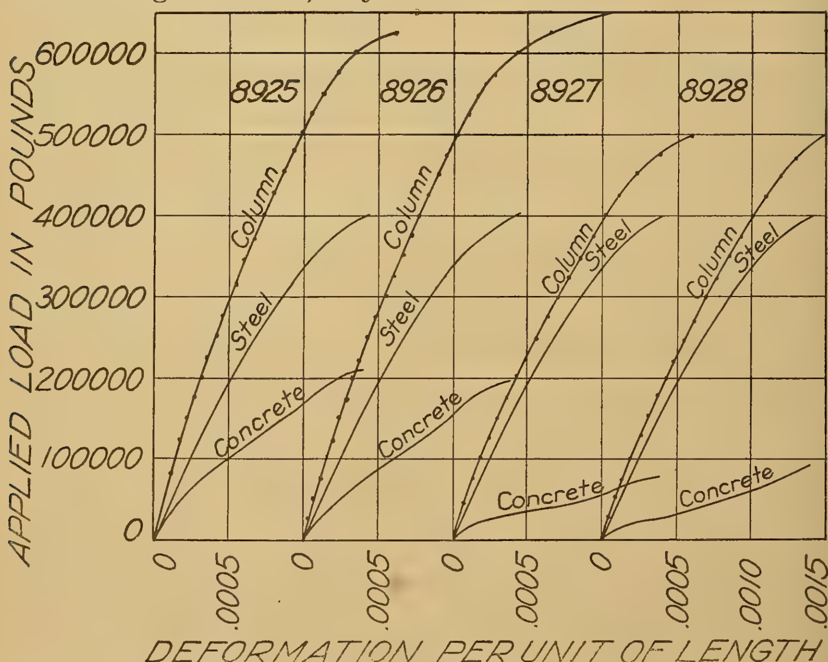

FIG. 15. LOAD-DEFORMATION DIAGRAMS FOR COLUMNS WITH LEAN AND RICH CONCRETE.

23. *Basis for Determining Load Taken by Concrete.*—In studying the strengthening effect of the concrete on the steel column it is necessary to decide upon some basis of division of the total load into the part considered to be carried by the steel and the part carried by the concrete of the columns. The load-deforma-

tion diagrams given for the tests of plain steel columns (Fig. 6, page 18) show that the compression deformations of these columns vary considerably from a straight line, which would be the form if a constant modulus of elasticity were assumed for the column. A straight-line relation does not take into account the adjustments in the bearing of the rivets and in the loading of the tie plates or lacing bars, the local flexure of the flanges, and the later changes in the stiffness of the material itself. Of course, it can not be told to what extent the presence of the core concrete in columns will overcome the agencies which cause curvature in a plain steel column before parts of the steel reach the yield point. It seems probable that there will be some such action and that the concreting of the column will add stiffness to the steel section itself. However, for the purpose of the discussion it seems best to consider that the steel section in any concreted column carried the same amount of load as was carried by a plain steel column of the same length. For this purpose the average load-deformation diagram for the plain steel columns of the same length are plotted on the load-deformation diagrams of the tests of columns of core type. (See Fig. 11 and 15). For any given unit-deformation we may then obtain the amount of load considered to be carried by the concrete by subtracting the load carried by the plain steel columns at this deformation from the total load carried by the concreted column. The results have been plotted on the line marked "Concrete". It seems possible that the values given by this line will be somewhat in excess of the part actually taken by the concrete, though of course it may be of little consequence whether this small part of the load is taken by the concrete or is carried by the steel by reason of the greater stiffness given it by the concrete. A similar method was used for determining the part of the load taken by the concrete at the maximum strength of the column. The values thus found are given in Table 6. These methods of dividing load between steel and concrete will be used in the later discussion of the ratio of steel stresses to concrete stresses.

24. *Development and Amount of Concrete Stress.*—A consideration of these derived curves for the concrete (Fig. 11 and 15) enables the unit-stress on the concrete to be determined (a comparative and not an absolute figure) and also the manner of development of this stress throughout the test. For all mixtures it was found that at the beginning of the test the concrete took the load rapidly, with constant speed in the testing machine,

much the same as does the concrete in the cube or the cylinder. In Fig. 16 the stress-deformation diagrams are shown for the concrete of several columns and for the cylinder made from the same batch of concrete. In the earlier stages of the test the curves are not dissimilar. As the deformation increases, however, the concrete in the columns takes proportionately less and less stress. In general, too, the concrete in the column may be said to be less stiff than the same concrete placed in the cylinder.

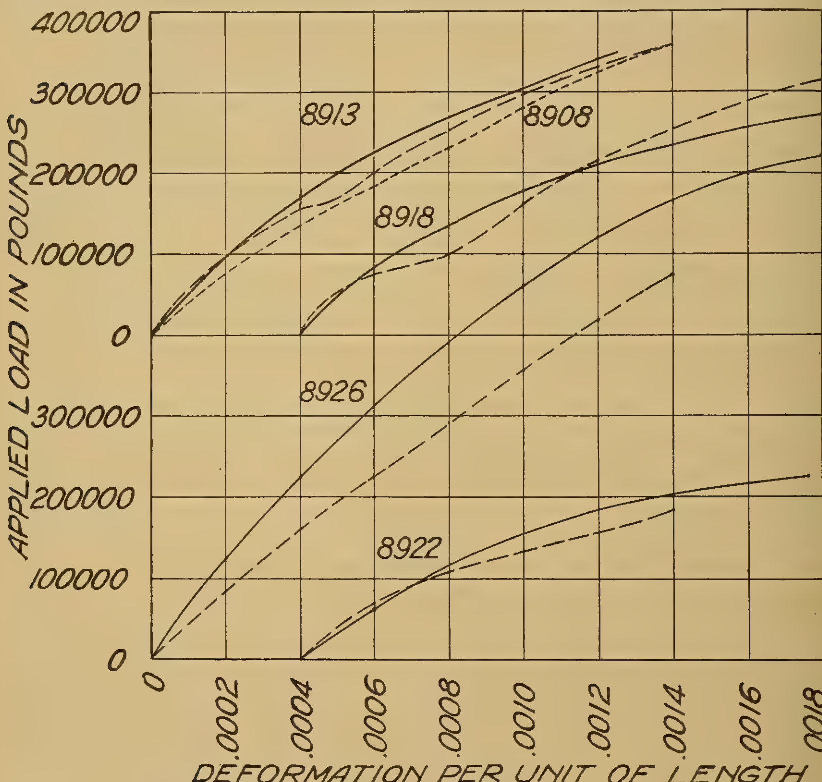

FIG. 16. COMPARISON OF STRESS-DEFORMATION IN COLUMN CONCRETE AND IN CYLINDERS.

For the 1–1–2 concrete there are no exceptions to this statement. For the 1–2–4 mixture 6 out of 8 columns follow the rule and the other two follow the rule up to a medium load. For the 1–3–6 mixture both columns show less stiffness up to a medium load and greater stiffness at the latter stages of the test than do the corresponding concrete cylinders. None of the concrete curves of Fig. 11 and 15 reaches its maximum value before the column as a whole reaches its ultimate load, but the curve becomes very flat at the higher deformations and shows a tendency for the concrete

stress to become nearly constant in value over a considerable range of shortening. This tendency is one to be considered in deciding upon permissible stresses for use in designing columns of the core type.

The amount of stress considered to be taken by the concrete, upon the assumptions previously discussed, is given in Table 6, page 15. It is to be noted that the columns with 1–2–4 concrete made on October 28 and 29 give concrete stresses much lower than those made on November 8, so much so that the concrete of the columns falls into two distinct groups. The cube and cylinder tests fall into two similar groups. The materials were taken from the same lot and it is reasonably certain that the measurements and weights of materials are correct. The tests indicate that the cement used early in the season acquired its strength more slowly than that used later, (the later specimens acquired a normal amount of additional strength between the ages of 60 and 90 days) and the room temperature the last of October and the first of November was lower than later in the year. It is thought then that part of the difference in strength is due to differences in rate of hardening. Under the circumstances it will be best to treat the 1–2–4 columns as made of two grades of concrete, dividing them into two groups, those made October 28 and 29 in group (a) (No. 8907, 8912, 8917 and 8922) and those made November 8 in group (b) (No. 8908, 8913, 8918 and 8923). The columns with 1–1–2 concrete gave a similar but smaller variation. The distinctions named above have been indicated in Fig. 12 and 14, the solid symbols representing group (a) and the open symbols group (b).

25. *Comparison of Cube and Cylinder Strength with Column Strength.*—In Fig. 17 the values of the ultimate stresses taken by the concrete determined as described in a preceding paragraph are plotted as abscissas and the strengths of the corresponding cubes and cylinders made from the same batch of concrete as ordinates. The results of the two groups of 1–2–4 concrete are shown by separate symbols. The relation between the cube and the column strengths seems to be well expressed by a straight line; it indicates that the column concrete developed about two-thirds the strength of the same concrete tested in 6-in. cubes. The relation between the cylinder and the column strengths seems more uncertain. For the 1–2–4 mixtures, the ratio seems to be about 1. For the 1–1–2 mixture the cylinder shows higher strength and for the 1–3–6 mixture the core concrete shows higher

strength. The average of all the ratios of cylinder-column strength is about 1.

26. *Values of E and n.*—An accurate determination of the modulus of elasticity of the column concrete can not be made, but for the purposes of comparison it may be proper to use the stresses obtained by the method given in "23. Basis for Determining Load Taken by Concrete". Assuming these concrete stress-

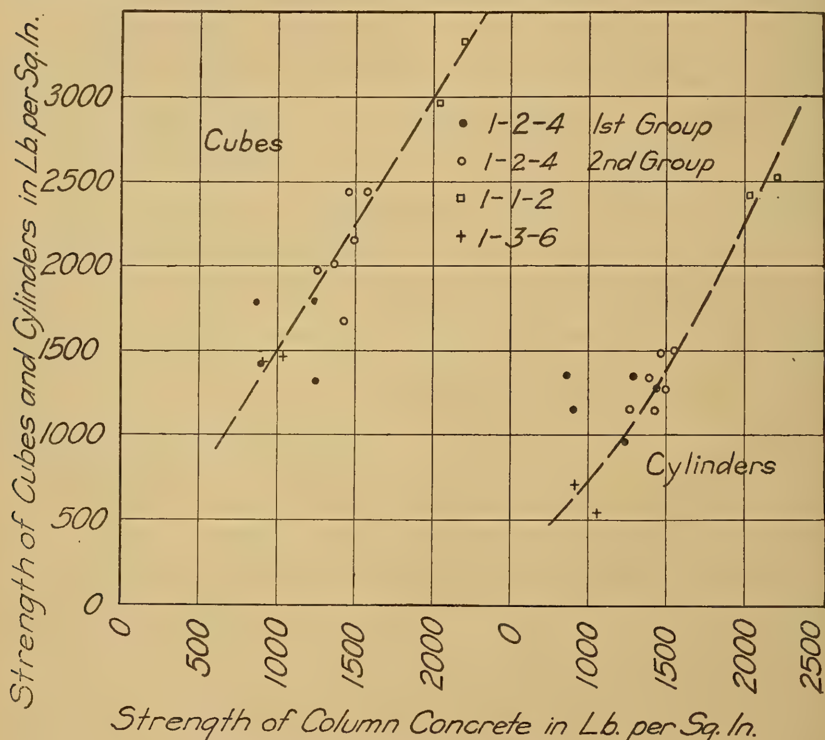

FIG. 17. COMPARISON OF STRENGTH OF CONCRETE IN CUBES, CYLINDERS AND COLUMNS.

es to be correct, the secant moduli of elasticity (E) have been calculated and are given in Table 8, page 22, for unit deformations of .0004, .0007, .0010 and for a deformation near the ultimate. The ratio between this modulus of elasticity and the modulus of elasticity of the plain steel column at the same deformation (values of n) are also given in Table 8. This ratio expresses the relation between the unit-stress taken by the steel and the unit-stress taken by the concrete. It is to be noted that these values of the ratio n are larger than ordinarily assumed in reinforced

concrete design. This is probably due largely to the fact that the concrete in the column is less dense than the same concrete would be if allowed to settle and shrink freely. A striking fact is that for widely different values of the unit-deformations the value of n varies but little.

If the columns with 1–2–4 concrete are divided into the two groups noted in "24. Development and Amount of Concrete Stress", we shall obtain the values for the stresses taken by the concrete and the values of the ratio n given in Table 8. In discussing the value of n to be used in design it must be borne in mind that the concrete in the columns was not as well seasoned as would ordinarily be the case in building construction. It should

TABLE 9.

AVERAGE STRENGTH OF SPECIMENS AND VALUES OF n.

Stresses are given in pounds per square inch.

Mixture	Compressive Strength			Average Values of n	Suggested Values of n
	Cube	Cylinder	Column Concrete		
1–3–6	1400	690	990	36	35
1–2–4 (a)	1500	1150	1050	35	
(b)	2200	1300	1450	23	25
1–1–2	3100	2475	2125	16	16

also be said that, as in this type of column the steel will be used as the basis of design, the value of the ratio n to be accepted in design should be greater than the average value found, rather than less, in order to be on the safe side. In Table 9 values of n for the different mixtures of concrete are given which seem reasonable for use in design.

Another Basis for Design.—Another basis for design which seems rational is to determine the strength of the steel column for the $\frac{l}{r}$ of the steel column, taking this from the straight-line equation on page 23, and then to use as the strength of the concrete of the core section (without reference to the length of the column for the column slenderness usual in buildings) a value taken from the strength of plain concrete, say two-thirds of the cube strength, in this way combining the strength of steel and concrete. This seems to be in accord with the results of tests within the limits of the $\frac{l}{d}$ here used. Of course a suitable factor of safety would then be applied.

D. Fireproofed Columns.

27. *Phenomena of the Tests.*—Three columns in which a 2-in. shell of concrete was added to the core section were tested to determine the additional strength afforded by this covering and to study the behavior of a fireproofed column under load. During the earlier stages of the test there was no difference between the behavior for this type of column and one of the core type except that the outer shell did not take the expected proportion of the load. The concrete shell remained intact until the ultimate deformation of the column was practically reached, the unit-deformation being .0018 or over at first crack. The steel was then evidently starting to yield and the concrete in the core was very close to its ultimate deformation and strength. When the shell cracked, the total load on the column dropped off about 65 000 lb. and the strength of the core itself was well evidenced by the length of time during which the load remained at this second ultimate although the machine was in operation in the meantime.

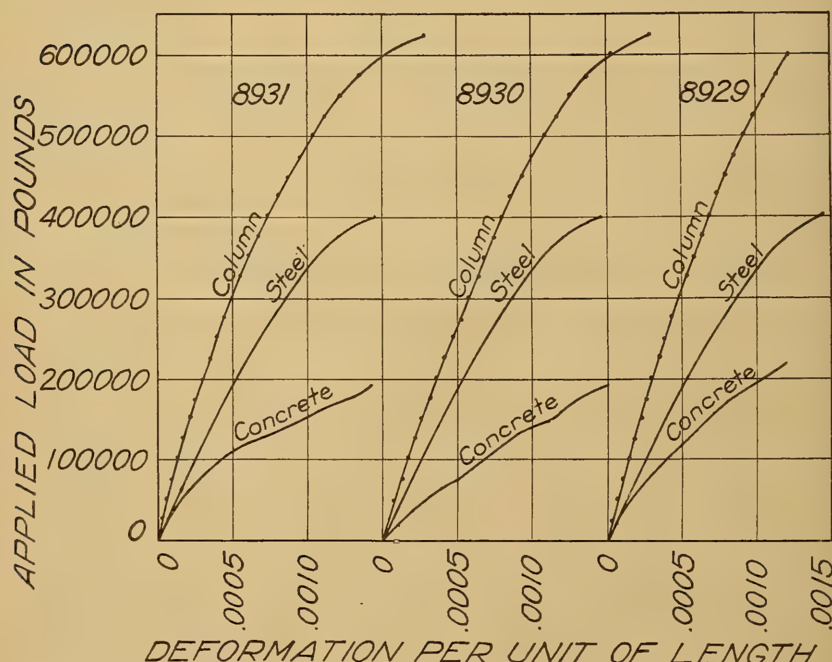

Fig. 18. Load-deformation Diagrams for Fireproofed Columns.

The load rose 2000 to 4000 lb. after its first drop, showing that the strength of the steel and of the enclosed concrete was not quite fully developed when the shell cracked. Fig. 4 gives a view after the maximum strength had been developed and the shell had cracked.

28. *Stress-deformation Relations.*—The stress-deformation diagrams are given in Fig. 18. The moduli of elasticity of the columns and the values of n are given in Table 8, page 22. Owing to the presence of the shell, which constituted 47% of the area of the total concrete section, and which it will be seen did not carry its full share of the load, the value of the modulus of elasticity of the column concrete is somewhat less for the fireproofed type than for the core type. Where n for the latter averaged about 23 for concrete of the same grade, it becomes more nearly 30 for the former. Since undoubtedly it is wisest not to figure on any load on the shell in designing, this value of n is significant only in showing that the use of values of n herein recommended does not threaten the safety and integrity of the shell.

29. *Comparison of Concrete Stresses on Gross Section and on Core Section.*—The maximum load carried by the fireproofed columns is given in Table 6, page 15. The division of load between steel and concrete, determined on the assumptions used in the discussion of the core type of column, is also given in this table.

Of the three fireproofed columns tested two were tested to failure. No. 8930 took a maximum load of 630 700 lb.; the load fell to 565 000 lb. when the shell failed, indicating that a load of 65 700 lb. was carried by the shell. For No. 8931 the maximum

TABLE 10.

STRESS CARRIED BY CONCRETE IN SHELL AND IN CORE OF FIREPROOFED COLUMNS.

Column No.	Stress in pounds per square inch		
	Gross Concrete	Shell Concrete	Core Concrete
8930	1 060	710	1 370
8931	1 090	680	1 440

load was 635 700 lb. and the core load was 572 000 lb., the difference, 63 700 lb., apparently being carried by the shell. Of the core load 418 000 lb. may be considered to have been carried by the steel (determined from the average strength of the plain steel col-

umns of the same length), leaving on the core concrete 147 000 lb. for No. 8930 and 154 000 lb. for No. 8931. The area of the shell was 93 square inches and of the core concrete 107 square inches.

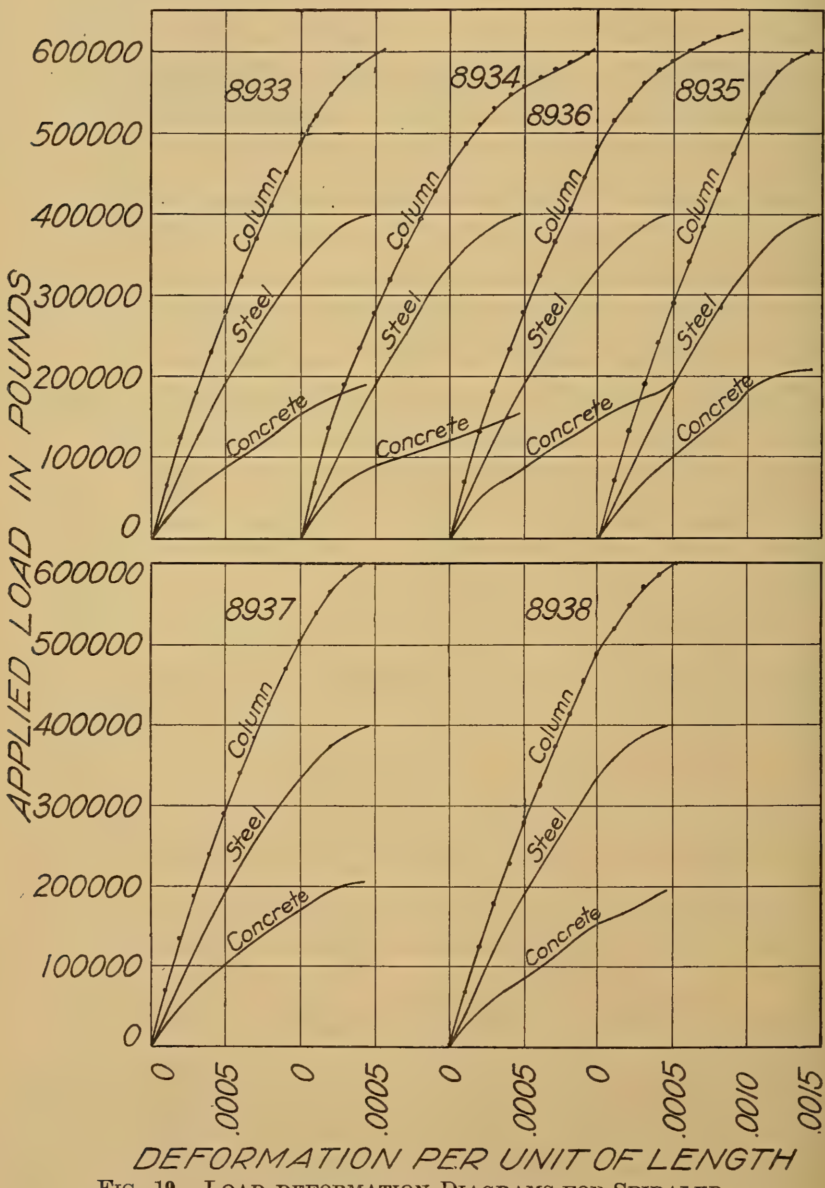

FIG. 19. LOAD-DEFORMATION DIAGRAMS FOR SPIRALED COLUMNS,—FIRST TEST.

The unit-stresses, as found by the method just outlined, are given in Table 10. These values indicate that the concrete shell carried roughly one-half as much load in pounds per square inch as the concrete of the core.

30. *Permissible Deformation as Governing Design.*—The question always arises in design as to the effect of large deformation in a column under load upon the integrity of the shell. If the shell goes to pieces at relatively low unit-deformations, it is evident that deformation rather than stress must govern in the selection of the working stresses to be used in design. As already noted, the shell did not crack or the load drop off until the unit-deformation exceeded .0018, and this is practically the deformation at the maximum load for the core type of column also. In other words, the action of the shell during the test did not seem to be such as to impose any restriction on the selection of working stresses.

31. *Need of a Tie to Prevent Stripping of Shell.*—Although the shell maintains itself intact under high unit-deformation, yet it seems that a tie of some sort (wire mesh, spiral, or other binder) should be imbedded in the outer shell to make its permanence certain. The backs of the flanges occupy one-fourth of the bonding surfaces between the shell and the core, and in practice this proportion might be even greater. It seems unwise to trust the fireproofing shell of the column to stand uninjured under collisions and accidents with so large a surface uncertainly supported. This tie would also prevent the rapid and complete failure of the shell at maximun load, as it occurred in the test of the columns, although this advantage is not very great in actual construction. The prevention of spalling away from the steel in case of a severe fire is a more important reason for requiring a metal binder to hold the exterior concrete in place. From the tests of spiraled columns it is concluded that a spiral is an excellent tie for this purpose.

E. Spiraled Columns.

32. *Phenomena of Tests and Stress-deformation Relations.*—For loads within the capacity of the University of Illinois testing machine (600 000 lb.) the load-deformation curves of the columns with $\frac{3}{4}$ % and with 1 % spiral reinforcement (see Fig. 19) are practically identical in nature with those for the core type. The tests were not carried to the point where the thin concrete coat-

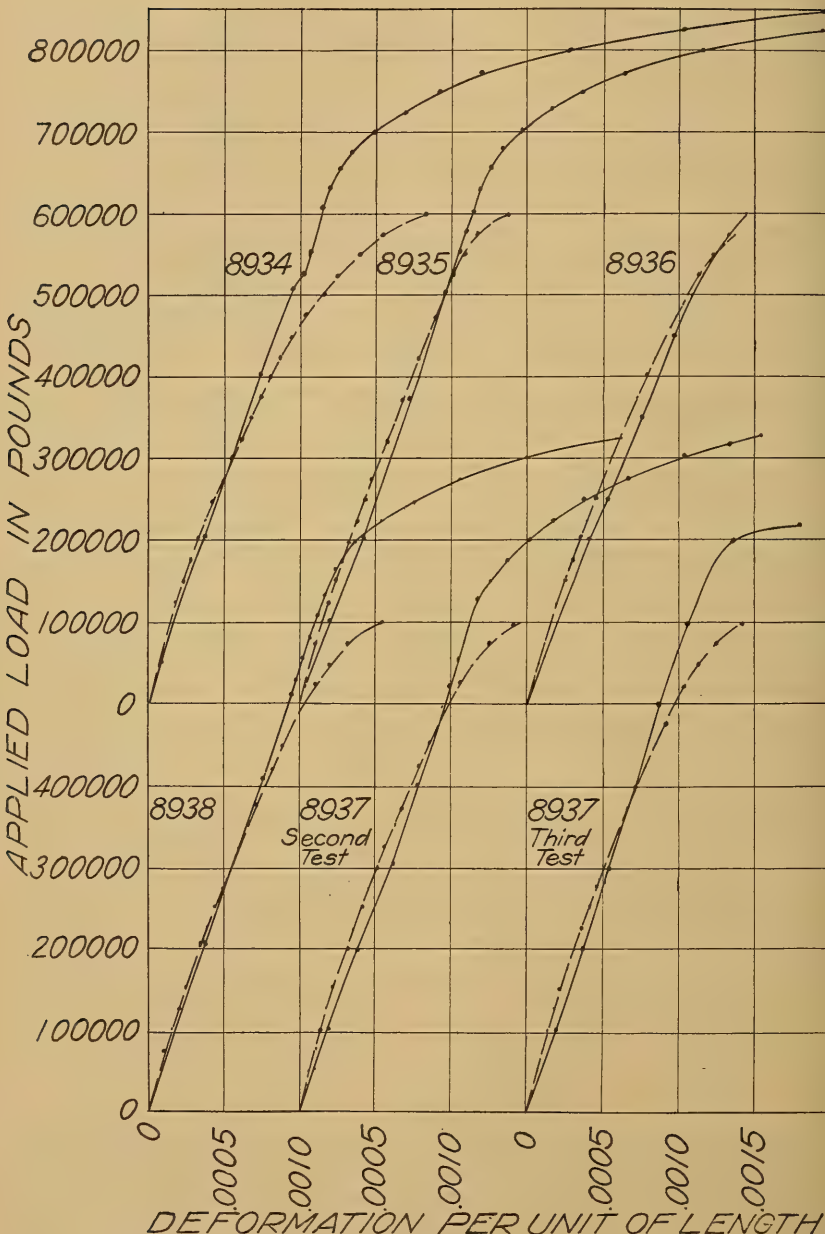

FIG. 20. LOAD-DEFORMATION DIAGRAMS FOR SPIRALED COLUMNS,—SECOND AND THIRD TESTS.

ing over the spiral would be expected to spall. In three of the spiraled columns the maximum load was applied three to five times, the result being increased deformations under maximum load and increased set upon the release of load.

Four of the spiraled columns were subsequently tested a second time at Lehigh University, where a load of 830 000 lb. was applied. Even with the heavier testing machine, the full strength of the columns was not developed, though No. 8934 evidently was loaded nearly to its maximum strength. The load-deformation diagrams are shown in Fig. 20. The deformations at the first tests of the same columns are also given by the broken lines. It must be borne in mind that the columns were about three months older at the second test than when first tested, and the increased strength of the concrete at a given unit-deformation may be accounted for by the increased age. To secure some conception of the amount of the added strength due to the greater age of the concrete, No. 8937 was tested a third time, the spiral first being removed, the outer concrete stripped off, and the column reduced to the section of the standard core type. The column in this condition carried a load of 714 000 lb., as compared with 547 000 lb. carried by the two corresponding columns of the core type at an age of 60 days, (see Fig. 20) and it is evident from this that a considerable portion of the added load carried in the second test was due to the greater age of the concrete. It is well to call attention to the fact that this increase of strength was gained after the concrete had been subjected to high stresses at an age of 60 days.

The large lateral deflection of the four columns tested at Lehigh University is of interest. In No. 8934, at a load of 752 100 lb. the deflection was .09 in. and at this load the concrete began to spall. At a load of 856 000 lb. the deflection was .26 in., a deflection set of .21 in. remaining when the load was released. At the fourth application of a load of about 850 000 lb. the deflection became .54 in., a set of .50 in. remaining with the release of the load. In No. 8935, at a load of 750 000 lb. the deflection was .084 in. At a load of 825 000 lb. the deflection became .31 in., and the concrete at the bottom scaled off between wires. Upon the release of load the set was .31 in. At a fifth application of the load, the deflection became .48 in. and the deflection set shown upon the final release of this load was .43 in. There was no marked bending in No. 8937 until the third application of a load of 830 000 lb., when it became .12 in.

In No. 8938, at a load of 776 000 lb. the deflection was .08 in., becoming .17 in. at a load of 825 000 lb., with a set of .16 in. shown upon release of load. At the fifth application of the load, the deflection became .25 in. with a resulting set of .22 in. This marked bending of the spiraled columns at loads above those which would be carried by an unspiraled column is in keeping with the large deflections found in hooped concrete columns of the usual type.

The phenomena of the second test were not essentially different from those described for the first test. At a load of about 750 000 lb. spalling of the outer concrete began on the columns having ¾ % spiral reinforcement, and this spalling continued during the remainder of the test. It seems evident from the action of the columns and the amount of deformation developed that these columns were very close to their maximum load at the end of the second test. The view in Fig. 5 shows the spalling of the concrete and the buckling of the spacing strip. With the columns having 1 % of spiral reinforcement the spalling was slight even at the highest load carried, and these columns evidently would have carried considerably more load. An examination of the structural steel after the second test showed no crimping of the flanges and no movement of the parts relatively to one another, although, of course, the total shortening of the column was not large. The spiral seems to have acted to hold the steel in alignment and to permit a greater shortening than would otherwise have taken place.

33. *Effect of Spiral.*—The maximum loads placed on the spiraled columns are given in Table 6, page 15. It must be borne in mind that the capacity of the testing machines used did not permit the maximum strength of these columns to be developed.

Within the limits of the first test (600 000 lb. load) the load-deformation diagrams do not show any effect which may be attributed to the variation in the percentage of spiral reinforcement or even to the presence of the spiral. A study of the stresses in these columns of the core type shows that the spiral has little apparent effect upon the action of the column within the load of 600 000 lb.

In the second test the maximum load applied evidently approached very closely to the maximum strength of the columns with ¾ % of spiral reinforcement, but it did not stress the col-

umns with 1 % of spiral reinforcement nearly as severely. The load-deformation curves (Fig. 20, page 38) show that the marked yielding of the column takes place at about the same load for both percentages (650 000 to 700 000 lb.) and that at this yield point the spiral begins to play an important part. During this later part of the test the difference in amount of spiraling becomes apparent and the heavier spirals show greater strength. Since the tests were not carried to destruction, the amount of this added strength can not be ascertained, but its existence seems well established.

For most purposes, then, the significant point on the stress-deformation curves is the point where the curve bends sharply and becomes flat and which may be considered to be the load which would be carried by an unspiraled column. Beyond this point any additional load is carried only by virtue of very greatly increased deformations. For the four columns tested this point lies at or below 750 000 lb., and it will be interesting to compare this with the strength of the column without spiral reinforcement. For the purpose of this comparison the results of the third test of No. 8937, after it was stripped of its spiral and reduced to the standard core type, may be used to estimate the strength of the concrete at the time of the later test. This column carried 714 000 lb. load as against 580 000 lb. load carried by columns of the core type at 60 days of age. If we consider all the concrete within the spiral in the second test to be as effective as the core concrete in No. 8937, the load carried without aid from the spiral would be about 800 000 lb. This figure is probably a little high, as the concrete outside the core, although inside the spiral, would probably take somewhat less load per square inch than the concrete in the core. This tends to confirm the view that the load at the point when the load-deformation curve bends sharply and becomes flat is approximately equal to the strength of the unspiraled column, and that only the part of the strength developed after this yield point is passed should be attributed to the spiral directly. In No. 8934, this added load amounts to about 100 000 lb. which is 715 lb. per sq. in. of inclosed concrete. This is at the rate of 950 lb. per sq. in. of column concrete per 1 % of spiral reinforcement. Applying this figure to the 1 % spiral columns, the computed probable maximum load becomes 900 000 lb., a value which the deformations of the second test seem to indicate as a reasonable expectation for these columns.

34. *Availability of Spiral Strength for Design.*—The tests of this series of columns indicate that up to a unit-deformation of .0015 no appreciable difference in the action of columns with or without spiral reinforcement is found. In building construction the safe unit-deformation may ordinarily be placed at .0007 or less. It would seem that any attempt to use an imaginary spiral strength at working loads could result only in very high actual unit-deformations. The marked tendency of the column to bend laterally after the yield point was passed and the large amount of set found are also evidence of the unavailability of the higher strengths. The spiral does afford protection against sudden failure, and gives a tougher and safer column, and these properties may be considered to warrant the use of higher unit-stresses in spiraled columns. In the columns of the core type tested the need of a spiral is much less than in the ordinary reinforced concrete column, since these columns are found to possess toughness and the flanges of the structural angles restrain the core concrete to some extent. It thus appears that the use of a large percentage of spiral reinforcement in columns of the type here considered is hardly justifiable. A light spiral may serve to tie the shell together securely and protect it from accident, but this spiral should not be directly considered in the computations for strength of column.

F. Summary.

35. *General Comments.*—The columns tested were of a form now frequently used in building construction. The percentage of steel used (area of steel section 10.8 % of the area of the octagon inclosing the structural shapes) is within the range used in building construction. The conclusions given in the discussion relate to the properties of columns which have the forms and sections of the columns tested, and variations in proportions of metal and concrete may give somewhat different results. The tests, however, may be expected to throw light on the properties of columns of the same general type within the limits of ordinary design. The principal conclusions found in the discussion are as follows:

1. The maximum load carried by the plain steel columns is expressed by the straight-line formula, $\frac{P}{A} = 36\,500 - 155\frac{l}{r}$, where $\frac{l}{r}$ is the ratio of length of column to radius of gyration of the section of the steel column.

2. Earlier in the test the effect of length upon load carried at a given unit-deformation was less proportionately than at maximum load, the coefficient of $\frac{l}{r}$ in the equation being only 55 for a unit-deformation of .0008, and 27 for a unit-deformation of .0005, as compared with 155 in the equation for maximum load.

3. The load-deformation diagrams diverge from a straight line at loads well below the maximum.

4. In the concreted columns of the core type, the effect of length upon strength of column was almost identical with that found in the tests of plain steel columns. In other words the stress taken by the concrete may be considered to be nearly independent of the slenderness ratio of the column, within the limits of the lengths tested, and the stress taken by the steel may be considered to be the same as that taken by a plain steel column of the same slenderness ratio.

5. In the tests the concreted columns of the core type showed considerable toughness, though at the maximum load there was no material lateral deflection. The final failure of the concrete generally occurred at or above tie plates. The discussion shows that the concrete of the columns was less strong than the concrete of the cubes and less stiff than the concrete of the cylinders.

6. The stress taken by the concrete within the core or within the spiral is approximately equal to the strength of concrete of the cylinders tested and to two-thirds of the strength found in the 6-in. cubes.

7. The values of the ratio of the modulus of elasticity of the steel column to that of the concrete, n, under the assumptions used in the analysis, are much larger than are commonly used in reinforced concrete design. Values of n for use in designing are suggested in Table 9.

8. A basis for design which seems rational is to determine the strength of the steel column by the use of the column formula for the $\frac{l}{r}$ of the steel column and then to consider the concrete of the core section (without reference to the length of the column

for any ordinary length ratio, say a length of 15 diameters) to have a stress value proportional to the strength of the plain concrete, say two-thirds of the cube strength. A suitable factor of safety would of course be somewhere applied.

9. In the test of the fireproofed type of column (which had a shell of concrete outside the steel) the concrete shell remained intact until a deformation was reached as great as that developed at the maximum load in columns of the core type. This integrity of section at high deformations indicates that the presence of the shell need not impose any restrictions upon the working stresses available for the steel and for the core concrete. Of course, there are good reasons for the use of a metal binder like wire mesh or spiral for holding the shell securely in place.

10. The discussion indicates that the stress carried by the concrete of the shell is only about half of that carried by the core concrete. This lower strength is not objectionable, since the shell is not considered in designing the column.

11. The action of the spiraled columns indicates that the spiral has little effect up to a deformation and load corresponding to the maximum load for an unspiraled column. Beyond this load the column compresses rapidly and the presence of the spiral adds materially to the strength of the column. The tests do not fix the exact amount of this added strength.

12. In view of the large shortening necessary to make the added strength due to spiraling available and the general toughness of columns of the core type, it would seem that for building construction the use of a large percentage of spiral reinforcement in columns made up of structural shapes and concrete is hardly justifiable. A moderate spiral may warrant the use of somewhat higher unit-stresses, since it adds to the toughness of the column and gives a possible higher ultimate strength, and it will also serve to tie the concrete of the shell together securely and protect it from accident, but it does not seem best to consider this spiral directly in the computations for strength of column.

13. The columns tested possess the qualities of a good structural member and seem well adapted to more general use in building construction.

These comments are made on the assumption that the concrete is placed in as workmanlike a manner as is obtainable in the construction of high-grade work in columns reinforced with longitudinal rods or with rods and spirals.

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(2) To make possible the establishment and maintenance of a trade circulation through the regular publishing houses.

(3) To regulate the distribution of the reserve or "out-of-print" supply.

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W. F. M. GOSS
Director.

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BULLETIN NO. 57

SUPERHEATED STEAM IN LOCOMOTIVE SERVICE

(A REVIEW OF PUBLICATION NO. 127 OF THE CARNEGIE INSTITUTION OF WASHINGTON)

BY

W. F. M. GOSS

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

BULLETIN NO. 57 APRIL 1912

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DEAN OF THE COLLEGE OF ENGINEERING
DIRECTOR OF THE ENGINEERING EXPERIMENT STATION
DIRECTOR OF THE SCHOOL OF RAILWAY ENGINEERING AND ADMINISTRATION

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PREFACE

Publication No. 127, of the Carnegie Institution of Washington, entitled, "*Superheated Steam in Locomotive Service*", is a publication of 144 pages, dealing with a research which was carried on in the laboratory of Purdue University, during the writer's connection with that University. It presents, in tabulated and graphical form, the full record of observed and derived results.

In this Review, the text of the original publication has been freely quoted, and the conclusions and arguments by which they are sustained, appear as given in the orginal publication. This Review, therefore, takes the form of a resumé of the research and its results, the complete record of which is available elsewhere.

Acknowledgments are due Mr. F. W. Marquis for the editorial work incident to this Review.

W. F. M. Goss

April, 1912

SUPERHEATED STEAM IN LOCOMOTIVE SERVICE

I. Introduction

1. *A Summary of Conclusions.*—The results of the study concerning the value of superheated steam in locomotive service, the details of which are presented in the succeeding pages, may be summarized as follows:

1. Foreign practice has proved that superheated steam may be successfully used in locomotive service without involving mechanism which is unduly complicated or difficult to maintain.

2. There is ample evidence to prove that the various details in contact with the highly heated steam, such as the superheater, piping, valves, pistons, and rod packing, as employed in German practice, give practically no trouble in maintenance; they are ordinarily not the things most in need of attention when a locomotive is held for repairs.

3. The results of tests confirm, in general terms, the statements of German engineers to the effect that superheating materially reduces the consumption of water and fuel and increases the power capacity of the locomotive.

4. The combined boiler and superheater tested contains 943 sq. ft. of water-heating surface and 193 sq. ft. of superheating surface; it delivers steam which is superheated approximately 150°. The amount of superheat diminishes when the boiler-pressure is increased, and increases when the rate of evaporation is increased, the precise relation being

$$T = 123 - 0.265\,P + 7.28\,H$$

where T represents the superheat in degrees F., P the boiler-pressure by gauge, and H the equivalent evaporation per foot of water-heating surface per hour.

5. The evaporative efficiency of the combined boiler and superheater tested is

$$E = 11.706 - 0.214\,H$$

where E is the equivalent evaporation per pound of fuel and H is the equivalent evaporation per hour per foot of water-heating and superheating surface.

6. The addition of the superheater to a boiler originally designed for saturated steam involved some reduction in the total area of heat-transmitting surface, but the efficiency of the combination, when developing a given amount of power, was not lower than that of the original boiler.

7. The ratio of the heat absorbed per foot of superheating surface to that absorbed per foot of water-heating surface ranges from 0.34 to 0.53, the value increasing as the rate of evaporation is increased.

8. When the boiler and superheater are operated at normal maximum power, and when they are served with Pennsylvania or West Virginia coal of good quality, the available heat supplied is accounted for approximately as follows:

	Per cent
Absorbed by water	52
Absorbed by steam in superheater	5
Utilized	57
Lost in vaporizing moisture in coal	5
Lost in CO	1
Lost through high temperature of escaping gases	14
Lost in the form of sparks and cinders	12
Lost through grate	4
Lost through radiation, leakage, and unaccounted for	7

9. The water consumption under normal conditions of running has been established as follows:

Boiler-pressure	Corresponding Steam per i. h. p. hr.
lb.	lb.
120	23.8
160	22.3
200	21.6
240	22.6

The minimum steam consumption for the several pressures is materially below the values given. The least for any test was 20.29 lb.

10. The coal consumption under normal conditions of running has been established as follows:

Boiler-pressure	Coal Consumed per i. h. p. hr.
lb.	lb.
120	3.31
160	3.08
200	2.97
240	3.12

11. Neither the steam nor the coal consumption is materially affected by considerable changes in boiler-pressure, a fact which justifies the use of comparatively low pressures in connection with superheating.

12. Contrary to the usual conception, the conditions of cut-off attending maximum cylinder efficiency are substantially the same for steam superheated 150° as for saturated steam. With superheated steam, when the boiler-pressure is 120, the best cut-off is approximately 50 per cent stroke, but this value should be diminished as the pressure is raised, until at 240 lb., it becomes 20 per cent.

13. Tests under low steam pressures, for which the cut-off is later than half stroke, give evidence of superheat in the exhaust.

14. The saving in water consumption and in coal consumption per unit power developed, which was effected by the superheating locomotive, *Schenectady No. 3*, in comparison with the saturated-steam locomotive *Schenectady No. 2*, is as follows:

Saving in Water Consumption				Saving in Coal Consumption			
Boiler-pressure	Locomotive			Boiler-pressure	Locomotive		
	Saturated Steam	Super-heating	Gain		Saturated Steam	Super-heating	Gain
lb.	lb.	lb.	per cent	lb.	lb.	lb.	per cent
120	29.1	23.8	18	120	4.00	3.31	17
160	26.6	22.3	16	160	3.59	3.08	14
200	25.5	21.6	15	200	3.43	2.97	13
240	24.7	22.6	9	240	3.31	3.12	6

15. The power capacity of the superheating locomotive is greater than that of the saturated-steam locomotive.

II. Foreign Practice in the Use of Superheated Steam in Locomotive Service

2. The Use of Superheated Steam in Locomotive Service.—In the year 1898, the first superheating locomotives, two in number, were placed in service upon the Prussian State Railway. As might have been expected in machines of new design, a number of difficulties were encountered in their operation, but one by one these were overcome. Special forms of pistons, of piston-valves, and of rod packing, designed better to withstand exposure to steam of high temperature, were introduced. In 1899, the two

original superheating locomotives were followed by two superheating express locomotives, and in 1900, by two superheating tank locomotives, the superheaters of all being of the same design. While these six trial engines were by no means perfect, they served to show that highly superheated steam might be generated and successfully employed in locomotive service. As a result of the experience thus gained, the Prussian State Railway has, since 1900, made large purchases of the new type of engine. So rapidly has their use increased that in April, 1907, there were 682 in service and 467 in the process of building, or covered by orders; while in the whole Empire of Germany there were 1320 locomotives of the new type running or on order. The locomotive builders of Germany draw their support from many different countries. While building superheating locomotives for the Empire, they have stimulated interest in and created a demand for the new type in other countries. Thus, Belgium, Russia, Austria, Sweden, Switzerland, Italy, France, Holland, England, Denmark, Spain, Greenland, Canada, and South America all have their German-designed and German-built superheaters, and at the time just quoted, April, 1907, the total number of superheating locomotives in service or on order for all countries approached closely to 2000. This rapid extension of a new practice expresses the degree of confidence which many engineers have in its value. In fact, the introduction of the superheater has become a world-wide movement, and as such it is entitled to the respect and the thoughtful attention of American engineers.[1]

3. *Types of Superheaters.*—The original Schmidt locomotive superheater was of the smoke-box type, and practically half of the superheating locomotives now (1907) in operation on the Prussian State Railway are of this design. The later introduction of the Schmidt fire-tube type of superheater has, however, proved so satisfactory that the manufacture of the earlier smoke-box type has in recent years been discontinued. All of the superheating locomotives of the Prussian State, now under construction, are to be equipped with this later type. Other forms of superheaters have been proposed and one of these has been used experimentally, but the practice of superheating in Europe as it exists to-day implies the use of the Schmidt fire-tube superheater. The introduction of this superheater (Fig. 1 and 2) requires that the

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[1] Supplementing the statements of this paragraph, which were formulated in 1907, it may be stated that the number of superheating locomotives in Europe is now (1911) reported as 7000 and that the number in this country, in service or under order, is approximately 2000.

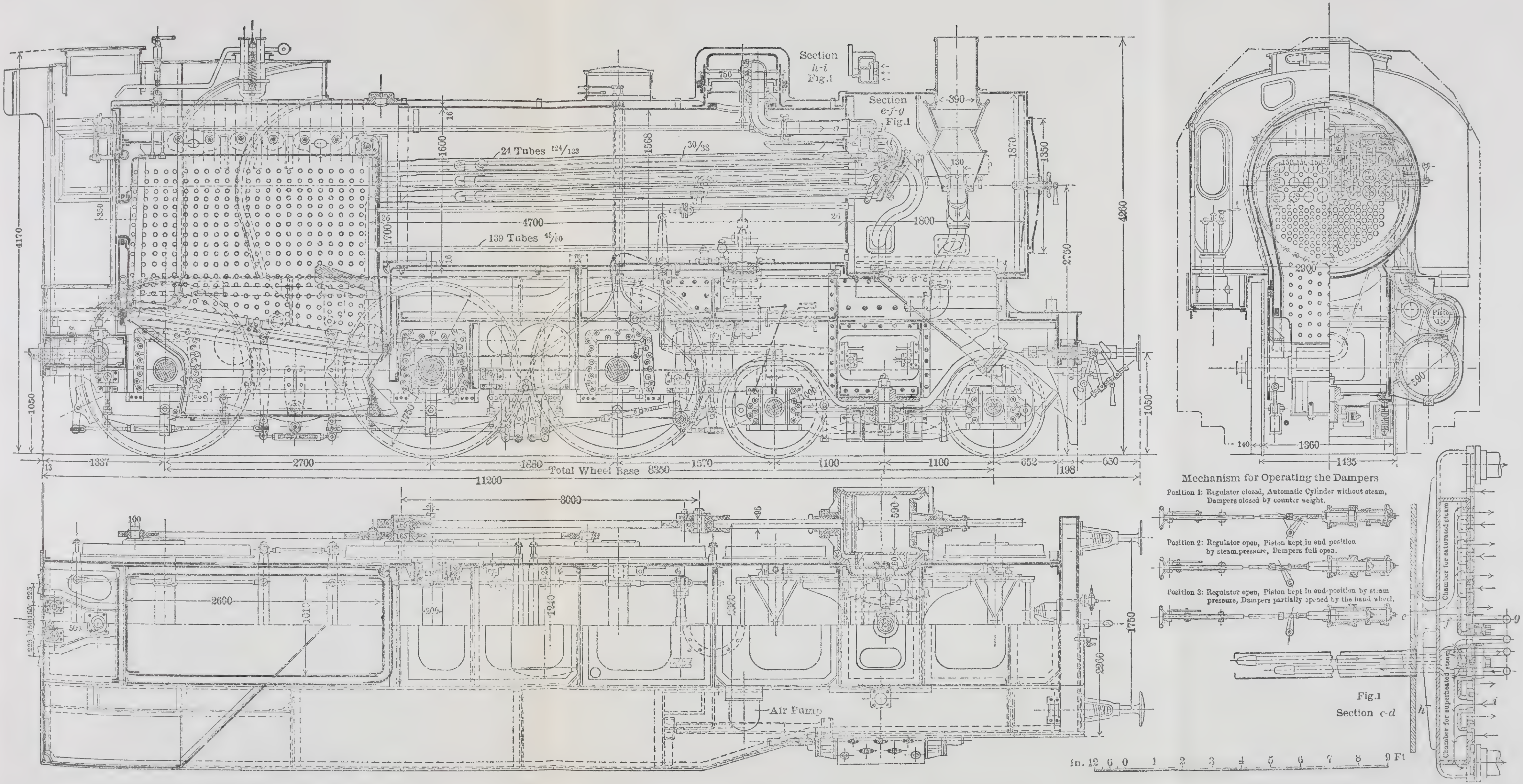


FIG. 1. A SUPERHEATED STEAM EXPRESS LOCOMOTIVE OF THE PRUSSIAN STATE RAILWAY

upper part of the boiler be fitted with from two to four rows of large smoke-tubes which are expanded into the fire-box and front tube-sheet of the boiler, in a special manner. These tubes have an inside diameter ranging from 4 to 5.25 in., which diameter is reduced somewhat near the fire-box end. Inserted in each of these large tubes is a superheater element or section consisting of a set of pipes bent in the form of a double U and connected at the smoke-box end to a header, the whole arrangement being such as to form a continuous double-looped tube. Each particle of steam in passing from the boiler to the branch-pipes has to traverse some one of these elements, making four passes in the movement[1].

The ends of each element extend into the smoke-box, where they are bent slightly upward and are expanded into a common flange which is secured to a steam-collector by a single central bolt. Two slightly different methods are employed in arranging the pipe-ends in the smoke-box. By the first method, the pipes are bent upward only (Fig. 2), as already described, in which case the flange-joints are horizontal, and the flanges are fastened by vertical bolts, the heads of which are movable in slots in the bottom of the collector casting. By the second method (Fig. 1) the pipes are carried forward and are bent upward and backward in such a manner as to connect with vertical flanges secured by horizontal studs to the steam-collector. Both methods have been extensively used, the latter being the one which has been finally selected by the Prussian State Railway. The construction of the steam-collector and the manner in which connection is made with the steam-pipes and with the branch-pipes is best shown by the figures.

By the construction which was adopted, the gases of combustion are divided, one part passing through the ordinary boiler-tubes and the other through the larger tubes. In the larger tubes, a portion of the heat is given up to the water surrounding the tubes, and a portion to the steam contained in the superheating elements inclosed. The flow of heat through the large tubes is controlled by dampers hinged or pivoted below the steam-collector in the smoke-box. As long as the throttle of the locomotive is closed, these dampers are kept closed, either by a counterweight or by a spring, but, as soon as the throttle is opened, they are opened

[1]In the original arrangement, each element consisted of two separate single-loops, but it has been found that the double looping of the superheating pipes, by increasing the velocity at which the steam travels, results in the better protection of the tubes against overheating and in the more effective superheating of the steam.

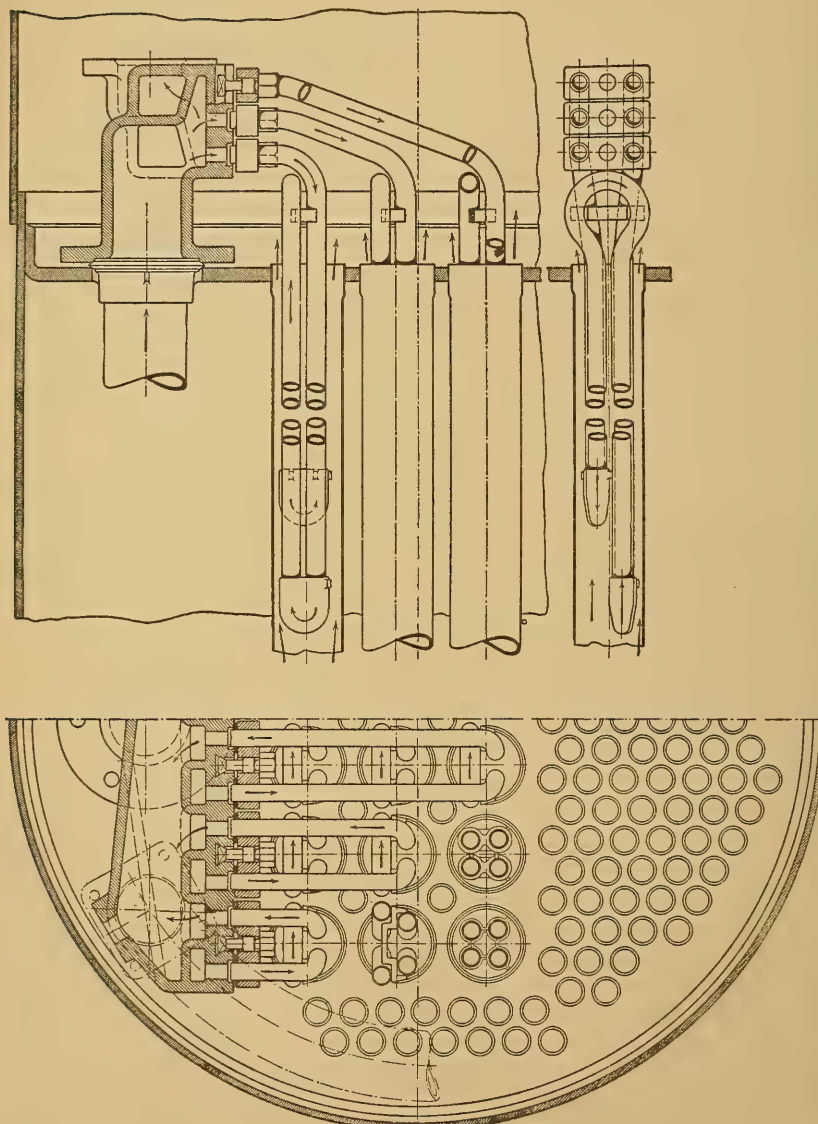


Fig. 2. Smoke-tube Superheater, Wilhelm Schmidt's Patent

simultaneously by means of a piston working in a small automatic steam-cylinder. Thus, while getting up steam or while standing at stations, under any conditions, in fact, for which no steam is passing the loops of the superheater to keep them cool, no gases of combustion can pass through the large smoke-tubes to heat them. This arrangement provides against the overheating of the superheating pipes. It is only when the throttle is open, and when, as a consequence, steam is passing through the superheating pipes, that the dampers which control the circulation of the heated gases open and permit them to have contact with the superheating elements.

The limited number of the superheating elements and their small diameter provide a comparatively small area through which the steam must pass from the boiler to the engine-cylinders. The degree of restriction constitutes one of the important elements in the design of the Schmidt superheater. It has been found that the superheating surface is made more effective as the flow of the circulating steam is made more rapid; the statement by a German authority being that for constant-temperature differences, the rate of heat transmission varies as the square root of the velocity of the steam. By maintaining high velocities through the superheating pipes, therefore, two important results are accomplished: first, a higher degree of superheat is obtained than would otherwise be possible; and, second, the protection of the superheating elements against overheating is made more complete. The degree of restriction employed in the Schmidt superheaters is such that when the engine is working at full power with the throttle wide open, the drop in pressure between the boiler and the valve-box is approximately 15 lb. It is stated that under these conditions the velocity through the superheating tubes varies from 325 to 400 ft. per sec.

The superheaters thus described have an abundant capacity. Locomotives fitted with them are provided with a dial thermometer showing the temperature of the steam in the valve-box. After starting, this temperature steadily rises until it exceeds 300° C. (572°F.), after which the dampers controlling the circulation of heat through the superheater-tubes are partially closed by means of mechanism which connects with a hand-wheel in the cab. This manipulation of the dampers is such as will check the rising temperature before the maximum safe limit of 350° C. (662° F.) is reached.

It is said that there is now no difficulty in designing, for any locomotive, a superheater which will give with certainty any desired degree of superheat within limits which are practicable. Rules governing all portions of the design have been formulated and are strictly adhered to by the Wilhelm Schmidt Company, Limited, in working out the details of their design. Such rules have not yet been published.

4. *The Maintenance of Superheating Locomotives.*—From such inspection in shops and roundhouses as could be had, and from the testimony of those concerned in the handling of locomotives at terminals, it appears reasonably certain that no special difficulty is experienced in the maintenance of those details which are peculiar to a superheating locomotive. This statement applies especially to the superheater itself. In a repair-shop, a boiler was inspected from which the superheater had been drawn bodily and laid upon blocking beyond the boiler. All of the ordinary small tubes of the boiler were being cut out in preparation for retubing. The superheater, meanwhile, had been judged to be in good condition, and it was to go directly back to its place without having any work done upon it. Even those German engineers who are not friendly to the superheating locomotive frankly say that there is no difficulty in maintaining the present type of fire-tube superheater.

With respect to the care bestowed in freeing the large flues containing the superheating tubes from deposits of fuel and ash, the superheating locomotive requires additional labor. The extent of this depends, however, upon the nature of the fuel used. The fact that solid matter entering the large tubes from the firebox comes upon the ends of the loops of the superheater, as upon an obstacle set in their path, naturally leads to such a result. With some grades of fuel little or no difficulty arises, while with other fuels it is difficult to get a locomotive over its division. Roundhouses, therefore, which handle superheating engines, are equipped with long pipe-nozzles through which a blast of air is delivered for the purpose of clearing the large flues. Upon at least one division inspected, an equipment of these nozzles was carried upon locomotives. In certain roundhouses a set of small tools from 3 to 6 ft. long, with sharp cutting-edges, usually at right angles to the axis, which could be inserted from the firebox, was in use for the purpose of cutting out the deposits in the tubes. A round of inspection gives abundant evidence that the stopping-up of flues is a serious matter. Engineers friendly to

the use of superheating locomotives admitted that there are coals which are serviceable in other classes of locomotives which can not be used by the superheating locomotives.

In the matter of maintaining valves and valve-gears, much has already been said. There appears to be no difficulty in maintaining lubrication in the presence of superheated steam, and it is repeatedly asserted that the wear of piston-rings and of valve-parts in the superheating locomotive is of less consequence than the similar wear which occurs in compound and simple engines using saturated steam.

It will be remembered that in Germany the natural competitor of the superheating locomotive is the compound, and that in passenger service the compound is of the four-cylinder type. This fact is often mentioned as important from a roundhouse point of view, since it is easier to wipe the superheating locomotives after a run than the compounds, a statement which, of course, grows out of the fact that there are fewer parts to receive attention. Whether the superheating passenger engines are on the whole easier to maintain than the balanced compounds is a matter concerning which little definite information can be obtained. It is admitted everywhere that the absence of balance for reciprocating parts in the superheating engines tends to increase the cost of maintenance, and it is not unlikely that the newness of the type also operates to its disadvantage. The compounds, on the other hand, with their duplication of parts and their higher boiler-pressures, demand attentions which are peculiar to their type. In Berlin, where superheating is in the ascendancy, it is generally agreed that the problems of maintaining the superheating engine are far more simple than those of maintaining the compound, while in Hanover, which is the home of the balanced compound, opinions are likely to be the reverse of this.

5. *The Economy Resulting from the Use of Superheating Locomotives.*—The degree of economy attending the operation of the superheating locomotive is probably not definitely understood in Germany. There are as yet no locomotive-testing plants in the Empire, and while the results of many road tests are reported, they are upon a comparative rather than upon an absolute basis. As the superheating locomotives are all of recent design, there are no simple locomotives using saturated steam whose performance can fairly be compared with them. Partisan advocates of the superheating locomotive not infrequently claim for it a sav-

ing in water of from 30 to 40 per cent and in coal from 25 to 35 per cent when compared with the simple locomotive, and some data are presented in support of such claims. It is also claimed that the superheating locomotives consume 25 per cent less water and 10 per cent less coal than the four-cylinder balanced compounds. Such statements, of course, reflect partisan opinions. One who is a close student of these matters and whose position is such as to make him quite independent in opinion believes that the saving in water could be taken at from 20 to 25 per cent and in coal at from 15 to 20 per cent, as compared with the consumption of simple locomotives.

When comparisons are made between the performance of a superheating locomotive and that of the compound, partisan advocates of the superheating claim an advantage for their type of machine. Conservative experts give the opinion that the performance of the superheating locomotives is without question equal to that of the compound. Others, whose opinions are perhaps entitled to equal attention, affirm that it is better, but how much better they are not willing to say. The performance sheet of a division operating balanced compounds and also superheating locomotives in the same service shows that the coal used per 1,000 kilometers run by superheating locomotives varied from 12.1 tons to 14.4 tons and for the balanced compounds from 8.9 to 14.2 tons. Upon the basis of these statements, it would appear safe for the American engineer to assume that the superheating locomotives are as economical in their use of water and fuel as are the highest types of compounds. This applies to a practice which involves locomotives using a high degree of superheat on the one hand, and a well-perfected type of compound on the other.

6. *Concerning the Trend of Foreign Practice with Reference to Superheating in Locomotive Service.*—In the development of this chapter, attention has been given thus far to the single type of locomotive which may properly be referred to as the Garbe type. Taking now a more general view of the tendency manifested in Europe with reference to superheating, mention may be made of several significant facts. First, it should be noted that while it is the opinion of the officials of the Prussian State Railway that success in superheating depends in large measure upon the adoption of those details in the design of machine parts which are peculiar to their special type, this view is not shared by the locomotive builders of Germany or by engineers not connected with

railway service. While, therefore, the high character of the details of the Garbe engine from the designer's point of view is unquestioned, it is probably true that the superheater may enter as a detail into the design of any well-considered locomotive without disturbing other details.

Among certain foreign engineers, the plan of combining the superheater with the compound engine has been favored, and a considerable number of locomotives have been constructed on this plan. In favor of such an arrangement, it is urged that the presence of the superheater will serve to reduce the coal consumption of the compound to the extent of approximately 7 per cent. Against it are urged the objections that the two systems are in the main antagonistic; that the compound, to work effectively, must be supplied with steam at high pressure, whereas it is counted as one of the advantages of the superheating locomotive that without sacrifice of its efficiency, it may employ much more moderate pressures. The American engineer is likely to concur in the opinion that the saving which can result from the combination is too small to justify the complication incident to the presence of both systems.

It is an interesting fact that in France, the birthplace of the balanced compound, there is to-day an extremely active interest in the practice of superheating as developed in Germany. A commission representing the leading railways of France has, after a careful investigation, made a report most favorable to the new practice. While it is not likely that superheating locomotives will, in France, be allowed to take the place of the highly developed types of balanced compound common in that country, and while it is not likely that superheating will be employed to any considerable extent in connection with the compounds, there are still in the freight and switching service of France many simple engines, and the hope is expressed that the superheater may be the means of improving them. Meantime, in Belgium, where the advantage of the balanced locomotive is well understood, and the objections of combining the superheater with the balanced compound engine are appreciated, practice is involving a four-cylinder balanced simple engine with the superheater, an arrangement which gives promise of very great success.

7. *Arguments Favoring the Adoption of Superheating.*—These, as based upon results derived from German experience, may be set forth as follows:

1. The advantages of superheated steam may be had in practice without involving undue complication in mechanism and without involving a degree of attention in maintenance in excess of that demanded by a simple engine.

2. The superheating locomotive will perform its service efficiently while employing a comparatively low steam-pressure, a condition which tends to reduce cost in maintenance. The presence of the superheater does not necessitate any qualification of this statement.

3. Superheating will materially reduce the consumption of water, which in bad-water districts constitutes a matter of importance.

4. The superheating locomotive will reduce the fuel consumption probably to that required by a first-class compound engine.

5. As to power and capacity, the superheating locomotive is to be compared with the compound rather than with the simple engine. It may be forced to limits of power far beyond those possible with simple engines.

6. In operation the degree of superheat increases with increased rate of power, which tends to conserve the steam supply as the demand for power is increased.

III. Tests to Determine the Value of Superheating in Locomotive Service

8. *Conditions Suggesting Tests.*—The fact that the efficiency of a steam engine may be improved by superheating the steam supplied it, is a matter which has long been understood and appreciated and the effect of such highly heated steam upon the process of heat interchange, which goes on in the engine cylinder, has been so carefully traced that the precise manner in which the improvement is brought about has been made a part of the common knowledge of the engineer. But the process of producing superheated steam is one which consumes heat and involves apparatus which has been expensive in first cost and difficult to maintain. Against the thermodynamic gain, therefore, which may be secured by the use of superheated steam is to be set the cost of fuel necessary to produce the superheating and the interest and maintenance charges arising from the presence of the superheater. Costs resulting from these accounts are in large measure functions of the design of the superheater and of the materials which enter into its con-

struction, so that the wisdom of adopting the superheater in any branch of steam-engine practice is a matter which involves very much more than the fundamental thermodynamic theory—a fact which greatly complicates the task of the present-day student in this particular field of research. In recent years, the problem of superheater design has received generous attention, and materials possessing qualities hitherto unobtainable have been made available to the designer, so that a practice which a quarter of a century ago was generally regarded as of doubtful expediency has gradually been advanced to a position of great promise.

Attention has been called to the extensive use of superheated steam in the locomotive practice of Germany and to the influence of this practice upon that of other European countries and of America. In America, especially, there are evidences of a strong professional interest which is doing much to secure for our country a more general introduction of superheating locomotives. Under the stimulus of these developing conditions, it was natural that the energies of the locomotive-testing laboratory of Purdue University should have been turned in the direction of superheated steam. The University's locomotive, designed originally for work under high steam pressure, was converted into a superheating locomotive, and with the aid of many friendly influences has since been subjected to an elaborate series of tests, the results of which define the performance which is to be expected from such a machine. This performance, when compared with that of a normal locomotive using saturated steam, should aid in making up an estimate of the gain to be secured by the use of the superheater in locomotive service.

9. *The Means Employed.*—The locomotive laboratory of Purdue University, established in 1891 for the instruction of students and for research, has been many times described[1]. The locomotive, which for a number of years had been operated upon this plant, is of the single-expansion American type, having a boiler designed to carry working pressures as high as 250 lb. per sq. in[2]. In preparation for a new program of tests, this locomotive was fitted with a Cole superheater, the boiler and other parts being rebuilt, so far as was necessary, to make the reconstructed machine a normal superheating locomotive which, from the time of reconstruction, has been known as *Schenectady No. 3.*

[1] "The Purdue University Locomotive Testing Plant;" also Locomotive Testing Plants" (A. S. M. E.).

[2] "For complete description with drawings of this locomotive, see High-Steam Pressures in Locomotive Service, Publication No. 66, Carnegie Institution of Washington.

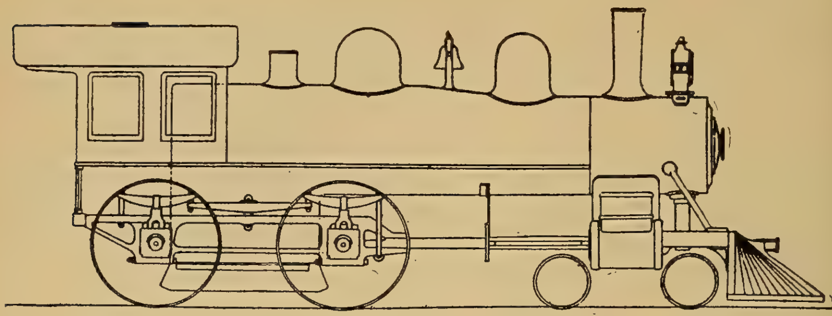


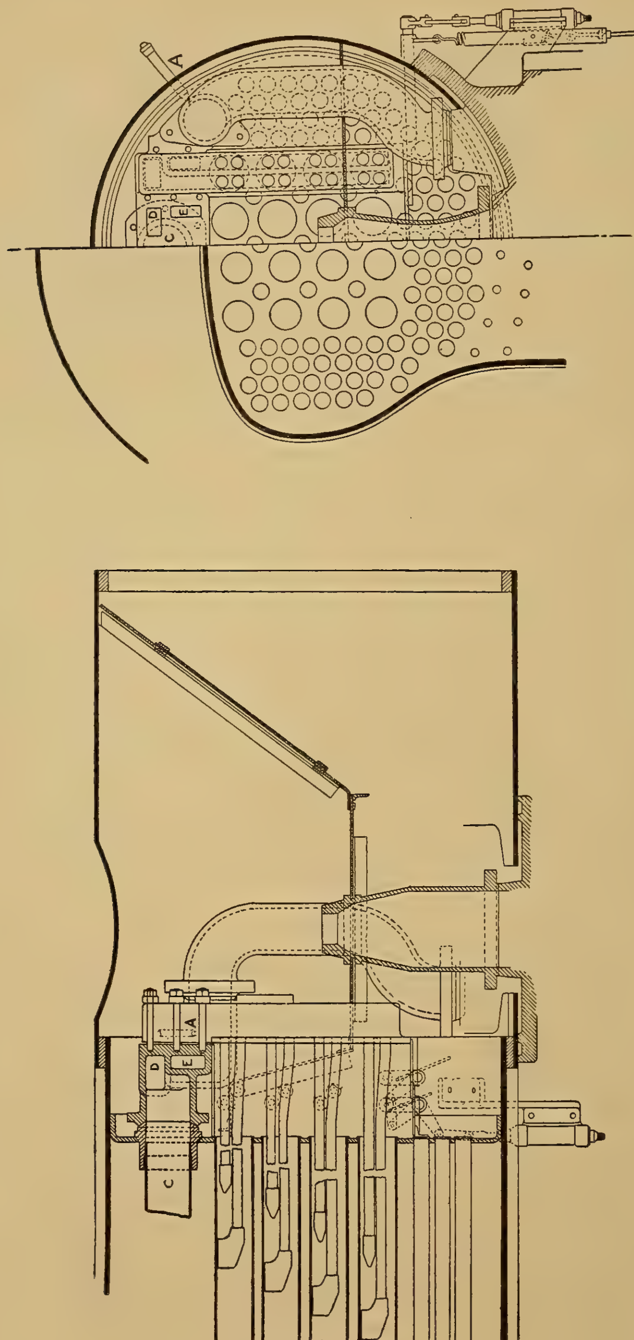
FIG. 3. OUTLINE ELEVATION OF LOCOMOTIVE

10. *The Principal Characteristics of Locomotive Schenectady No. 3* are as follows:

Type	4-4-0
Total weight (pounds)	109,000
Weight on four drivers (pounds)	61,000
Valves (type, Richardson balance):	
Maximum travel (inches)	6
Outside lap (inches)	1⅛
Inside lap (inches)	0
Ports:	
Length (inches)	12
Width of steam-port (inches)	1½
Width of exhaust-port (inches)	3
Total wheel-base (feet)	23
Rigid wheel-base (feet)	8½
Cylinders:	
Diameter (inches)	16
Stroke (inches)	24
Drivers, diameter outside of tire (inches)	69¼
Boiler (type, extended wagon-top):	
Diameter of front-end (inches)	52
Length of fire-box (inches)	72$\frac{1}{16}$
Width of fire-box (inches)	34¼
Depth of fire-box (inches)	79
Number of 2-inch tubes	111
Number of 5-inch tubes	16
Length of tubes (feet)	11.5
Heating-surface in fire-box (square feet)	126
Heating-surface in tubes, water side (square feet)	897
Heating-surface in tubes, fire side (square feet)	817
Total water-heating surface, including water side of tubes (square feet)	1,023
Total water-heating surface, including fire side of tubes (square feet)	943
Superheater; type, Cole return tube:	
Outside diameter of superheater tubes (inches)	1¼
Number of loops	32
Average length of tube per loop (feet)	17.27
Total superheating surface based upon outside surface of tubes, surface of headers neglected (square feet)	193
Total water and superheating surface, including water side of boiler tubes (square feet)	1,216
Total water and superheating surface, including fire side of boiler-tubes (square feet)	1,136
Total water and superheating surface, accepted for use in all computations (square feet)	1,216

Ratio of heating-surface based on water side to that based on fire side........	1.074
Thickness of crown sheet (inch)..	$\frac{7}{16}$
Thickness of tube-sheet (inch)..	$\frac{9}{16}$
Thickness of side and back sheet (inch)..	1
Diameter of radial stays (inches)..	1⅛
Driving-axle journals:	
Diameter (inches)..	7½
Length (inches)..	8⅛

11. *The Cole Superheater*, as applied to the locomotive, is well shown in Fig. 4. It consists chiefly of a series of return-tubes extending inside of certain of the flues which make up a portion of the water-heating surface. To make room for the superheater, the upper central portion of the usual flue-space is taken by sixteen 5-inch flues, which are reduced to a diameter of 4 in. for 7 in. of their length at the fire-box end and increased to a diameter of 5 $\frac{1}{16}$ in. at the front tube-sheet. They have a length between flue sheets of 138 in. In each of these sixteen flues, there is an upper and a lower line of superheating tubes. Each line extends from a steam-pipe header in the smoke-box back into its flue to a point near the back tube-sheet, where it meets and is screwed into a return-pipe fitting of special design. From the second of the two openings in this fitting, a similar pipe extends forward through the flue and into the smoke-box to a second header, from which branch-pipes lead to the cylinders. All together, there are 32 of these loops. In 13 of the flues, the lower loops are 116 $\frac{3}{8}$ in. long, extending into the flue within 2 ft. 5 in. of the back of the tube-sheet. In the other 3 flues the loops are, respectively, 3 ft., 2 ft., and 1 ft. shorter than the normal. The upper loop in each flue is, in all cases, approximately 9 in. shorter than the lower loop. The headers to which the pipes of the superheater connect at the smoke-box end are of cast steel. They have walls three-eighths of an inch thick and are cored in such a manner that all steam passing the throttle-valve must traverse some one of the several loops. In its passage from the boiler, the steam leaves the dry-pipe *C*, Fig. 4, and passes into the headers through the openings *D* in the top part of the tee-head. It then flows downward through the passage in one side of this header and passes back toward the fire-box through the 8 tubes which are joined to it. At the castings which form the return bends, its direction is reversed and it passes back through the return tubes to the passage in the other side of the header. It then passes upward into the lower half of the tee-head *E*, and from there into the branch steam-pipes.



G. 4. Cole Superheater

12. *A Comparison of the Dimensions* of the boiler as set forth above with those of *Schenectady No. 2*, follows. The exhibit shows the extent of the change brought about by the installation of the Cole superheater.

Number of 2-in. flues displaced by sixteen 5-inch flues, necessary to give place to the superheater	89
Reduction in water-heating surface (square feet)	299
Reduction in water-heating surface (per cent)	22.6
Heating-surface replaced by the installation of the superheater (square feet)	193
Heating-surface replaced by the installation of the superheater (per cent of surface removed)	64.5
Reduction in total transmitting-surface (water and superheating) (square feet)	106
Reduction in total transmitting-surface (water and superheating) (per cent)	8

13. *The Tests* were begun in November, 1906, and were completed, so far as the series at present under consideration is concerned, in July, 1907. During this period of 8 months, the experimental locomotive ran 1,417,995 revolutions, which is equivalent to 4,851 miles.

The tests were run at pressures of 240, 200, 160, and 120 lb., respectively, the number of tests under each pressure being sufficient to disclose the performance when running at several different speeds and cut-offs. The exhibit is especially full for a pressure of 160 lb., a pressure which is a close approach to that commonly used in connection with superheating locomotives. Altogether, 38 tests were run. A concise statement of the pressure, speed, and cut-off represented is set forth by Fig. 5 to 8, in which each

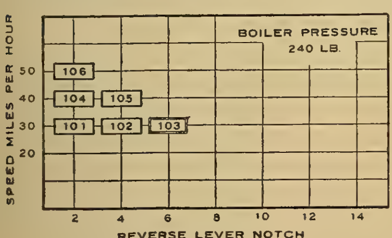


FIG. 5. TESTS AT 240-LB. PRESSURE

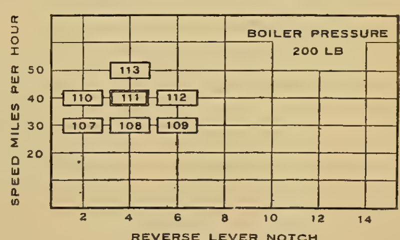


FIG. 6. TESTS AT 200-LB. PRESSURE

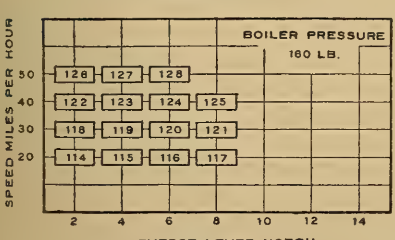


FIG. 7. TESTS AT 160-LB. PRESSURE

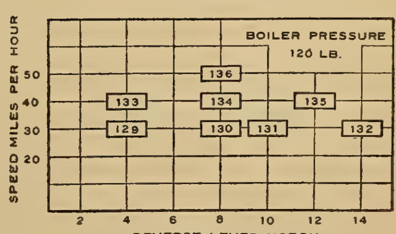


FIG. 8. TESTS AT 120-LB. PRESSURE

rectangle represents the general conditions of one test. Concentric rectangles represent conditions for which duplicate tests were run; the enclosed numeral is the serial number of the tests.

All results presented are directly comparable with corresponding results obtained when using saturated steam, which are set forth in the published account of a previous research.[1]

IV. Performance of Boiler and Superheater

14. Attention should early be called to the fact that because of threatened interruptions in the running of the tests, it was thought expedient to use two grades of coal and to rely upon the results disclosed by the heat-balance of the several tests as a basis for final comparisons. Of the two fuels used, the Youghiogheny coal has been accepted as standard, and where results obtained from the Pocahontas coal have been needed in formulating conclusions, they have been reduced to equivalent results which would have been obtained had the standard coal been used throughout the work. One man served as fireman for all tests.

In the paragraphs which follow, an attempt will be made to state briefly some of the more significant facts which may be developed from the data; in some cases graphic methods are employed to emphasize their importance.

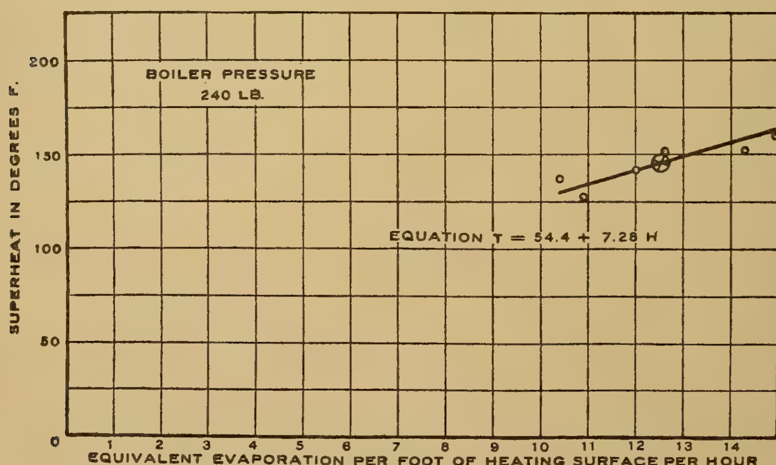


Fig. 9. Superheating As Affected by Rate of Evaporation, Boiler Pressure 240 lb.

[1] *High Steam-Pressures in Locomotive Service*, by W. F. M. Goss, Publication No. 66, Carnegie Institution of Washington, reviewed and presented as Bulletin No. 26, *High Steam-Pressures in Locomotive Service*, Engineering Experiment Station, University of Illinois.

15. *Superheating.*—The observed temperature of the steam delivered from the superheater was measured by a high-grade mercurial thermometer placed in the header at a point near its opening into the left-hand branch-pipe, point *A*, Fig. 4. Fig. 9 to 12 present the extent of the superheating effect for the different rates of power to which the boiler was worked, (equivalent

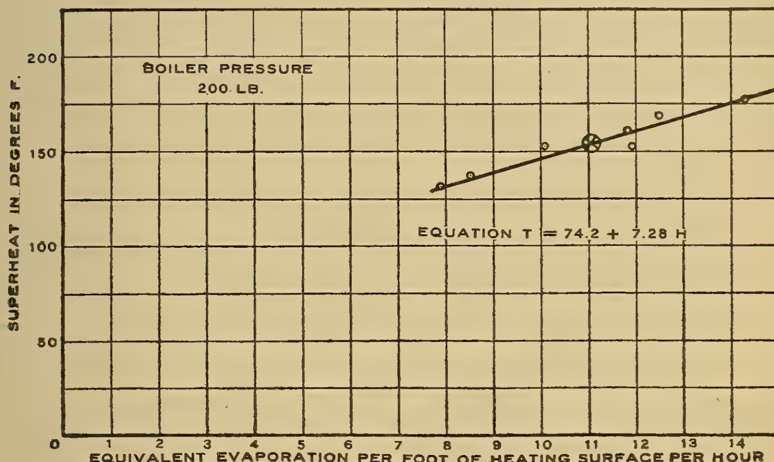


FIG. 10. SUPERHEATING AS AFFECTED BY RATE OF EVAPORATION
BOILER PRESSURE 200 POUNDS

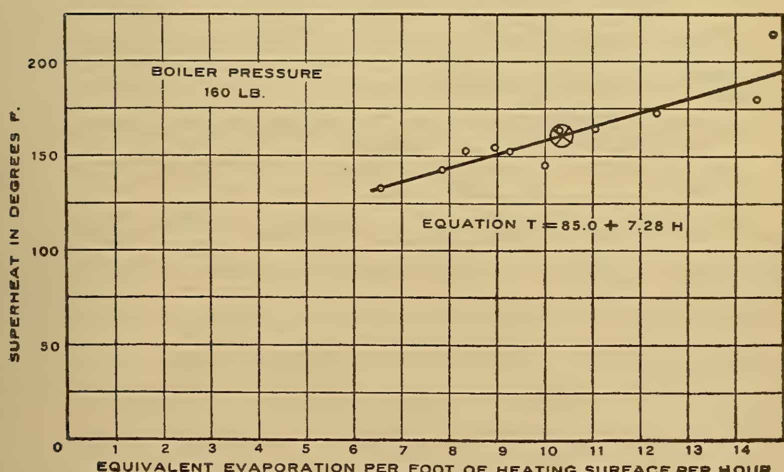


FIG. 11. SUPERHEATING AS AFFECTED BY RATE OF EVAPORATION,
BOILER-PRESSURE 160 POUNDS

evaporation per square foot of water-heating surface per hour), each diagram representing some one of the several pressures under which the boiler was operated. The ordinates and the abscissas of all points in each diagram have been averaged, and from values thus obtained, an average point, designated as a cross in a circle, has been located. Through this a straight line has been drawn which represents, with a fair degree of accuracy, all of the experimental points involved. It happens that the slope of these lines is the same for all of the diagrams under consideration. They define the change in the degree of superheat attending changes in the rate of evaporation when the boiler-pressure has the value stated.

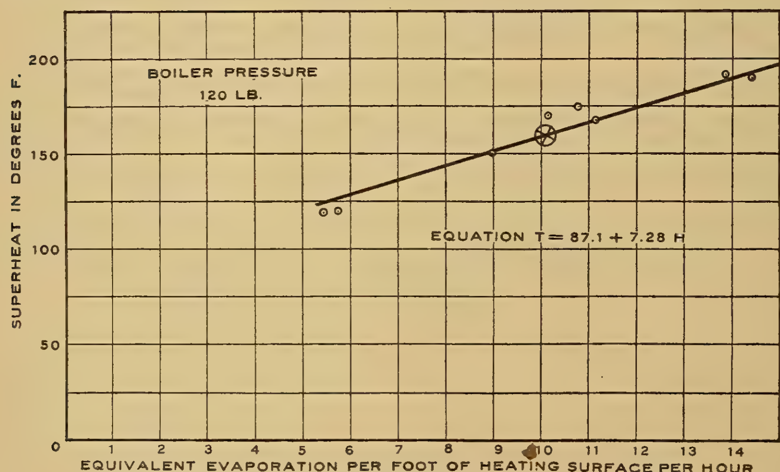


FIG. 12. SUPERHEATING AS AFFECTED BY RATE OF EVAPORATION, BOILER-PRESSURE 120 POUNDS

A comparison of the several diagrams will show that if the boiler were operated under a constant rate of evaporation for each of the several boiler-pressures, the degree of superheat would be different in each case. For example, if a comparison is based upon a rate of evaporation of 11 lb. of water per sq. foot of water-heating surface per hour, the degree of superheat will be:

170° when the boiler pressure is 120 lb.
165° " " " " " 160 "
154° " " " " " 200 "
135° " " " " " 240 "

These values are shown graphically in Fig. 13. Referring to this figure, it will be noticed that the superheat varies more rapidly for a given increment at the higher pressures than at the lower pressures, and that the line connecting the experimental points is a curve. A straight line may, however, be drawn which will represent all the experimental points with an error which in no case will be greater than 2 per cent. In the same manner, a series of straight lines may be determined, each showing the re-

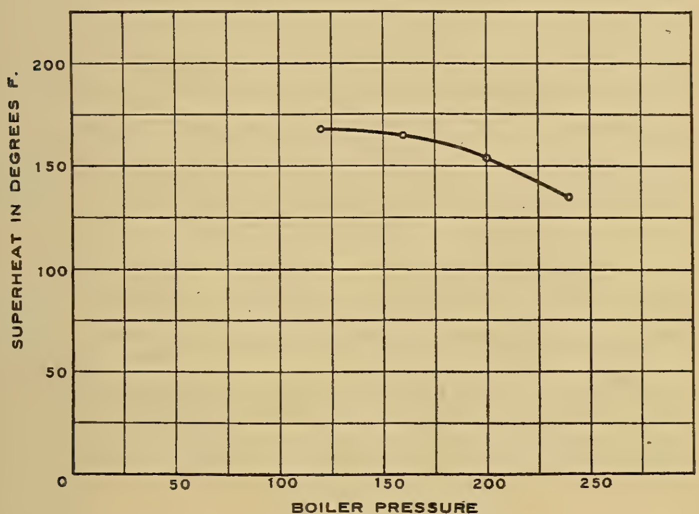


FIG. 13, DEGREES OF SUPERHEAT UNDER ALL CONDITIONS OF PRESSURE WHEN RATE OF EVAPORATION IS 11 POUNDS PER SQUARE FOOT OF HEATING-SURFACE PER HOUR

lation between boiler pressure and superheat at a different rate of evaporation. This series of straight lines may be represented by an equation, which defines the performance of the superheater for any pressure between 120 and 240 lb. gauge, with a maximum error of less than 2 per cent. The equation thus determined is

$$T = 123 - 0.265\,P + 7.28\,H$$

where T equals the number of degrees superheat; P the boiler-pressure by gauge, and H the equivalent evaporation per square foot of water-heating surface in the boiler.

16. *Draft.*—The draft produced in the front-end of the locomotive was measured at a point directly in front of the diaphragm. The rate of increase in draft values with increased rates of evaporation is well shown by Fig. 14. This figure gives the results

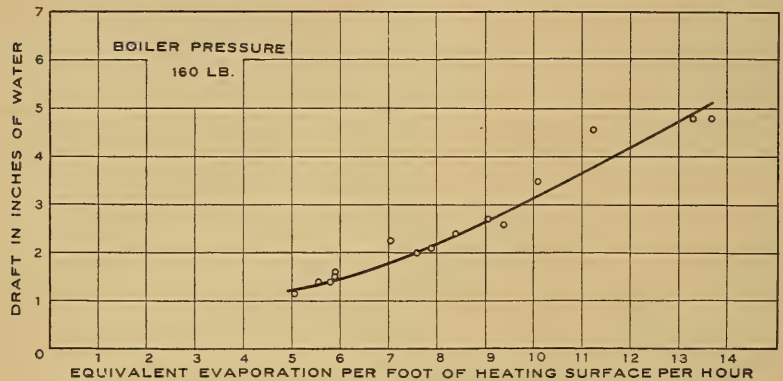


FIG. 14. DRAFT IN FRONT OF DIAPHGRAM

of tests at 160 lb. only. Curves representing points obtained at other pressures are practically identical with those shown, the fact being that changes in boiler-pressure, within the limits of the experiments, have practically no influence upon draft values. As would be expected, these depend entirely upon the rate of evaporation required.

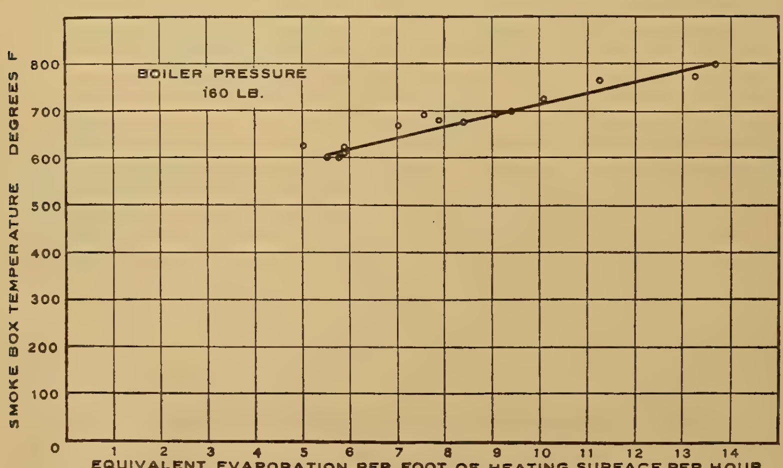


FIG. 15. SMOKE-BOX TEMPERATURE, BOILER-PRESSURE 160 POUNDS

17. *Smoke-Box Temperatures.*—The temperatures of the smoke-box gases were read by a mercurial thermometer placed midway between the diaphragm and the front tube-sheet. Fig. 15 shows the effect upon the smoke-box temperature of changes in the rate of power for a boiler pressure of 160 lb. It will be seen that the smoke-box temperature increases as the rate of evaporation is increased, an effect the significance of which is well understood. For example, when the rate of evaporation equals 6 lb. of water per sq. foot of heating-surface, the smoke-box temperature is approximately 600° F. When the rate of evaporation is increased to 12, the temperature of the smoke-box approaches 800° F. It is not far from the truth to say that a change of 1 lb. in the rate of evaporation produces a change of 20° in the temperature of the smoke-box. The smoke-box temperature shows a slight tendency to increase with increase of pressure, other things being the same, but the differences are too slight to be accepted as material.

18. *Evaporative Efficiency of the Combined Boiler and Superheater.*—The relation between the pounds of water evaporated

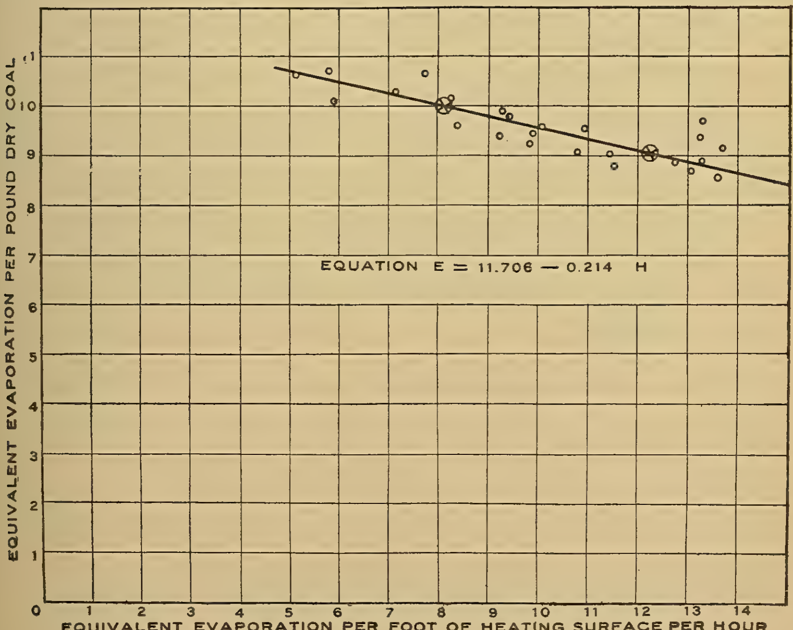


FIG. 16. EQUIVALENT EVAPORATION PER POUND OF COAL, UNDER ALL CONDITIONS OF PRESSURE; COMBINED BOILER AND SUPERHEATER

from and at 212°, equivalent to the weight of superheated steam delivered per pound of dry coal, and the equivalent evaporation per sq. ft. of surface in the boiler and superheater per hr., under all conditions of pressure, is given in Fig. 16. The data show that, if the discussion is allowed to concern itself with very small differences, the highest efficiency is obtained when the boiler pressure is lowest; conversely, the lowest efficiency results when the boiler pressure is highest. But, except in the case of tests at 120 lb., the results of which do not compare closely with those for other pressures, the differences are hardly more than measurable. In a larger sense, it seems to be true that changes in boiler pressure between the limits of 120 lb. and 240 lb., have practically no effect upon the evaporative efficiency of the boiler.

Proceeding on this basis, it is clear that a general expression for the evaporative efficiency of the combined boiler and superheater may be based upon the results of all tests, regardless of the pressure at which they were run. Such expression is represented by the line drawn through the plotted points of Fig. 16.

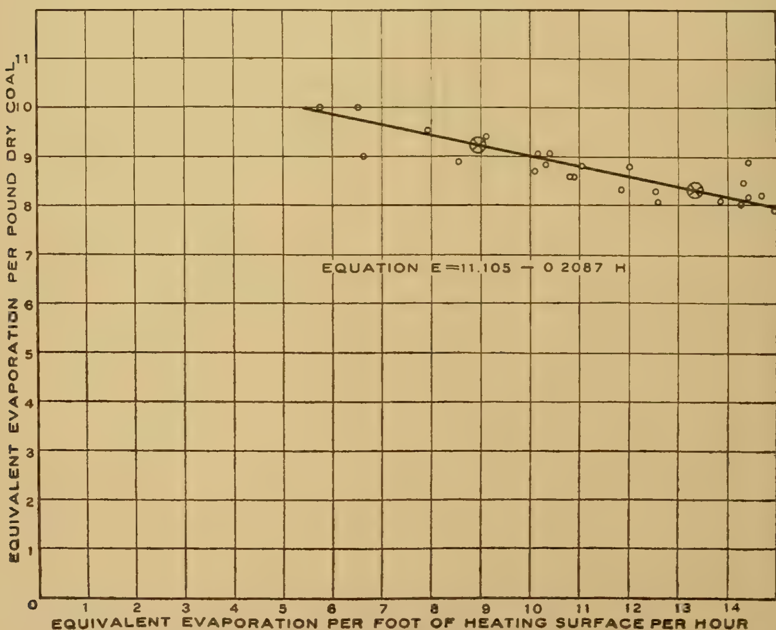


FIG. 17. EQUIVALENT EVAPORATION PER POUND OF COAL UNDER ALL CONDITIONS OF PRESSURE, EXCLUSIVE OF THE SUPERHEATER

The equation for this line, and consequently one which defines in general terms the performance of the combined boiler and superheater, is

$$E = 11.706 - 0.214\,H$$

where E is the equivalent evaporation from and at 212° F. per pound of dry coal, and H is the equivalent evaporation per square foot of water and superheating surface.

19. *Evaporative Efficiency of the Boiler, Exclusive of the Superheater.*—The equivalent evaporation of the boiler per pound of dry coal, in terms of the equivalent evaporation per square foot of waterheating surface in the boiler per hour is shown in Fig. 17. The equation for the mean line drawn through these points is:

$$E = 11.105 - 0.2087\,H$$

This curve is substantially of the same slope as that which represents the performance of the combined boiler and superheater (Fig. 16), but it represents values which are lower, a result due to the fact that the basis of the comparison practically assumed that the heat which is normally absorbed by the superheater is in this case lost.

20. *The Division of Work between Water and Superheating Surface.*—The ratio of the heat absorbed per square foot of superheating surface to that absorbed per square foot of water-heating surface may be accepted as an expression of the relative efficiency of the water and superheater surface. Fig. 18 represents

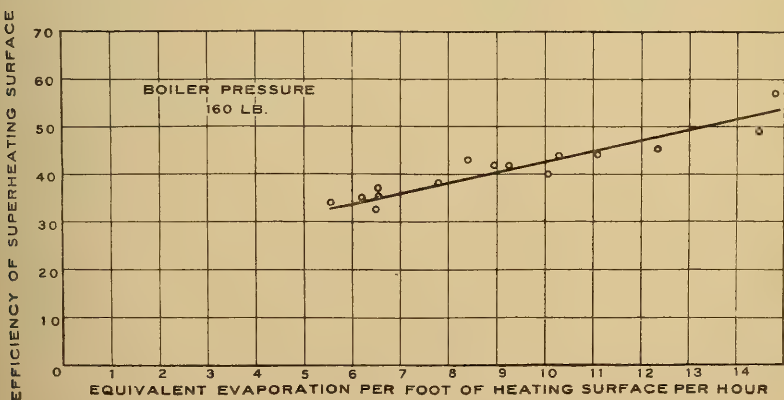


FIG. 18. RELATIVE EFFICIENCY OF THE SUPERHEATING SURFACE, THAT OF THE WATER-HEATING SURFACE BEING 100; BOILER-PRESSURE 160 POUNDS

this quantity plotted against equivalent evaporation per square foot of water-heating surface for a boiler pressure of 160 lb. Referring to this figure, it will be seen that as the rate of evaporation increases, there is a corresponding increase in the ratio of the heat absorbed per square foot of superheater surface to that absorbed per square foot of boiler surface. Thus, the ratio has a value of 34 per cent when the rate of evaporation is 6 lb. of water per square foot of water-heating surface and 53 per cent when the rate of evaporation is increased to 14 lb. The value of this ratio is independent of the boiler-pressure.

21. *Smoke-Box Gases.*—The percentage of excess air is in all cases small (between 20 and 25 per cent in most tests) and it distinctly tends to diminish as the rate of evaporation is increased. The reason for this is to be found in the fact that in locomotive service higher rates of evaporation necessarily involve the use of thicker fires, which offer greater resistance to the admission of air.

The percentage of carbon dioxide (CO_2) present in the smoke-box gases ranges from 10.8 to 14.6. The significance of these results as factors in any general comparison is impaired by the variable quality of the fuel used. Taken as they stand, they do not disclose any well-defined law governing the changes in their value with changes in the rate of combustion. The highest values are, however, those which were obtained in tests under the higher pressures, the average value for all tests at 240 lb. being 14.25, while the average value for all tests at 120 lb. is but 11.70. This may be accepted as evidence that, for some reason not defined

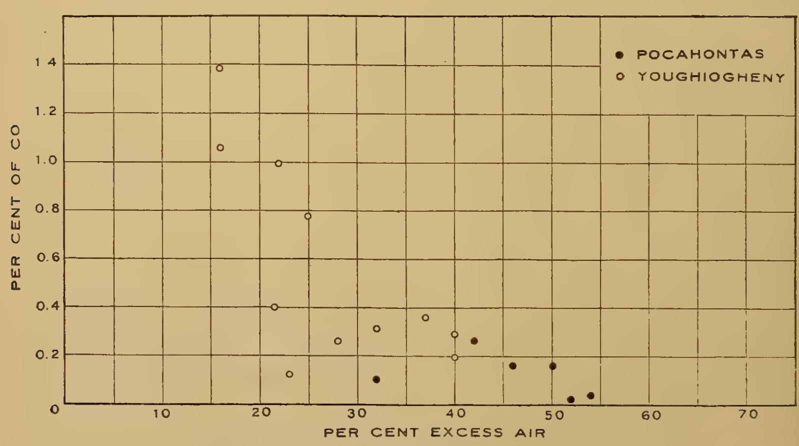


FIG. 19. CARBON MONOXIDE AND EXCESS AIR

the fire was maintained in a more efficient condition during the tests under high pressure than during those at lower pressures.

The percentage of carbon monoxide (CO) present in the smoke-box gases is never great notwithstanding the low percentage of excess air present. At the same time, there are no tests that do not show the presence of a trace or more than a trace of this gas. Its tendency to increase as the percentage of excess air diminishes is well shown by Fig. 19. This figure shows also that under similar conditions, the combustion of the Pocahontas coalis less perfect than that of the Youghiogheny, a result which is more likely to be due to the presence of a greater percentage of fine coal in the Pocahontas than to differences in composition. The tendency of carbon monoxide to increase with increased rates of evaporation is shown by Fig. 20. This tendency is doubtless due to mechanical conditions. It may be accepted also as a function

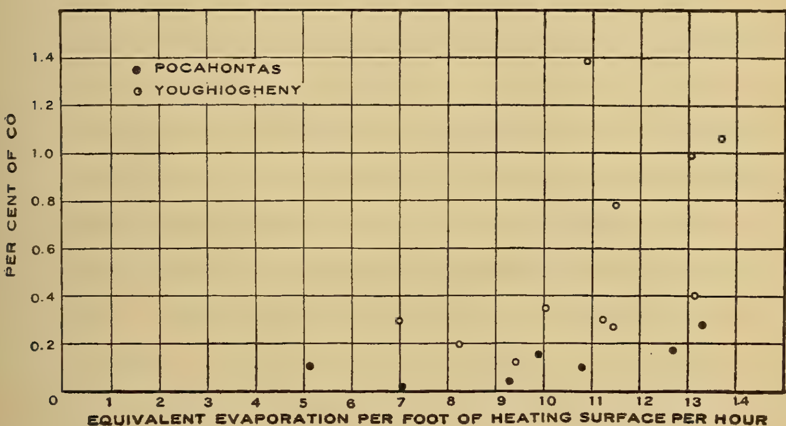


FIG. 20. CARBON MONOXIDE AND RATE OF EVAPORATION

of that tendency to which attention has already been called. Thus, increased rates of evaporation demand higher rates of combustion, and these in turn require more air, which must be supplied by an increase in the strength of the draft-action. In the presence of a stronger draft, the bed of the fire must be thickened, and the thicker fire throttles the passage of air into the fire-box to such an extent that the supply is not commensurate with the increase in the draft-action; hence a reduction in the amount of excess air, and this, as has already been shown, leads to an increase in the percentage of unconsumed gas.

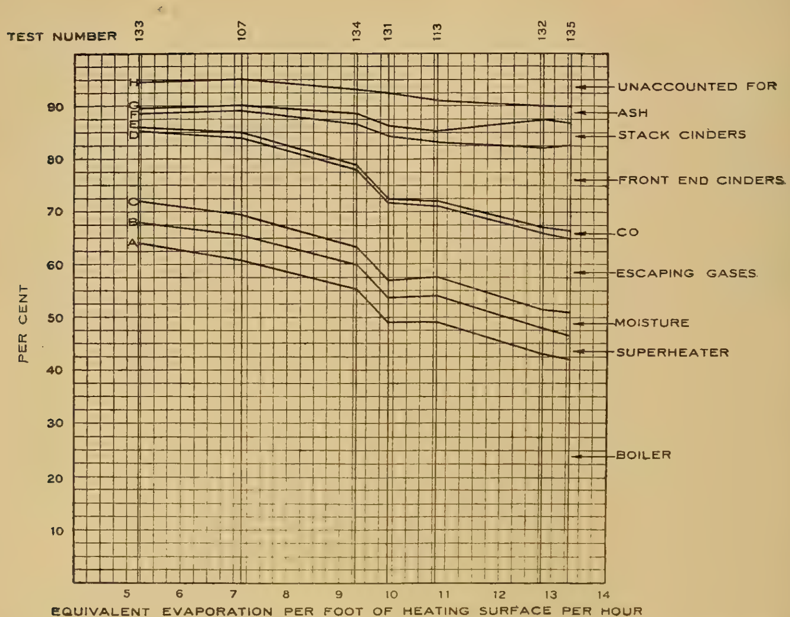


FIG. 21. HEAT-BALANCE OF COMBINED BOILER AND SUPERHEATER AS DERIVED FROM TESTS OF POCAHONTAS COAL

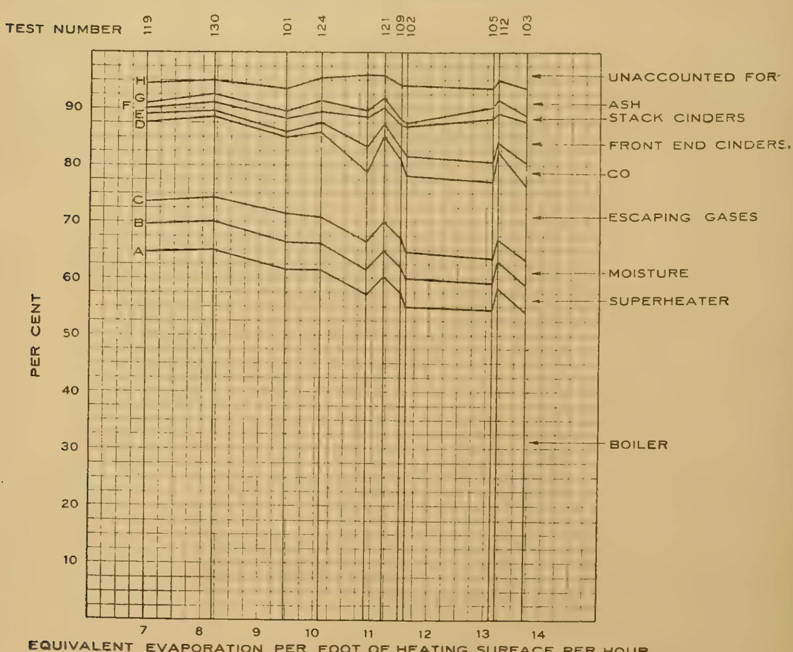


FIG. 22. HEAT-BALANCE OF COMBINED BOILER AND SUPERHEATER AS DERIVED FROM TESTS OF YOUGHIOGHENY COAL

22. *Heat Balance.*—From data obtained, it has been possible to complete a heat-balance for 18 tests. Graphic representations of the heat absorbed and of the heat lost, plotted in terms of the rate of evaporation, are presented by Fig. 21 and 22. The necessity for two diagrams is to be regretted. It has been no part of the purpose of the present work to make tests of coals, and thus far in the discussion, it has been possible to avoid bringing into direct comparison the results obtained from the two varieties employed. The process of making up a heat-balance, however, admits of no compromise, and the discussion which follows necessarily defines the behavior of the coals. In the consideration given this portion of the work, it will be well to remember that the commercial grading of the two coals was not the same. This is well brought out by the following summarized facts concerning them.

The Pocahontas coal used was run-of-mine, and, as such it contained a considerable amount of slack. It was fairly uniform throughout.

The Youghiogheny coal was obtained from two different sources and was less uniform in quality. All tests involving this fuel, run prior to April 12, were fired with a so-called Virginia lump, while tests run after this date were fired with fuel delivered as run-of-mine, but which was screened at the laboratory before being used. Averages of all results obtained from samples of the Pocahontas and Youghiogheny coal are shown in the following statement.

	Pocahontas	Youghiogheny
Moisture (per cent)	3.10	1.89
Volatile matter (per cent)	15.23	31.94
Fixed carbon (per cent)	72.75	57.71
Ash (per cent)	8.92	8.46
Heating value per pound of dry coal (B. t. u.)	14,347	14,047
Heating value per pound of combustible (B. t. u.)	15,802	15,372

In the diagrams, Fig. 21 and 22, the term "heating-surface", as employed in designating the abscissas, includes the heat-transmitting surface of both boiler and superheater. The ordinates of the diagrams represent the percentage of heat in the fuel supplied. Distances measured on ordinates between the axis and the first line *A* represent the percentage of the total heat supplied which is absorbed by the water of the boiler. The line *A* is in fact a definition of the efficiency of the boiler under the varying

rates of evaporation represented by the series of tests. While based upon a different unit, it is, as it ought to be, similar in form to curves defining the evaporative efficiency of the boiler, which shows the pounds of water evaporated per pound of coal used. The inclination of all such lines shows the extent to which the efficiency of the boiler suffers as the rate of evaporation is increased. The nature and extent of the losses leading to such a result are to be found in the areas above the line *A*. The fact that the points representing different tests, through which this line *A* is drawn, do not result in a smooth curve is due to irregularities in furnace conditions which were beyond the vigilance of the operator, an explanation which applies equally to other lines, *B*, *C*, *D*, etc., of the same diagrams.

The percentage of the total heat which is absorbed by the superheater is measured by distances on ordinates between the line *A* and the line *B*. It is apparent from the record that the percentage of the total heat absorbed by the superheater is practically constant, whatever may be the power to which the boiler is driven. The normal maximum power of a locomotive may, for present purposes, be assumed to be that power which is represented by an evaporation of 12 lb. of water per sq. foot of heating-surface per hour. Basing a statement on the record as it appears from the rate of power, the superheater, which contains 16 per cent of the total heat-transmitting surface, receives approximately 8 per cent of the total heat absorbed. Distances between the broken line *B* and the axis represent the efficiency of the combined boiler and superheater. Distances above this line *B* account for the various heat-losses incident to the operation of the furnace, boiler, and superheater.

Losses of heat arising from the presence of accidental and combined moisture in the fuel, the presence of moisture in the atmospheric air admitted to the fire-box, and of moisture resulting from the decomposition of hydrogen in the coal are represented by distances measured on ordinates between the lines *B* and *C*. It is of passing interest to note that the heat thus accounted for is practically equal to that absorbed by the superheater.

Losses of heat in gases discharged from the stack are represented by distances measured on ordinates between the lines *C* and *D*. The distances between the lines *D* and *E* represent that portion of these losses which is due to the incomplete burning of the combustible gases. The record shows that this loss is necessarily large, but does not increase with the increased rates of com-

bustion, as has commonly been supposed. In other words, the loss in evaporative efficiency with increase of power does not occur in any degree through the channel of the smoke-box gases. That portion of this loss which is chargeable to incomplete combustion is small under low rates of combustion, but may increase to values of some significance under the influence of very high rates of combustion, as will be seen from the record of the Youghiogheny coal.

Losses of heat through the discharge from the fire-box of unconsumed fuel are represented by distances measured on ordinates between the lines *E* and *H*. The loss thus defined is separated into three parts, viz., the heat lost by partially consumed fuel in the form of cinders collecting in the front-end (*E F*); the heat lost by partially consumed fuel in the form of cinders or sparks thrown out of the stack (*F G*); the heat lost by partially burned fuel dropping throught the grate into the ash-pan (*GH*).

The first two of these losses increase with the rate of power developed. They are in fact the chief cause of the falling off of the evaporative efficiency of a locomotive boiler with increased rates of power. This is well shown by a comparison of the two diagrams. In the case of tests with the Pocahontas coal (Fig. 21) the cinder loss is comparatively heavy and the boiler efficiency diminishes in a marked degree under high rates of power, while tests under similar conditions with the Youghiogheny coal (Fig. 22), involving less loss by cinders, show an efficiency of the boiler under high rates of power which is much better sustained.

The cinder loss, expressed as a percentage of the total weight of coal fired, is shown by Fig. 23, and the heating value of the material thus accounted for by Fig. 24. It will be seen that cinders from the Pocahontas coal have more than double the weight, and that each pound has nearly double the heating value of those resulting from the Youghiogheny coal, a result doubtless due in part to the large percentage of fine coal in the Pocahontas and to the absence of such material in the Youghiogheny. The stack cinders from both coals have a higher calorific value than those caught in the smoke-box. Under the practice of the laboratory, in no case was the coal wetted previous to its being fired. Concerning the general significance of the results, it will be well to remember that the fuel used in all tests was of high quality. Lighter and more friable coals are, as a rule, more prolific producers of stack and front-end cinders.

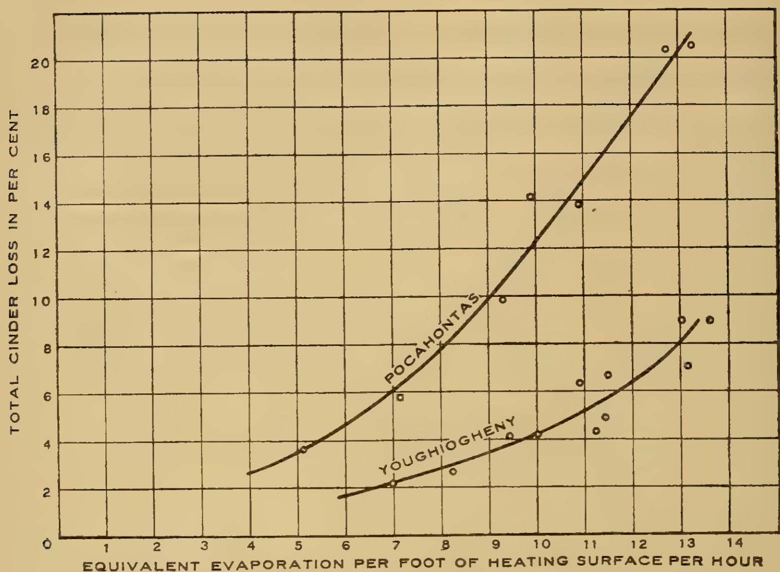


FIG. 23. STACK AND FRONT-END CINDER LOSS, PER CENT OF COAL FIRED

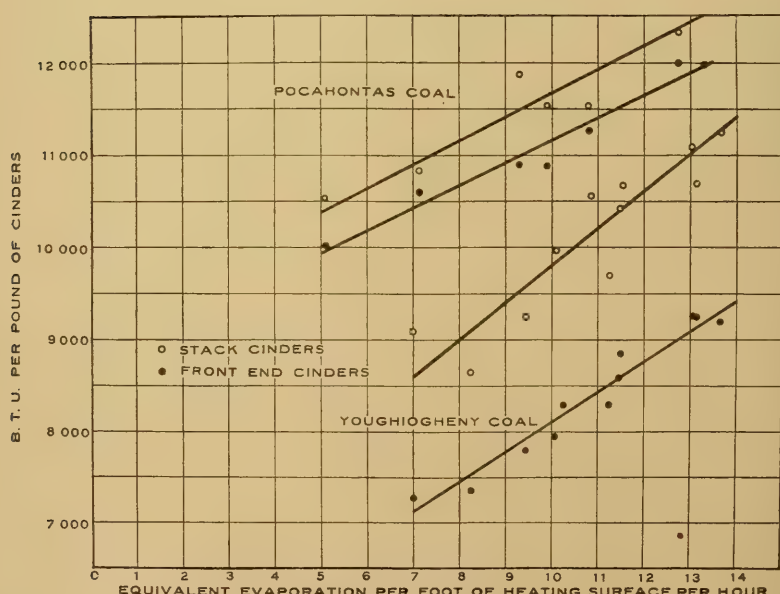


FIG. 24. HEAT VALUE OF STACK AND FRONT-END CINDERS

Radiation, leakage, and all other losses unaccounted for are represented by distances measured on ordinates between the line *H* and the 100 per cent line of the diagram. The radiation losses are probably from 1 to 2 per cent of the total heat available, the remainder equaling from 2 to 4 per cent, representing leakage of steam or water, or inaccuracy in the determination of quantities already discussed.

23. *A Summarized Statement with Reference to the Distribution of Heat in the Locomotive Experimented upon.*—It is sometimes convenient, for the purpose of fixing values in one's mind, to have an elaborate statement of fact summarized into a few representative values, the relation between which may be easily remembered. Such a summary may be framed in the present case by assuming that the normal maximum power of the locomotive tested is that which involves a rate of evaporation of 12 lb. of water per square foot of heating surface per hour, and by averaging values for this rate of power, from the diagrams, Fig. 21 and 22. The result may be accepted as showing in general terms the action of such a locomotive as that tested when fired with a good Pennsylvania or West Virginia coal. It is as follows:

	Per cent
Total heat available, absorbed by water in boiler	52
Total heat available, absorbed by steam in superheater	5
Total heat available, lost in vaporizing moisture in coal	5
Total heat available, lost through discharge of CO	1
Total heat available, lost through high temperature of escaping gases, the products of combustion	14
Total heat available, lost through unconsumed fuel in the form of front-end cinders	3
Total heat available, lost through unconsumed fuel in the form of cinders or sparks passed out of stack	9
Total heat available, lost through unconsumed fuel in ash	4
Total heat available, lost through radiation, leakage of steam and water, etc	7
Total heat available accounted for	100

V. Performance of the Engine and of the Locomotive as a Whole

24. *Indicated Horse-Power.*—The range in the values of the indicated horse-power for all pressures falls between the limits of 163 and 599 h. p. The maximum horse-power (599) was developed under a boiler pressure of 160 lb. and a speed of 40 miles per hr., with the reverse lever in the eighth notch, corresponding to a cut-off of 35 per cent of stroke. The following table shows the effect

of changes in boiler pressure upon the power output of the engine when run with the reverse lever in the fourth notch at a speed of 40 miles an hour.

Pressure	Corresponding horse-power
lb.	
120	179
160	343
200	445
240	551

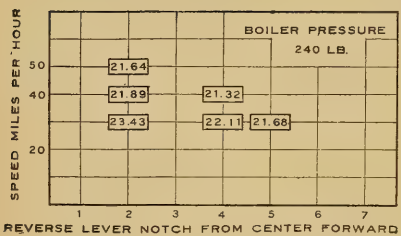


Fig. 25. Steam per Indicated Horse-power Hour; Boiler Pressure 240 Pounds

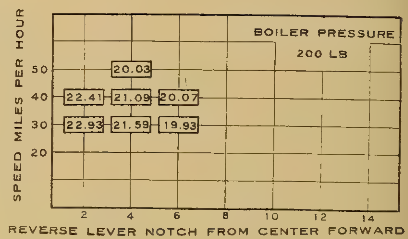


Fig. 26. Steam per Indicated Horse-power Hour; Boiler Pressure 200 Pounds

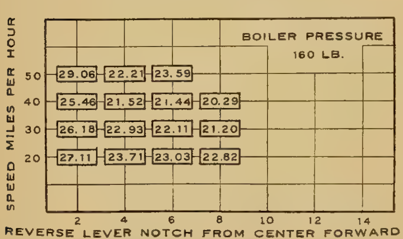


Fig. 27. Steam per Indicated Horse-power Hour; Boiler Pressure 160 Pounds

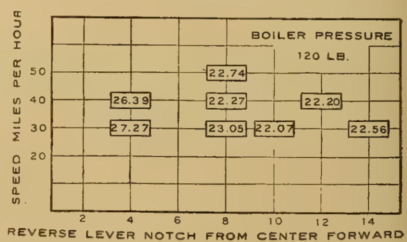


Fig. 28. Steam per Indicated Horse-power Hour; Boiler Pressure 120 Pounds

25. *Steam Consumption per Indicated Horse-Power.*—The steam consumption per indicated horse-power is presented graphically by Fig. 25 to 28. These diagrams show clearly the effect of speed and cut-off on the steam consumption of the cylinders. The most complete exhibit is that of tests run at a pressure of 160 lb. In this series, the full range of speed and cut-off possible under a wide-open throttle has been carried out, and, consequently, the exhibit of results for this pressure discloses a record of the maximum and minimum performance under a wide-open throttle. The maximum steam consumption for any test of record is 29.06 lb. and the minimum is 20.29.

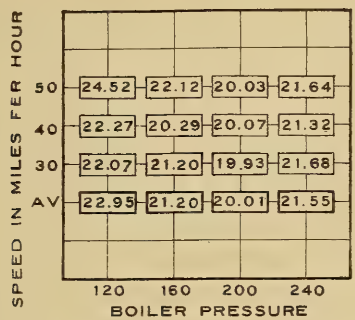


FIG. 29. LEAST STEAM CONSUMPTION FOR EACH OF THE SEVERAL SPEEDS AT DIFFERENT PRESSURES

The least steam consumption per horse-power hour for each of the several pressures at which tests were run is shown by Fig. 29.

For the purpose of securing a statement of the steam consumption of the engine, as set forth by the data presented, values for all pressures have been plotted upon a single sheet. In making up this figure, values for the second-notch tests at 160 lb. have been excluded, since these tests were at a very low power and there are no corresponding tests at other pressures. In a few cases, values for other pressures, not covered by the data, have been derived by extrapolation. The least steam consumption for each of the several pressures as represented by the average values given in Fig. 29, is indicated by points designated by crosses, while the average of all accepted values for each given pressure is represented by circles. In order that the steam consumption of the engine may be defined by a single series of values, a line (AB, Fig. 30) has been drawn through the average points represented by the circle. It is proposed to accept this line as representing the steam consumption of the experimental engine under the several pressures employed. It should be noted that it is not the least consumption or the maximum, but that it is the average of a group of results all of which represent normal working conditions, and none of which represents a consumption more than 4 lb. above the minimum.

From the curve it appears that the minimum normal consumption is obtained under a pressure of 200 lb., and that at this pressure it amounts approximately to 21.6 lb. per indicated horsepower hour.

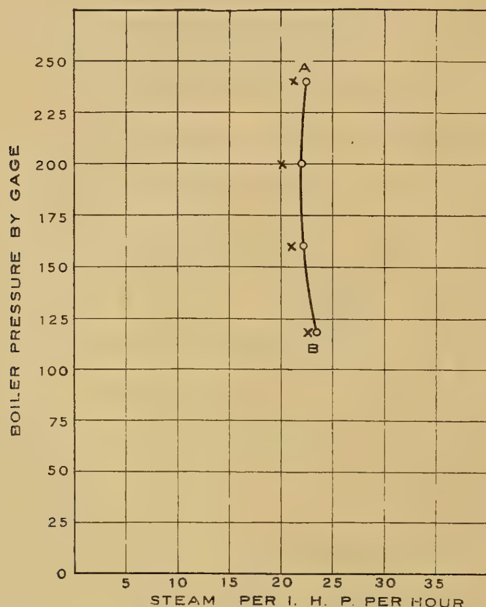


FIG. 30. STEAM CONSUMPTION UNDER DIFFERENT PRESSURES

26. *Steam Shown by Indicator.*—If, for any test, it should appear that there is present in the cylinder more than 100 per cent of mixture, it is to be accepted as evidence that for that test the steam was superheated at release. The data show that such a condition is not closely approached for any of the high-pressure tests, all of which were necessarily run under comparatively short cut-off. As the pressure is reduced, and the cut-off is lengthened, the percentage of steam accounted for increases, and for two tests under a pressure of 120 lb. the actual presence of superheated steam is shown. This applies to tests for which the cut-off was not less than half stroke.

For these tests, also, the percentage of steam accounted for by the indicator was greater than that accounted for by the tank, which is conclusive evidence that the exhaust was superheated.

A comparison of the percentages of the total mixture, which are shown by the indicator at release with similar values taken from the performance of the saturated-steam locomotive Schenectady No. 2, is presented in Table 1.

TABLE 1

PERCENTAGE OF MIXTURE SHOWN AS STEAM AT RELEASE BY INDICATOR-CARDS (SPEED, 30 MILES PER HOUR)

Cut-off, Reverse-lever Notch from Center Forward	Percentage of Mixture Shown As Steam at Release by Indicator-cards							
	Boiler-pressure 240 lb.		Boiler-pressure 200 lb.		Boiler-pressure 160 lb.		Boiler-pressure 120 lb.	
	Super-heate	Sat-urated	Super-heated	Sat-urated	Super-heated	Sat-urated	Super-heated	Sat-urated
I	II	III	IV	V	VI	VII	VIII	IX
2	82.8	75.0	85.2	72.9	85.7	....	....	....
4	82.2	76.7	86.2	77.5	87.4	75.7	89.8	72.0
6	....	....	89.7	75.5	86.0	77.0	....	....
8	....	....	....	....	90.9	80.0	89.7	74.6
12	....	....	....	....	....	....	93.5	....
14	....	....	....	....	....	....	106.0	84.7

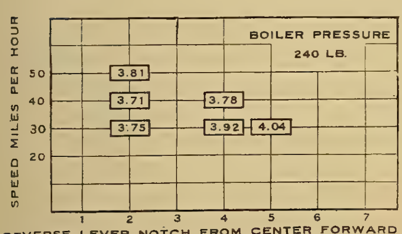


FIG. 31. COAL PER DRAW-BAR HORSE-POWER HOUR; BOILER PRESSURE 240 POUNDS

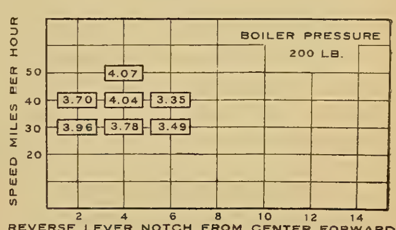


FIG. 32. COAL PER DRAW-BAR HORSE-POWER HOUR; BOILER PRESSURE 200 POUNDS

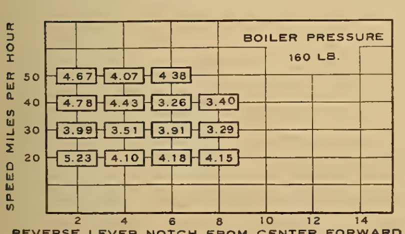


FIG. 33. COAL PER DRAW-BAR HORSE-POWER HOUR, BOILER PRESSURE 160 POUNDS

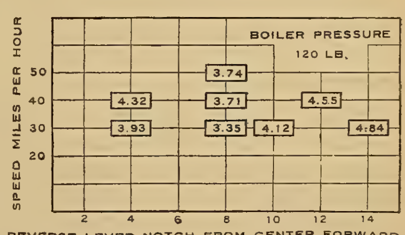


FIG. 34. COAL PER DRAW BAR HORSE-POWER HOUR; BOILER PRESSURE 120 POUNDS

27. *Coal Consumption.*—The coal consumption per indicated horse-power hour is, under favorable conditions, approximately 3 lb., the minimum value of record being 2.8 lb. In 2 tests only, of the 38 tests of record, does it reach a maximum of 4 lb. The coal consumption per draw-bar horse-power hour appears graphically in Fig. 31 to 34. These values are based upon direct observations. They include no accounting for differences in the quality of fuel; these and irregularities arising from other sources are dealt with in paragraph 28.

28. *Comparing the Performance of the Locomotive, Assuming Incidental Irregularities in the Tests To Have Been Eliminated.*—It is apparent that any series of values based directly upon experimental observations will present irregularities. In the course of the preceding discussion, it was sought to eliminate the effect of certain of these irregularities, and to define the performance of the boiler, of the superheater, and of the cylinders of the locomotive experimented upon, in terms which have resulted from a careful summarization of all the data available. Making use of the statements of performance thus secured, it is possible to compile Table 2, which is a table of engine performance based upon the experimental data but freed from its inconsistencies.

Obviously, the exhibit of such a table will have the highest value for purposes of comparison. Thus, the equation defining the performance of the boiler and superheater combined is $E = 11.706 - 0.214\,H$, and that defining the performance of the superheater is $T = 123 - 0.265\ P + 7.28\,H$, and the performance of the engine is defined by the curve AB, Fig. 30.

Column 1.—Test number.

Column 2.—Laboratory symbol.—The first term of this symbol represents the speed in miles per hr., the second term represents the position of the reverse lever upon its quadrant, expressed in notches from the center forward, and the third represents the steam pressure.

Column 3.—Equivalent steam to engine per hr., feed-water at a temperature of 60° F. = steam supplied the engine per hour × (B. t. u. taken up by each pound of water in the boiler and superheater + temperature of feed-water, in degrees F. − 60) ÷ 965.8.

Column 4.—Equivalent evaporation per lb. of dry coal, assuming the evaporative efficiency of the boiler to have been represented by the equation $E = 11.706 - 0.214$ H, where E is the equivalent evap-

oration per lb. of coal and H is the rate of evaporation per ft. of water and superheating surface per hr. For values in question, H = columns 3 ÷ 1216.

Column 5.—Dry coal fired per hr., assuming the evaporative efficiency to be that shown by the equation = column 3 ÷ column 4.

Column 6.—Dry coal per indicated horse-power hour = column 5 ÷ indicated horse-power.

Column 7.—Equivalent steam per indicated horse-power hour = column 3 ÷ indicated horse-power.

Column 8.—Machine friction in terms of mean effective pressure.—The purpose of this column is to eliminate irregularities in action due to variations in lubrication, etc. The values given are those determined by the previous experimental work upon *Schenectady No. 2*[1].

Column 9.—Machine friction horse-power is the power equivalent, assuming the friction M. E. P. to have been that shown by column 8.

Column 10.—Machine friction, per cent of indicated horse-power = 100 × column 9 ÷ indicated horse-power.

Column 11.—Dynamometer horse-power = indicated horse-power − column 9.

Column 12.—Draw-bar pull = 33,000 × column 11 ÷ (18.063 × r. p. m.,)

Column 13.—Coal per dynamometer horse-power hour = Column 5 ÷ column 11.

Column 14.—Equivalent steam per dynamometer horse-power per hr. = column 3 ÷ column 11.

[1] "*High Steam-Pressures in Locomotive Service*", Publication No. 66, Carnegie Institution of Washington. Abstracted in Bulletin No. 26, Engineering Experiment Station.

TABLE 2

COMPARATIVE PERFORMANCE OF THE LOCOMOTIVE ASSUMING IRREGULARITIES IN THE RESULTS OF INDIVIDUAL TESTS TO HAVE BEEN ELIMINATED.

Designation of Tests		Corrected Locomotive Performance											
No.	Laboratory Symbol	Equivalent Steam to Engine per hour Feed Water at 60°F.	Equiv. Evap. per lb. of dry coal by Equation $E = 11.706 - 0.214\ H$	Dry Coal Fired per hour Corrected by Equation	Dry Coal per i. h. p. per hour	Equiv. Steam per i. h. p. per hour	Machine Friction: M. E. P.	Machine Friction: H.P.	Machine Friction: Per cent i. h. p.	Dynamometer horse-power	Draw-bar pull	Coal per D. H. P. per hour	Equivalent Steam per D. H. P. per hour
1	2	3	4	5	6	7	8	9	10	11	12	13	14
		lb.		lb.	lb.	lb.					lb.	lb.	lb.
101	30-2-240	11,308	9.716	1164	3.14	30.59	6.5	46.45	12.58	323.31	4020	3.60	34.97
102	30-4-240	13,853	9.268	1495	3.13	29.07	8.5	58.49	12.27	418.19	5396	3.58	33.13
103	30-5-240	15,187	9.034	1681	3.07	27.79	8.9	61.95	11.33	472.12	6022	3.56	32.17
103*a*	30-5-240	......	.....	......	......	......	......	......	..	......	......	......	......
104	40-2-240	11,786	9.632	1224	2.95	28.40	6.5	61.12	14.72	353.97	3343	3.46	33.30
105	40-4-240	15,467	8.984	1722	3.12	28.06	8.5	79 69	14.45	471.67	4465	3.87	34.77
106	50-2-240	13,129	9.396	1397	3.01	28.31	6.5	77.24	16.65	386 63	2865	3.61	33.96
107	30-2-200	8,312	10.242	812	2.90	29.64	6.5	46 38	16.54	234.06	2910	3.47	35.51
108	30-4-200	10,815	9.802	1103	2.88	28.21	8.5	60.37	15.74	323.07	4037	3.41	33.48
109	30-6-200	13,571	9.318	1457	2.82	26.26	9.3	66.86	12.94	450.11	5559	3.24	30.15
110	40-2-200	9.007	10.121	890	2.87	29.10	6.5	61.42	19.84	248.16	2331	3.59	36.30
111	40-4-200	12,865	9.442	1363	2.92	27.58	8.5	81.29	17.43	385.22	3579	3.54	33.40
111*a*	40-4-200	.. ...	......	......	......	......	.....	......	. ....	......	......	....	.....
112	40-6-200	15,622	8.957	1744	2.96	26.54	9.3	89.82	15.26	498.81	4584	3.50	31.33
113	50-4-200	12,803	9.453	1355	2.78	26.27	8.5	96.99	19.90	390.47	3039	3.47	32.80

114	20-2-160	5,929	10.911	414	2.48	35.55	6.5	30.67	18.39	136.10	2559	3.04	43.56
115	20-4-160	7,070	10.462	676	2.91	30.45	8.5	40.08	17.26	192.07	3611	3.52	36.81
116	20-6-160	8,969	10.128	885	2,95	29.88	9.3	44.02	14.66	256.14	4802	3.45	35.01
117	20-8-160	10,811	9.803	1103	3.01	29.49	8.4	39.83	10.86	326.88	6115	3.37	33.08
118	30-2-160	6,421	10.824	463	2.42	33.53	6.5	46.24	24.14	145.29	1811	3.19	44.91
119	30-4-160	8,443	10.220	826	2.89	29.58	8.5	60.06	21.04	225.33	2830	3.67	37.47
120	30-6-160	11,156	9.743	1145	2.96	28.81	9.3	66.01	17.04	321.35	4019	3.56	34.72
121	30-8-160	13,363	9.354	1429	2.97	27.74	8.4	59.69	12.39	422.13	5272	3.39	31.66
122	40-2-160	6,886	10.760	500	2.36	32.56	6.5	62.72	29.66	148.74	1362	3.36	46.30
123	40-4-160	9,587	10.019	957	2.79	27.93	8.5	80.94	23.58	262.34	2447	3.65	36.54
124	40-6-160	12,011	9.592	1252	2.91	27.95	9.3	88.21	20.52	341.59	3199	3.67	35.17
125	40-8-160	15,944	8.900	1792	2.99	26.63	8.4	79.68	13.31	519.02	4857	3.45	30.72
126	50-2-160	6,688	10.784	486	2.70	37.10	6.5	76.78	42.60	103.46	778	4.70	64.63
127	50-4-160	9,882	9,967	992	2.89	28.81	8.5	100.30	29.24	242.75	1827	4.09	40.71
128	50-6-160	16,344	8.830	1851	3.56	31.43	9.3	110.04	21.16	409.99	3076	4.52	39.87
129	30- 4-120	5,649	10,712	527	3.23	34.60	8.5	60.37	36.97	102.93	1287	5.12	54.89
130	30- 8-120	9,924	9,960	996	2.97	29.60	8.4	60.13	17.93	275.14	3412	3.62	36.07
131	30-10-120	11.766	9,637	1221	2.98	28.72	6.9	48.65	11.87	361.07	4546	3.38	32.59
132	30-14-120	15,296	9,014	1697	3.28	29.56	3.0	21.28	4.11	496.21	6208	3.42	30.83
133	40- 4-120	5,989	10,652	562	3.15	33.54	8.5	80.32	44.97	98.26	922	5.72	60.96
134	40- 8-120	11,074	9,757	1135	2.96	28.93	8.4	79.39	20.74	303.50	2851	3.74	36.49
135	40-12-120	15,925	8,904	1789	3.27	29.09	5.0	47.27	8.63	500.30	4698	3.58	31.83
136	50- 8-120	12,159	9,566	1271	3.09	29.53	8.4	99.51	24.17	312.29	2339	4.07	38.94

It is now possible to determine the coal consumption per indicated horse-power per hour by assuming the efficiency of the locomotive to be that defined in the above relationship. The derived results are given as Table 3, which follows.

TABLE 3

LOCOMOTIVE PERFORMANCE UNDER DIFFERENT PRESSURES.

Boiler-pressure	Pounds of Superheated Steam per i. h. p. per hr.; Values from Curve	B. t. u. per lb. of Steam; Feed 60° and Superheat from Equation	Equivalent Pounds of Steam per i. h. p. hr.	Equivalent Pounds of Water per Pound of Dry Coal	Pounds of Coal per i. h. p. hr.
1	2	3	4	5	6
lb					
240	22.6	1258.7	29.45	9.426	3.12
220	21.8	1261.8	28.48	9.501	3.00
200	21.6	1263.1	28.25	9.518	2.97
180	21.9	1261.7	28.61	9.491	3.01
160	22.3	1259.3	29.07	9.455	3.08
140	22.9	1256.4	29.79	9.399	3.17
120	23.8	1252.7	30.87	9.316	3.31

Column 1 in Table 3 gives the boiler-pressure.

Column 2 gives the steam consumption per indicated horse-power per hour for the several pressures, as defined by the curve *A B*, Fig.

Column 3 gives the number of thermal units in the pounds of steam at the several pressures, assuming the feed-water temperature at 60° F. and the degrees superheat that represented by the equation $T = 123 - 0.265\ P + 7.28\ H$.

Column 4 gives the number of pounds of water from and at 212° F. per indicated horsepower hour. It equals column 2 times column 3 ÷ 965.8.

Column 5 gives the pounds of water evaporated from and at 212° F. per pound of coal and is calculated as follows: Assuming that a fair average load for the locomotive tested is 440 horse-power and that this unit of power is developed under all pressures, the corresponding rate of evaporation may be found by multiplying this value by those of column 4 and dividing by the area of water-heating surface plus superheating surface; that is, rate of evaporation = 440 × column 4 ÷ 1216. The equivalent pounds of water per pound of coal is found by substituting the rate of evaporation found for H in the equation $E = 11.706 - 0.214\ H$.

Column 6 gives the pounds of coal per indicated horse-power hour and equals column 4 ÷ column 5.

From the values given in the table it will be seen that the coal consumption per indicated horse-power hour varies from 2.97 to 3.31. The minimum value 2.97 is found at 200 lb. boiler-pressure.

VI. ECONOMY RESULTING FROM THE USE OF SUPERHEATED STEAM

29. *Comparisons Involving Boiler and Superheater.*—The whole discussion, as presented in the preceding chapters, has been developed with a view to establishing in concise terms the performance of the locomotive experimented upon, while operating under

superheated steam. The method of expressing results and the units of measurement employed have been so chosen that a comparison may readily be made with those which have previously been derived for the same locomotive when, as *Schenectady No. 2*, it was operated with saturated steam. The changes in the extent of heat-transmitting surface resulting from the application of the superheater are described in detail in paragraph 12. Data concerning the performance under saturated steam, which are made a basis for comparison, are drawn from a previous report[1]. Youghiogheny coal or its reduced equivalent has been used in all cases.

30. *Boiler Performance.*—The boiler of *Schenectady No. 2*, designed for delivering saturated steam, gave an efficiency expressed by the equation

$$E = 11.305 - 0.221\, H$$

while the boiler as equipped with a Cole superheater, *Schenectady No. 3*, gave an efficiency expressed by the equation

$$E = 11.706 - 0.214\, H$$

Obviously, on the basis of these equations, the superheating boiler has the advantage. The comparison is, however, not a fair one, since in both cases the equations are based on the extent of heat-transmitting surface, and in *Schenectady No. 3* such surface was sacrificed in making room for the superheater. To make the comparison fair, the term in the equation representing equivalent pounds of water per square foot of heating surface must be expressed in terms of total power delivered by the boiler. Comparisons on this basis, showing the performance of the boiler in one case, and of the boiler and superheater in the other case, expressed in terms of the equivalent evaporation, are shown diagrammatically by Fig. 35.

It will be seen that even upon this basis the efficiency of the combined boiler and superheater is superior to that of the boiler alone, the increase averaging between 3 and 4 per cent. The reason for this is not entirely apparent. An examination of related data suggests that the lines of Fig. 35 should not be far apart. Draft values plotted in terms of the rate of evaporation are lower for the superheating locomotive than for the locomotive using saturated steam, but when these are reduced to equivalent values representing an equal amount of power, they are identical for

[1]Publication No. 66, Carnegie Institution of Washington; Abstract in Bul. 26, Eng. Ex. Sta.

both locomotives—a condition which implies equality in the fuel lost in the form of cinder and spark. Similar comparisons involving smoke-box temperature lead to identical conclusions.

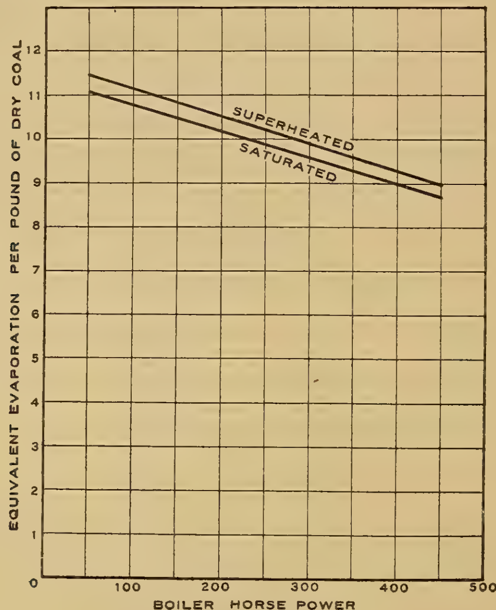


FIG. 35. BOILER EFFICIENCY

Upon the basis of these statements, the relation defined by Fig. 35 is not confirmed by collateral evidence. This statement, however, does not discredit the record, which is in fact one of no small significance. The line of performance for the superheating locomotive (Fig. 35) depends upon results of 38 tests and that for the saturated-steam locomotive upon results of 40 tests. It is therefore difficult to see how either could have been affected to the extent indicated by any incidental cause or causes. Whatever the conclusion may be with reference to this matter, it is clear that the combined boiler and superheater of *Schenectady No. 3* are not less efficient than the boiler of *Schenectady No. 2*, while being worked at the same rates of power, and the face value of the data shows its efficiency to be higher by 4 per cent.

31. *Comparisons Involving the Performance of the Engine.*—The steam consumption per indicated horse-power hour for the superheating locomotive, as determined by the results of 38 tests, has been defined as the line *A B*, Fig. 30. A similar line based upon the results of 100 tests of the saturated-steam locomotive estab-

lishes the cylinder performance of that machine. Replotting the results upon a single sheet gives the diagram Fig. 36. This exhibit (or better, perhaps, the numerical values given by columns 2 and 4, Table 4) shows well the saving in water realized by substituting steam superheated approximately 150° F. for steam which is saturated. The saving ranges from 18 per cent when the boiler-

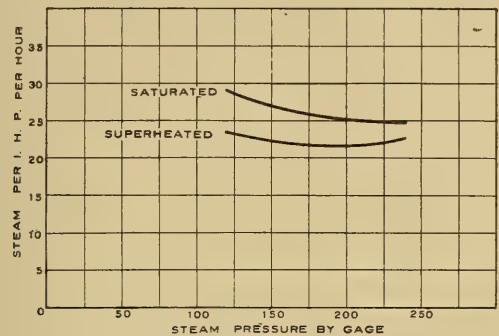


FIG. 36. STEAM PER INDICATED HORSE-POWER HOUR

pressure is 120 lb. to 9 per cent when the boiler pressure is 240 lb. It appears, also, from the diagram that with superheating, the least consumption of water, 21.6 lb. per horse-power hour, is secured when the boiler-pressure is approximately 200 lb., and that variations in the consumption resulting from changes in pressure are slight (column 4, Table 4). For example, the water consumption for all pressures between 160 and 220 lb. ranges between 21.6 lb., the minimum value obtained, and 22.3 lb., a range of approximately 4 per cent.

The saving of water in locomotive service is always a matter of moment; it diminishes the exactions of certain conditions in operation; and in some districts, where water is bad or hard to obtain, it tends to simplify difficult problems either in locomotive maintenance or in the maintenance of the water-supply. The fact, therefore, that superheating affords a material saving in the amount of water required, is not to be overlooked in estimating the value of superheating as a practice. But the saving in heat is not proportional to the saving in water, for each pound of superheated steam must have more heat imparted to it than a pound of

saturated steam at the same pressure. As an indication of the thermal advantage to be derived from the use of superheated steam in comparison with that of saturated steam, it is desirable to reduce the steam in each case to the same thermal basis. This has been shown graphically by Fig. 37 and numerically by columns 3 and 5, Table 4.

TABLE 4

STEAM PER INDICATED HORSE-POWER HOUR

Boiler-pressure lb.	Saturated Steam		Superheated Steam	
	Pounds of Steam per i. h. p. per hr.	B. t. u. per i. h. p. per min.	Pounds of Steam per i. h. p. per hr.	B. t. u. per i. h. p. per min.
1	2	3	4	5
240	24.7	483	22.6	474
220	25.1	491	21.8	459
200	25.5	498	21.6	455
180	26,0	507	21.9	461
160	26.6	517	22.3	468
140	27.7	537	22.9	481
120	29.1	563	23.8	497

Upon this basis the saving effected by the use of superheated steam is 12 per cent when the pressure is 120 lb., and 2 per cent when the pressure is 240 lb. Under a boiler-pressure of 180 lb. the substitution of superheated steam improves the efficiency of the engine 9.1 per cent.

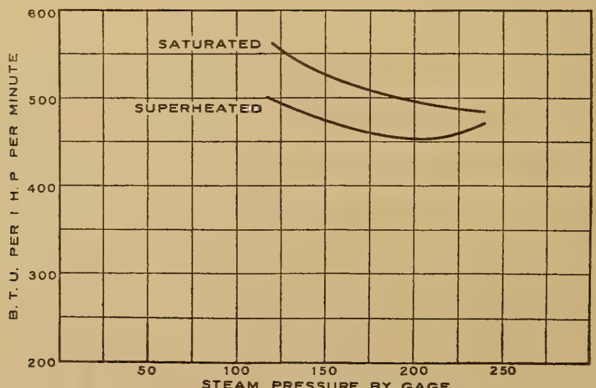


FIG. 37. THERMAL UNITS CONSUMED PER HORSE-POWER PER MINUTE

32. *Comparisons Involving the Performance of the Locomotive As a Whole.*—The performance of the locomotive as a whole, as expressed in terms of coal consumed per indicated horse-power hour, both for saturated steam and superheated steam, and the saving effected by the substitution of superheated for saturated steam, is given as Table 5. These results, since they combine the performance of both engine and boiler, represent a definition of the improvement in the performance of the locomotive experimented upon as the result of the substitution of superheated for

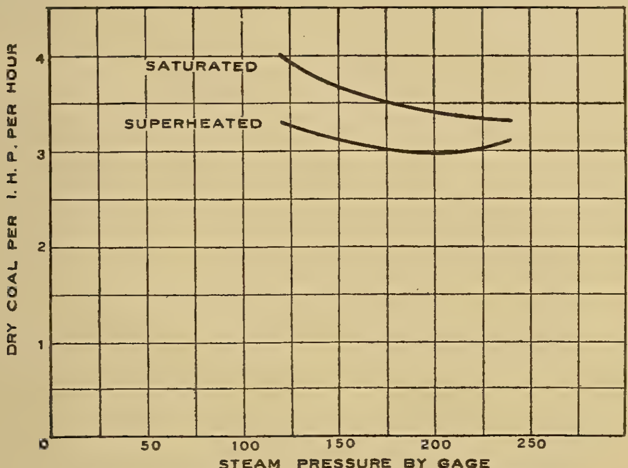


FIG. 38. COAL PER INDICATED HORSE-POWER HOUR

TABLE 5

SAVING IN COAL EFFECTED BY THE USE OF SUPERHEATED STEAM

Boiler-pressure lb.	Pounds of Coal per i. h. p. per hr.		Saving Effected by the Use of Superheated Steam			
			Over Values Obtained with Saturated Steam at Same Pressure		Over Values Obtained with Saturated Steam at 180 lb. Pressure	
	Saturated Steam	Superheated Steam	Pounds per i. h. p. per hr.	Per cent	Pounds per i. h. p. per hr.	Per cent
1	2	3	4	5	6	7
240	3.31	3.12	0.19	5.72	0.38	10.86
220	3.37	3.00	.37	10.98	.50	14.29
200	3.43	2.97	.46	13.31	.53	15.15
180	3.50	3.01	.49	14.00	.49	14.01
160	3.59	3.08	.51	14.21	.42	12.00
140	3.77	3.17	.60	15.98	.37	10.57
120	4.00	3.31	.69	17.25	.19	5.43

saturated steam. They show that the gain is most pronounced at the lower pressure; thus, at a pressure of 120 lb. it is 17 per cent, while at a pressure of 240 lb. it is but 6 per cent. They show also that the performance of the locomotive using superheated steam is only slightly affected by changes of pressure; for the entire range of pressure from 120 lb. to 240 lb., the difference in coal consumption from minimum to maximum is but a third of one pound, while for pressures between 175 lb. and 225 lb., it is practically constant and always near the minimum value. The least coal consumption per indicated horse-power hour, as it appears in the summarized statement, is 2.97 lb., and was obtained under a steam-pressure of 200 lb.

The results sustain a claim which has been put forward by advocates of the practice of superheating, to the effect that the adoption of such practice permitted the steam-pressure to be materially reduced over that now employed in locomotives using saturated steam without material sacrifice in efficiency. A detailed numerical statement showing the saving in coal resulting from a change from saturated to superheated steam is set forth by columns 4 to 7, Table 5. Columns 4 and 5 present results obtained by comparisons based on equal pressures, and columns 6 and 7 those obtained by comparing values obtained with superheating under the several different pressures employed with those obtained from saturated steam at a pressure of 180 lb.

33. *Comparisons Involving the Capacity of the Locomotive.*—The maximum power presented by the data derived from the locomotive using superheated steam is not to be accepted as a measure of its capacity. Except in the case of the series of tests run at 160 lb. pressure, the number of tests was insufficient to permit the establishment at each speed of a maximum cut-off for which the boiler could be made to supply steam. But while direct evidence is lacking, the data contain much which goes to show that the superheating locomotive is a more powerful machine than the locomotive using saturated steam. For example, it has been shown that for the development of equal amounts of power, the combined boiler and superheater of the superheating locomotive have an efficiency which equals or exceeds that of the saturated-steam boiler; hence the boiler-power which it may be made to deliver as a maximum equals or exceeds that which the boiler of the saturated-steam locomotive can be made to deliver. But each unit of power delivered from the boiler in the form of superheated steam is more effective in doing work in the cylinders than a similar unit of

power delivered in the form of saturated steam; hence, at the limit, the superheating locomotive is more powerful than the one using saturated steam, and the difference is that which measures the difference in the economy with which the cylinders use steam.

The same question may be dealt with through another series of facts, as follows: It can be shown that the power of any locomotive is limited by its capacity to burn coal, and coal-burning capacity is a function of the draft. The data show that for the development of a given cylinder power, the draft values of *Schenectady No. 3* (superheating) were in all cases less than those of *Schenectady No. 2* (saturated). The extent of these differences

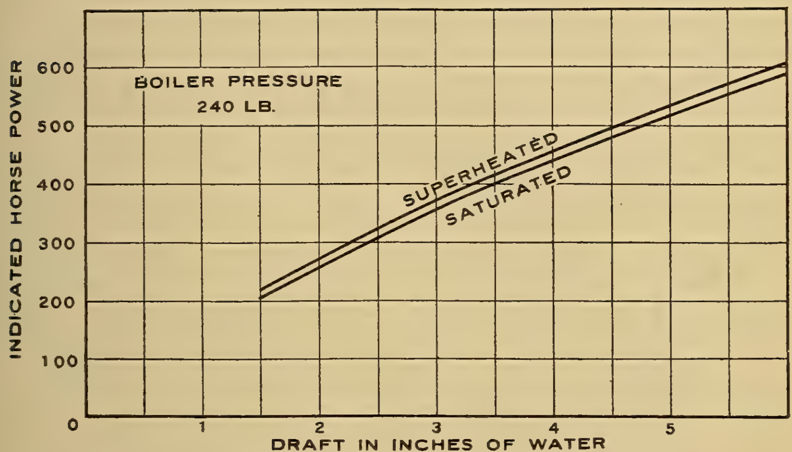


FIG. 39. INDICATED HORSE-POWER: BOILER PRESSURE 240 POUNDS

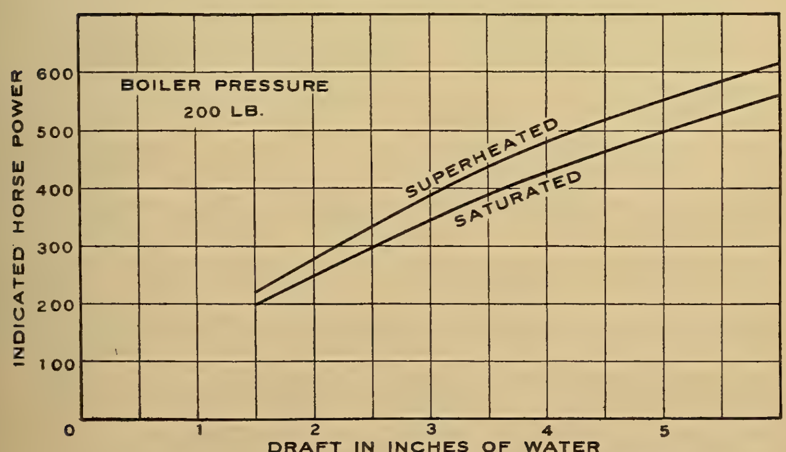


FIG. 40. INDICATED HORSE-POWER: BOILER PRESSURE 200 POUNDS

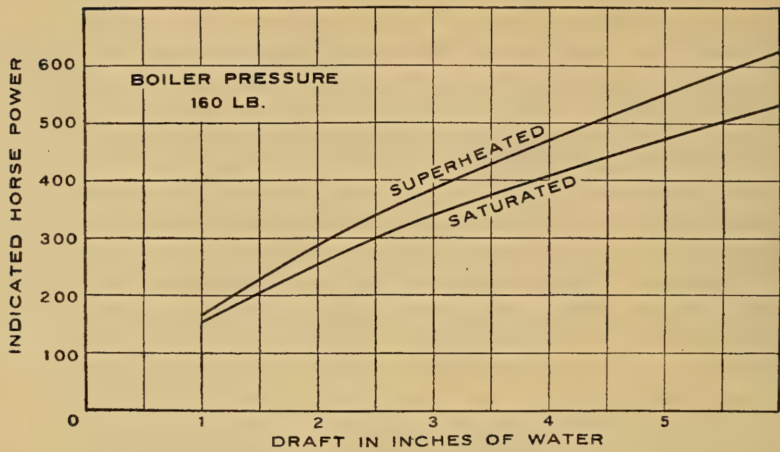


FIG. 41. INDICATED HORSE-POWER; BOILER PRESSURE 160 POUNDS

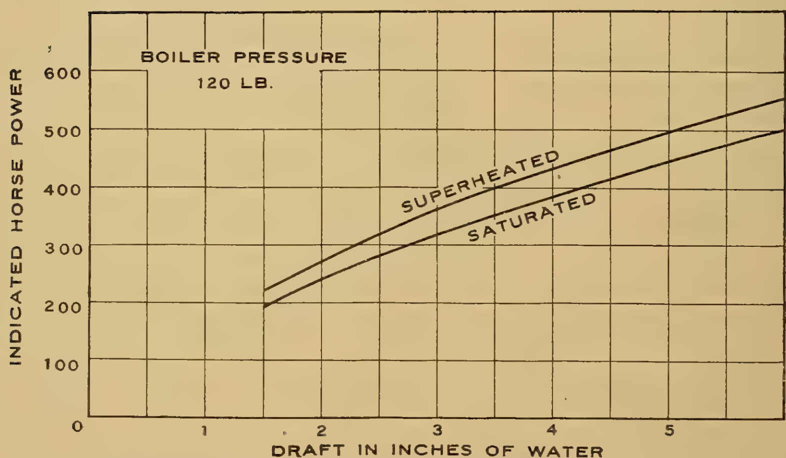


FIG. 42. INDICATED HORSE-POWER; BOILER PRESSURE 120 POUNDS

is well shown by Fig. 39 to 42. They are of small value for tests under high pressure, but as a rule they increase as the pressure is reduced. Tests at 160 lb. (Fig. 41) show that the power developed in return for a given draft is from 10 per cent to 16 per cent greater for the superheating locomotive than for the saturated-steam locomotive. Obviously, there is no reason why the draft for the former should not be increased to limits practicable with the latter, and when this is done the power developed by the superheating locomotive will exceed that which is possible with the saturated-steam locomotive.

34. *The Possible Economy Which May Result from the Use of Superheated Steam in Locomotive Service.*[1]—In the preceding paragraphs an attempt has been made to define with accuracy the increased efficiency resulting from the substitution in locomotive service of steam superheated to approximately 150° for steam which is saturated. The facts upon which comparisons have been based have been derived by careful processes, and the results can safely be accepted as the measure which has been sought. All discussion might well end with the presentation of the facts referred to, were it not that out of them arises a group of questions of great practical significance. To some of these attention must be given.

As a general proposition, the gain which in any service will result from the introduction of a superheater is a function of the degree of superheat employed, and this in turn is limited by the ability of the materials composing the superheater and the exposed parts of the engine to withstand the temperatures which are involved. The Prussian State Railway prescribes a boiler-pressure of 180 lb. and a temperature of steam of 300° C., which temperature may rise above 300°, but must never be allowed to exceed 350°. That is, a degree of superheating of 190° F. is regarded as satisfactory, while the maximum limit never to be exceeded is fixed at 280° F. Under normal running conditions, the degree of superheating is considerably above 200° F.

Comparing the superheating effects described by these statements with the degree of superheat obtained from the Purdue locomotive when working under a pressure of 180 lb. (Fig. 13), it appears that those of the latter may be increased by at least 33 per cent of their present value without exceeding the limit which has been proved practicable in the every-day practice of German railroads. The means to be employed in securing such a degree of superheat are, of course, matters of detail which concern the design and proportions of the superheater. The savings in water and fuel resulting from the presence of the superheater, as set forth by data already presented, would have been greater had the degree of superheat been higher. In the absence of data derived from experiments, it may be assumed that the possible increase in the savings will be proportional to the increase in the degree of superheat.

[1] Information resulting from later experiments, tending to confirm the statements of this section, is presented in the Appendix.

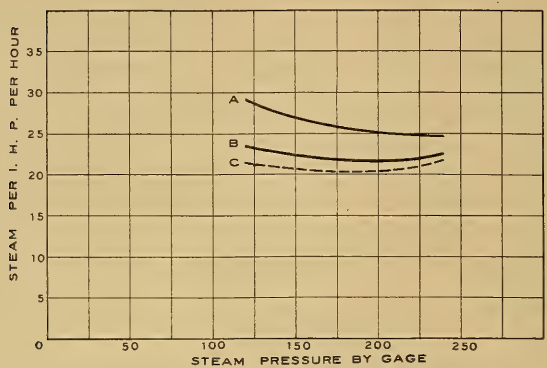


FIG. 43. STEAM PER INDICATED HORSE-POWER HOUR, SHOWING POSSIBLE GAIN BY INCREASING SUPERHEATING 33 PER CENT

On the basis, therefore, of the experimental results already presented and of these statements, the possible gain in water and fuel which may result from the adoption of the superheater is seen by Fig. 43 and 44, respectively. In these figures, the upper line *A* is that of saturated steam as derived from tests of locomotive *Schenectady No. 2;* the next below, *B*, is that of superheated

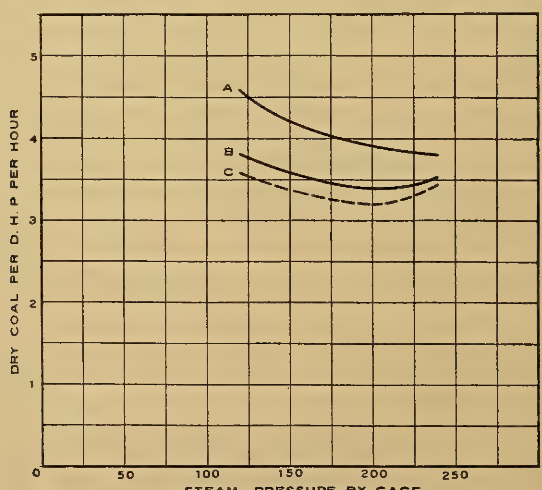


FIG. 44. COAL PER DRAW-BAR HORSE-POWER HOUR, SHOWING POSSIBLE GAIN BY INCREASING SUPERHEATING 33 PER CENT

steam as derived from tests of locomotive *Schenectady No. 3*, and the dotted line C is that which is assumed to represent the performance which *Schenectady No. 3* would have given had the degree of superheating been 33 per cent greater than that actually obtained. These are not maximum savings, but are such as are to be expected under normal conditions of continuous full-power operation. From this exhibit, it appears that for boiler-pressure of 180 lb., the substitution of superheated steam for saturated steam may result in a reduction of water consumption from 26 lb. to 20.5 lb. a saving of 21 per cent, and in a reduction of coal consumption per draw-bar horse-power of from 4 lb. to 3.25 lb., a saving of 19 per cent. These values may be accepted as representing what should reasonably be expected of superheating in American locomotive service, so far as the experiments herein described define them.

It will be a mistake, however, for anyone to assume that a railway company's bills for locomotive fuel may be diminished by the percentages set forth in the preceding paragraph merely by the introduction of the superheater. It should be clear, for example, that no part of the fuel used in raising the steam of a locomotive or of its wastes which occur between the round-house and the starting of the locomotive at the head of its train can be saved by the application of a superheater to a locomotive. Assuming that the fuel thus used is 15 per cent of the total for the run, a conservative estimate, the amount which remains subject to the influence of the superheat is 85 per cent of 19 per cent, or 16 per cent.

Again, the fuel used in maintaining a normal temperature of all parts of the machine when the locomotive is at rest at stations and at passing-points is fuel over which the superheater can exert no influence. The amount of fuel thus used is a function of the schedule of the train. The results set forth in Chap. IV show that in some classes of service upon American railways it will be so small as to be negligible, but in other classes of service it will constitute a considerable percentage of the total coal used. A review of Chap. IV will suggest the difficulty which confronts one in an attempt to fix numerical values covering fuel thus to be accounted for. Again, fuel used in generating steam which is discharged through safety-valves can not in any way be affected by the presence of a superheater. In none of the experimental work, the results of which are recorded in the preceding chapters, has there been any loss by safety-valves. This loss in practice is

necessarily indefinite. In some classes of service it is so small as to be negligible, and in others it involves a considerable percentage of the total coal used. Finally, attention should be called to the fact that the question at issue involves the whole problem of maintenance. Steam leaking past valves and pistons or coming out by leaky glands or through leaky cylinder-cocks or steam-joints wherever located causes losses which remain undiminished in the presence of the superheater.

Summarizing the preceding statements, and making such deductions from the known performance of the superheater as will suffice to remove from the calculations all expenditures of heat normal to the American locomotive, which are beyond the influence of the superheater, the actual net reduction in the amount of fuel needed for locomotive use, by a railroad having all its locomotives equipped with satisfactory superheaters, over that which would be required if all employed saturated steam, will not be far from 10 per cent. This value is not to be accepted as of strictly scientific import, but merely as an estimate based upon such facts as have appeared in the course of a rather careful study of the problem.

APPENDIX

APPENDIX

A Comparison of Results Obtained With Saturated Steam and With Four Different Degrees of Superheated Steam[1]

35. The tests reported in the preceding pages were completed in the summer of 1907. Since that date, the Purdue Laboratory has been engaged in investigations, involving the use of different superheaters, the results of which are to be accepted as the latest information available concerning the value of superheated steam in locomotive service. A brief abstract of a paper presented jointly by Dean Benjamin and Professor Endsley follows.

36. *Superheaters.*—Tests have been run with locomotive *Schenectady No. 3* equipped with four different superheaters, which have been designated as follows:

Cole A—193 sq. ft. of heating surface in the superheater
Cole B—151 sq. ft. of heating surface in the superheater
Cole C—109 sq. ft. of heating surface in the superheater
Schmidt—324 sq. ft of heating surface in the superheater

Cole A was the superheater used during the tests reported in the preceding pages.

The boiler dimensions were the same for all the Cole superheater tests, but in order to install a Schmidt superheater, with a larger amount of superheating surface, the number of small 2-in. flues was reduced from 111 to 107, and the large 5-in-flues were increased in number from 16 to 21. This change in the number of flues increased the water-heating surface from 897 sq. ft. to 956.5 sq. ft. With the above exceptions, the boiler and engine were the same for all the testing upon the four different superheaters.

37. *Basis of Comparison.*— It seems logical to compare the results obtained with the four different degrees of superheated steam and with saturated steam, since all the series of the tests so far run have been under the same steam pressures and cut-offs, developing approximately the same horse-power, the only difference being the area of the superheating surface and the area of the water-heating surface. As the area of the water-heating surface

[1] An abstract of a paper presented before the American Master Mechanics' Association at Atlantic City in June 1911 by Dean C. H. Benjamin and Professor L. E. Endsley, of Purdue University.

of the boiler with the Schmidt superheater is approximately only 47 sq. ft. greater than with the Cole superheater, it would seem that this difference would not be enough to affect the relative efficiency of the boiler. In the comparisons which follow, therefore, no allowance is made for differences resulting from different water-heating surfaces.

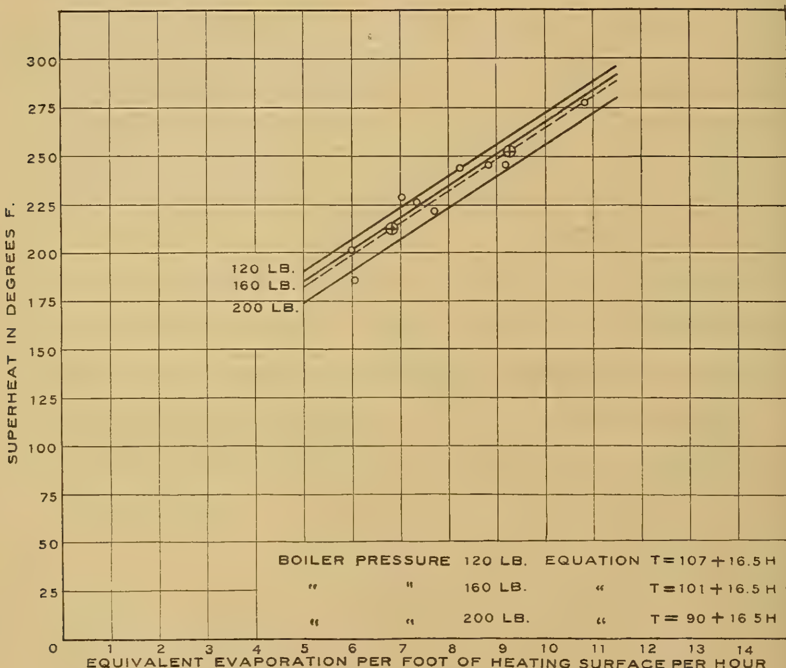


FIG. 45

38. *Superheating.*—The degree of superheating as affected by the rate of evaporation, at the different boiler pressures, is shown for the Schmidt superheater by Fig. 45. The same thing, for the Cole A superheater, has been shown in Fig. 9 to 12. The degrees of superheating plotted against the corresponding boiler pressures, at a rate of evaporation of 8.5 lb. per sq. ft. of heating surface per hour, is shown, for all four superheaters in Fig 46.

39. *Comparison of Engine Performance.*—The steam consumption of the locomotive operated under saturated steam and the four different degrees of superheated steam represented by Cole A, Cole B, Cole C and Schmidt are shown graphically in Fig. 47.

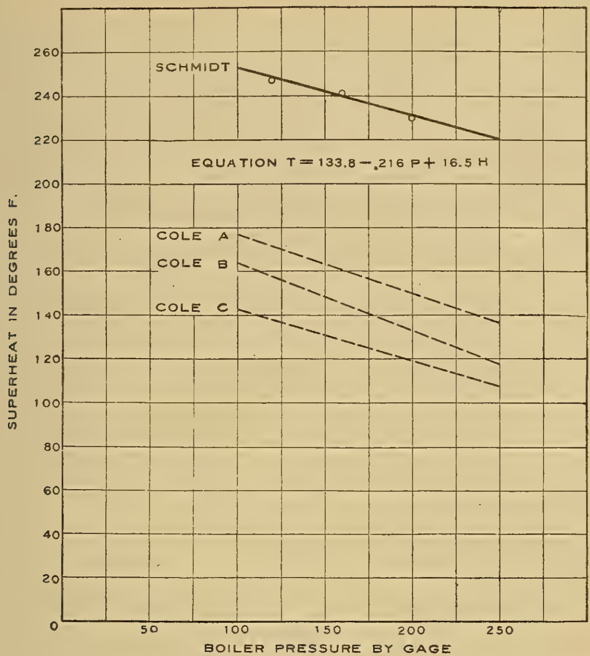

SUPERHEAT IN DEGREES F.
260
240
220
200
180
160
140
120
100
80
60
40
20
0
SCHMIDT
EQUATION T = 133.8 − .216 P + 16.5 H
COLE A
COLE B
COLE C
50
100
150
200
250
BOILER PRESSURE BY GAGE

FIG. 46

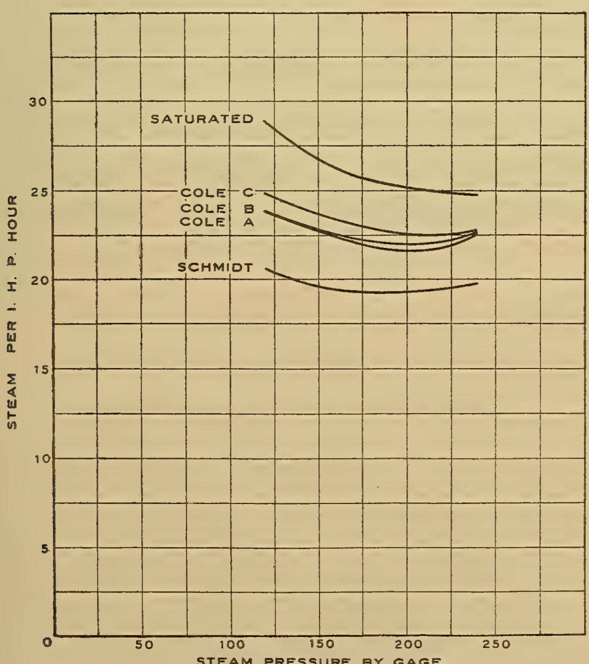

STEAM PER I. H. P. HOUR
30
25
20
15
10
5
0
SATURATED
COLE C
COLE B
COLE A
SCHMIDT
50
100
150
200
250
STEAM PRESSURE BY GAGE

FIG. 47

The numerical values are given in Table 6. From an inspection of these curves, it is seen that the tests with the Schmidt superheater, i. e., the one giving the highest degree of superheat, gave the lowest water consumption.

The curves showing the relation between the B. t. u. per i. h. p. per minute for the different conditions of tests are given in Fig. 48.

TABLE 6

STEAM PER INDICATED HORSE-POWER PER HOUR

Superheater	Boiler Pressure lb. by gauge	Superheat ° F.	Pounds Steam per i. h. p. per hr.	B. t. u. per i. h. p. per min.
I	II	III	IV	V
Schmidt A	240	222.2	19.5	421.4
Schmidt A	220	226.5	19.0	410.7
Schmidt A	200	230.8	18.9	408.3
Schmidt A	180	235.1	18.7	404.0
Schmidt A	160	239.4	18.9	408.0
Schmidt A	140	243.8	19.5	419.8
Schmidt A	120	248.6	21.0	452.3
Cole A	240	139.7	22.6	474
Cole A	220	145.0	21.8	459
Cole A	200	150.3	21.6	455
Cole A	180	155.6	21.9	461
Cole A	160	160.8	22.3	468
Cole A	140	166.1	22.9	481
Cole A	120	171.4	23.8	497
Cole B	240	120.6	22.6	469
Cole B	220	126.8	22.1	460
Cole B	200	133.0	21.8	454
Cole B	180	139.2	22.1	460
Cole B	160	145.4	22.5	469
Cole B	140	151.5	23.0	479
Cole B	120	157.7	23.8	496
Cole C	240	109.9	22.7	469
Cole C	220	114.6	22.5	465
Cole C	200	119.4	22.6	467
Cole C	180	124.2	22.8	472
Cole C	160	128.9	23.5	486
Cole C	140	133.7	24.0	496
Cole C	120	138.4	24.8	512
None	240	0	24.7	483
None	220	0	25.1	491
None	200	0	25.5	498
None	180	0	26.0	507
None	160	0	26.6	517
None	140	0	27.7	537
None	120	0	29.1	563

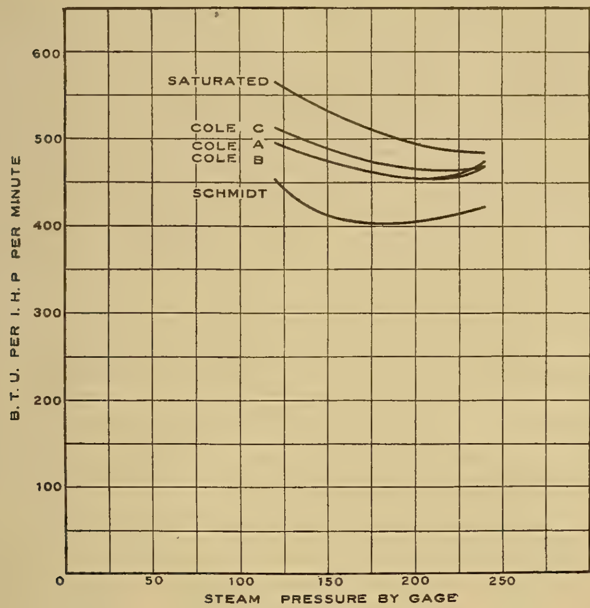

600
500
400
300
200
100
0
50
100
150
200
250
B. T. U. PER I. H. P PER MINUTE
STEAM PRESSURE BY GAGE
SATURATED
COLE C
COLE A
COLE B
SCHMIDT

FIG. 48

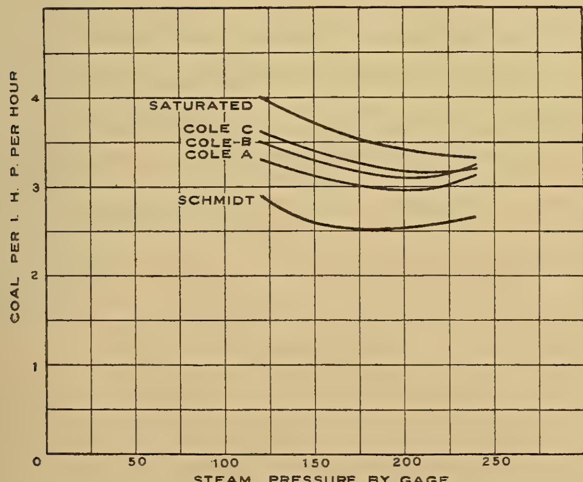

4
3
2
1
0
50
100
150
200
250
COAL PER I. H. P. PER HOUR
STEAM PRESSURE BY GAGE
SATURATED
COLE C
COLE-B
COLE A
SCHMIDT

FIG. 49

The relation in coal consumption per i. h. p. per hour for the four different superheaters and for the saturated steam is shown graphically in Fig. 49, the numerical values being in Table 7. Here again the Schmidt superheater results are the smallest, going as low as 2.5 lb. per indicated horse-power per hour.

TABLE 7.

COAL CONSUMPTION UNDER DIFFERENT PRESSURES AND SUPERHEATERS

Boiler Pressure Pounds Gauge	Pounds of Coal per Indicated Horse Power per Hour				
	Saturated Steam	Superheater Cole A	Superheater Cole B	Superheater Cole C	Superheater Schmidt A
I	II	III	IV	V	VI
240	3.31	3.12	3.24	3.20	2.63
220	3.37	3.00	3.16	3.16	2.57
200	3.43	2.97	3.11	3.18	2.55
180	3.50	3.01	3.16	3.22	2.51
160	3.59	3.08	3.24	3.35	2.55
140	3.77	3.17	3.33	3.45	2.63
120	4.00	3.31	3.48	3.60	2.89

The consumption of water per indicated horse-power, as affected by the degree of superheat, is well shown in Fig. 50, in which the pounds of steam per indicated horse-power per hour are plotted against the degrees of superheat. The pounds of steam per indicated horse-power per hour were obtained from the curves shown in Fig. 47. The degree of superheat was obtained from the lines shown in Fig. 46. It will be seen that the comparisons are made at 160, 180 and 200 lb. steam pressure, these being the pressures that fall in the center of the field of experiment, and for that reason would be more likely to represent correct results.

It would seem that this relation could be approximately represented by a straight line as shown. It is also seen that the water consumption for all pressures between 160 and 200 lb. for the Schmidt superheater is practically the same.

40. *Coal Consumption.*—The pounds of coal per indicated horse-power per hour plotted against degrees of superheat are shown in Fig. 51. The pounds of coal per indicated horse-power per hour were obtained from the curves of Fig. 49, and the degree of superheat was obtained in the same manner as for Fig. 50.

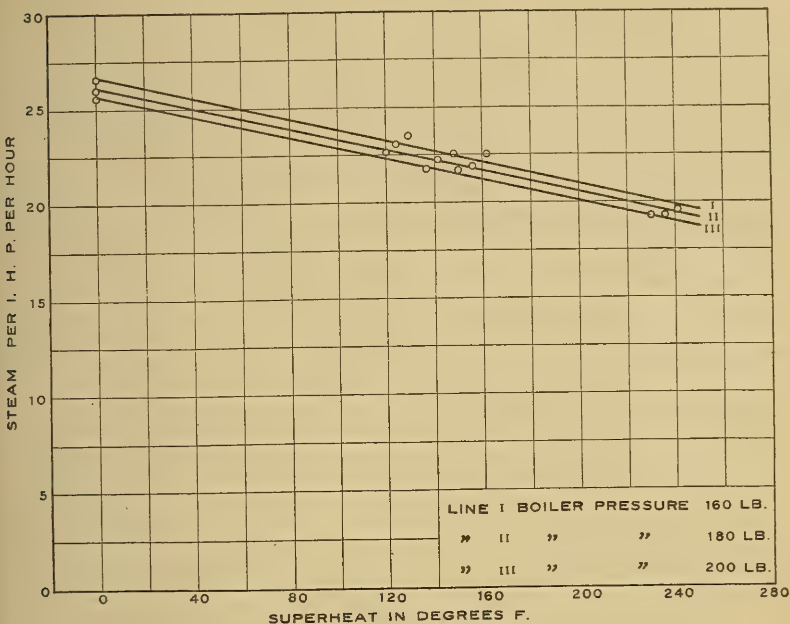

STEAM PER I. H. P. PER HOUR
30
25
20
15
10
5
0
0
40
80
120
160
200
240
280
SUPERHEAT IN DEGREES F.
I
II
III
LINE I BOILER PRESSURE 160 LB.
" II " " 180 LB.
" III " " 200 LB.

FIG. 50

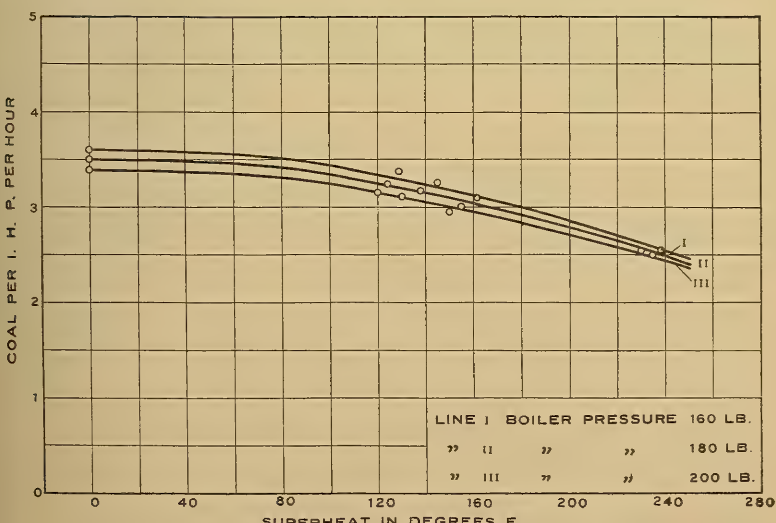

COAL PER I. H. P. PER HOUR
5
4
3
2
1
0
0
40
80
120
160
200
240
280
SUPERHEAT IN DEGREES F.
I
II
III
LINE I BOILER PRESSURE 160 LB.
" II " " 180 LB.
" III " " 200 LB.

FIG. 51

The same pressures of 160, 180 and 200 were used in this comparison as in the comparison for steam consumption. This relation between the coal per indicated horse-power per hour and the degree of superheat for pressures of 160, 180 and 200 would seem to indicate that it could be represented by a curve as shown. In other words, the first 80 or 100° of superheat do not make the same proportionate decrease in coal consumption as the second 80 or 100°, and, in like manner, the third 80° increase makes a still greater reduction in the coal consumption. For instance, the coal consumption per indicated horse-power per hour at 180 lb. steam pressure for the locomotive using saturated steam was 3.50 lb., and for 80° of superheat it was 3.4 lb. a gain in efficiency of 2.8 per cent; while the consumption at 160° superheat is 3.05 lb., a gain of 12.8 per cent, and the coal consumption at 240 degrees superheat is only 2.47 lb., a saving of 29.4 per cent over that of the locomotive using saturated steam. Thus, if we take the locomotive using saturated steam as consuming 100 per cent of coal, it might be said that the first 80° superheat will reduce this 2.8 per cent, the second 80°, 10.0 per cent, and the third 80°, 16.6 per cent, making the total reduction for 240° superheat, at 180 lb. pressure, 29.4 per cent. Practically the same results would be obtained for the curves representing 160 and 200 lb. steam pressure.

41. *Conclusions.—a.* A locomotive equipped with a superheater giving from 200 to 240° of superheat will during the time of running, effect a saving in coal consumption of from twenty to thirty per cent over that of the same locomotive using saturated steam.

b. It would seem that the total gain in efficiency which can be obtained from superheat in a locomotive would not be reached until the temperature becomes too high for practical purposes.

PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

**Bulletin No. 1.* Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

**Circular No. 1.* High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

**Bulletin No 2.* Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks 1905 *None available.*

**Circular No. 2.* Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

**Circular No. 3.* Fuel Tests with Illinois Coal. (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907,) by L. P Breckenridge and Paul Diserens. 1909. *Thirty cents.*

**Bulletin No 3.* The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

**Bulletin No. 4.* Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

**Bulletin No. 5.* Resistance of Tubes to Collapse, by Albert P. Carman. 1906. *Fifteen cents.*

**Bulletin No. 6.* Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *Thirty-five cents.*

**Bulletin No 7.* Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr and Henry B. Dirks. 1906. *Thirty-five cents.*

**Bulletin No. 8.* Tests of Concrete: I. Shear; II. Bond, by Arthur N. Talbot. 1906. *None available.*

**Bulletin No. 9.* An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Reprint available July 1, 1912. *Fifty cents.*

**Bulletin No. 10.* Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N Talbot. 1907. *None available.*

**Bulletin No. 11.* The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

**Bulletin No. 12* Tests of Reinforced Concrete T-beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

**Bulletin No. 13* An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *Fifty cents.*

**Bulletin No. 14.* Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907 *None available.*

**Bulletin No. 15.* How to Burn Illinois Coal without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

**Bulletin No. 16.* A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

**Bulletin No 17* The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908 *Twenty cents.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

**Bulletin No. 19.* Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *Twenty-five cents.*

**Bulletin No. 20* Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908 *None available.*

**Bulletin No 21* Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

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Bulletin No 24. The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*

Bulletin No 25. Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Twenty cents.*

Bulletin No. 26. High Steam-Pressures in Locomotive Service. A Review of a Report to the Carnegie Institution of Washington, by W. F. M. Goss 1908. *Twenty-five cents.*

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BULLETIN NO. 58

A NEW ANALYSIS OF THE CYLINDER PERFORMANCE OF RECIPROCATING ENGINES

BY

J. PAUL CLAYTON

UNIVERSITY OF ILLINOIS

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 58 MAY 1912

A NEW ANALYSIS OF THE CYLINDER PERFORMANCE OF RECIPROCATING ENGINES

BY J. PAUL CLAYTON, ASSISTANT, MECHANICAL ENGINEERING DEPARTMENT, ENGINEERING EXPERIMENT STATION

CONTENTS

A NEW ANALYSIS OF THE CYLINDER PERFORMANCE OF RECIPROCATING ENGINES

Introduction

Preliminary.—The cylinder performance of a reciprocating engine using an elastic fluid for the working medium may be considered from two points of view: (1) the performance as a heat engine, or the efficiency of transformation of the available heat into indicated work; and (2) as a mechanism, or the measure of perfection attained in the distribution of the working fluid, and in preventing its leakage past the valves, piston, or piston rods.

Our knowledge of cylinder performance is obtained almost entirely from indicator diagrams. These diagrams provide a measure of the work performed in one cycle of operation, thus giving a means for determining what proportion of the heat available has been transformed into work, this proportion being expressed as a thermal, potential or other efficiency.

The results of methods used at present in the analysis of cylinder performance may be divided into two classes: (1) those which are relatively exact and satisfactory; and (2) those which are relatively inexact and unsatisfactory. Under these two headings the following results may be enumerated:

Class 1.

1. Indicated work.
2. Aid in valve setting.
3. Rough location of cyclic events.
4. Hirn's analysis for steam cylinders.
5. Measure of initial condensation in steam cylinders.
6. Detection of leakage, if very large, during expansion or compression of any elastic medium.
7. Measure of the diagram factor for the purposes of design.

Class 2.

1. Accurate approximation of clearance volume from all cylinders using elastic media.
2. Close location of cyclic events.
3. Reliable detection and approximation of moderate leakage with the engine in regular operation.
4. Measure of the actual steam consumption from steam diagrams.

5. Separation of the initial condensation and leakage in steam cylinders.
6. Division of feed between the two ends of a steam cylinder for the application of Hirn's analysis.

It has generally been thought by engineers that a satisfactory solution of these last-mentioned problems is impossible because the indicator diagram has not been supposed to contain in itself the evidence necessary for their solution.

The investigation described in this bulletin is the result of an extensive analytical and experimental study of the forms of the expansion and compression curves which occur in indicator diagrams. The analytical study was carried on by means of transferring the indicator diagram to logarithmic cross-section paper and then drawing a figure which will be called the logarithmic diagram.

It is well-known that the equation of the polytropic curve $PV^n = C$ becomes a straight line when plotted on logarithmic cross-section paper. Conversely, when the expansion or compression curve of an indicator diagram becomes a straight line in the logarithmic diagram, the curve is of the form $PV^n = C$, the value of n being the slope of the line.

By means of the logarithmic diagram, it has been found that, free from certain abnormal influences, expansion or compression of an elastic medium takes place in the cylinders of reciprocating engines substantially according to the law $PV^n = C$.

From the fact that the law $PV^n = C$ holds for expansion and compression curves from practice, there have been developed rational methods of approximating the clearance volume, of closely locating the cyclic events, and of detecting moderate leakage when the engine is in regular operation. These methods apply, however, only to those indicator diagrams which are taken from the cylinders of reciprocating engines using any elastic fluid for the working medium and having, as a part of the cycle, an expansion or compression of the medium.

It has been discovered that the value of n for the expansion curves of steam diagrams bears a definite relation in any given cylinder to the proportion of the total weight of steam mixture which was present as steam at cut-off. This proportion or quality will be called x_c in this investigation, and its value will be expressed in parts of unity. The relation of the value of n to the value of x_c for the same class of cylinder as regards jacketing has

been found to be practically independent of engine speed and of cylinder size.

The practical significance of finding this relation is that there is now available an accurate method of approximating the value of x_c and therefore the actual weight of steam and water present at cut-off from the indicator diagram alone. As a result, a new analysis of cylinder performance has been developed.

Acknowledgments.—Acknowledgment of valuable assistance is made to Dean W. F. M. Goss, Professor G. A. Goodenough Professor O. A. Leutwiler, Mr. C. M. Garland, to Purdue University, and to various firms which furnished indicator diagrams and tests for analysis.

Part I. The Approximation of the Actual Steam Consumption From Indicator Diagrams

I. Preliminary Work

1. *Study of Indicator Diagrams.*—The preliminary work consisted of an examination of the nature and form of the expansion and compression curves of the indicator diagrams from a number of steam engines. This work was accomplished by means of transferring the indicator or PV-diagram to logarithmic cross-section paper, and thus drawing a figure which will be called the logarithmic diagram. It was found, after repeated experiments, (see p. 55) that free from certain abnormal influences, expansion or compression of steam takes place in cylinders substantially according to the law, $PV^n = C$. The values of n which were obtained, however, exhibited a large range of variation, the range being from 0.70 to 1.34. The engines from which the values were obtained differed in type, size, speed, steam pressure, ratio of expansion and back pressure. Obviously, comparisons could not be made of these examples because of the number and magnitude of the variables.

Indicator diagrams taken from the same cylinder, with different cut-off positions, showed that the value of n was higher as the cut-off was lengthened. There was a large variation in the value of n where the conditions of cylinder size, speed, steam pressure, and steam distribution were the same. The only variable shown by the diagrams was the length of the cut-off. It was thought that there might be some relation, in any one cylinder, between the value of n for the expansion curve and the quality of the steam mixture at cut-off, as this quality was known to be higher as the length of cut-off increased. One fact that seemed to

confirm this hypothesis was that with superheated steam (under the same general conditions, except the kind of steam used), when the quality at cut-off was high, the value of n for expansion was always much higher than with saturated steam. The only important variable between the use of saturated and superheated steam to account for the change in the value of n was the quality of the steam mixture at cut-off, or the proportion of the total weight of mixture in the cylinder which was present as steam at cut-off. This quality or proportion will be called x_c in this investigation, and its value will be expressed as decimal parts of 1.

All cases examined of engines using superheated steam at normal cut-off showed n to be higher than 1.0, and as high as 1.34; and all cases of small engines using saturated steam showed n to be lower than 1.0 and as low as 0.70. These facts led to the conclusion that the value of x_c was the most important single factor in the accompanying value of n. Tests were, therefore, planned in which the effort was made to vary the value of x_c between the widest practicable limits.

II. Laboratory Tests

2. *Equipment.*—A single-cylinder long-range cut-off, 12 in. x 24 in.-Corliss engine, located in the Mechanical Engineering Laboratory of the University of Illinois, was selected for the tests. A Corliss engine was selected because of the fact that in this type all the steam used passed through the cylinder.

3. *Plan of Tests.*—It was planned to observe the effect upon the value of n of varying the value of x_c under different conditions of pressure and speed. The value of x_c was varied through a large range by the use of saturated and superheated steam, in conjunction with different lengths of cut-off, under the same conditions of pressure and speed. The values of x_c obtained ranged from 0.413 to 0.943, covering the range usually found in practice with the type of engine used.

The values of n for the expansion curves were obtained by means of the logarithmic diagram (see Appendix 1). The value of x_c given in the log is the average of the results obtained from one set of head-end and crank-end diagrams for each test. The unit of measurement, therefore, was the revolution, as the values of x_c for the head and crank-ends cannot be measured separately when one exhaust pipe is used for both ends of the cylinder. The value of n given for one test is the average of the separate values from the expansion curves of the head-end and crank-end diagrams, taken

from the set of diagrams already mentioned.

4. *Number of Tests.*—Seventy five tests in 16 series were run. Of this number, 60 tests in 14 series were selected as fulfilling the requirements decided upon to give reliable data. Each series consisted of 4 to 5 separate tests, differing from each other only in the length of cut-off, with the same conditions of pressure and speed. All tests were run with the steam exhausting from the cylinder at about atmospheric pressure into a surface condenser. The length of cut-off was varied in nearly uniform steps from about 5 per cent to 45 per cent of the length of the stroke, and was the means of varying the value of x_c when using either saturated or superheated steam.

The 14 series were divided into 2 divisions of 7 series each; one division being run with saturated steam, and the other with steam superheated to 500° F. at the superheater. Each division consisted of 5 series run at different gauge pressures at constant speed, and 2 series run at different speeds with constant pressure. The steam pressures used were 57.5, 76.5, 95, 113, and 132 lb. gauge with the engine running at 120 r. p. m. The other speeds employed were 90 r. p. m. and 150 r. p. m. at the gauge pressure of 113 lb. Each division, therefore, gave the effect of the use of 5 steam pressures at constant speed, and 3 speeds at constant pressure.

The governor change-speed device was always set to give the desired speed with the engine running at no-load. As the load was increased, the speed decreased through the action of the governor in about the same proportion for all initial speeds. Whenever speed is mentioned, the no-load speed is the one referred to, the exact speed for any one test being given in the general log.

The general log of the 29 tests run with saturated steam is given in Table 1. Table 2 contains the results of the 31 tests run with superheated steam. Table 3 gives the averages of similar series, called groups, run at the same pressure and speed, with both saturated and superheated steam. For any one group of the two series of tests, as has already been pointed out, the only variables are the length of cut-off, and the value of x_c.

1. Values of n obtained under different conditions from the same engine cylinder.—All the simultaneous values of x_c and n obtained from the 60 tests were plotted in Fig. 1. A study of this figure shows beyond question that as x_c increases in value, n increases also. The values all lie in a region which has a definite trend towards higher simultaneous values of x_c and n. Observing the

TABLE 1. GENERAL LOG—TESTS WITH SATURATED STEAM

Date of Test	Series	No. of Test	Laboratory Designation	Pressure lb. per sq. in. At Throttle by Gauge	Barometer	Absolute Pressure at Throttle	Average Absolute Pressure of Series at Throttle	Revolutions per Hour	Weight of Condensate per Hour, lb.	Weight of Condensate per Revolution, lb.	Lb. of Steam per Revolution Shown by Indicator at Cut-off	Lb. of Compression Saved per Revolution by Indicator	Lb. of Steam Mixture Actually Present per Revolution	Quality of Steam Mixture at Cut-off. Parts of Unity	Absolute Pressure at Cut-off, lb.	Average Value of n from Expansion Curves	Cut-off, per cent of Stroke	Average Absolute Pressure at Cut-off for Series, lb.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
4-11-10	I	2	S 60-12-120	57.8	14.3	72.1		6428	1054	0.1640	0.0929	0.0248	0.1888	0.492	60.3	0.926	14.6	
4-16-10		3	16	57.3	14.2	71.5	71.3	6100	1104.5	0.1811	0.1037	0.0221	0.2032	0.510	61 3	0.882	16.5	60.6
5-12-10		4	32	55.8	14.5	70.3		6302	2010	0.3189	0.2099	0.0276	0.3465	0.606	60.2	0 961	41.2	
4-13-10	II	6	S 80-13-120	77.0	14.3	91.3		6846	1010	0.1475	0.0846	0.0235	0.1710	0.495	76.8	0.927	8.1	
4-16-10		7	25	76.7	14.2	90.9	90.5	6440	1384	0.2149	0.1281	0.0230	0.2379	0.539	80 2	0.895	15.7	77.9
5 12-10		8	38	75.6	14.5	90.1		6640	2104	0 3168	0.2089	0.0253	0 3421	0.610	77.4	0.949	30.9	
5-12-10		9	45	75.0	14.5	89.5		6282	2559	0.4073	0.2727	0.0260	0.4333	0.629	77.4	0.950	42 6	
4-8-10	IV	15	S 100-15-120	95.8	14.3	110.1		6940	1048	0.1510	0.0903	0.0236	0.1746	0.517	95.6	0.910	6.1	
4-16-10		16	34	96.0	14.2	110.2		6510	1530	0.2350	0.1462	0.0226	0.2756	0.531	95.6	0.887	14.6	
4-16-10		17	33	94.9	14.2	109.1	109.1	6432	1584	0.[illegible]462	0 1533	0.0233	0.2695	0.569	96.2	0.914	15.5	95.3
5-11-10		18	47	94.5	14 4	108.9		6750	2298	0.3404	0.2145	0.0246	0.3650	0.585	92.5	0.996	25.4	
5-11-10		19	57	93.6	14.4	108.0		6410	2758	0 4302	0.2952	0.0263	0.4565	0.647	95.6	1.002	37.0	
5-10-10		20	60	94.0	14.4	108.4		5958	3008	0.5048	0.3484	0.0248	0.5296	0.658	96.1	0.963	44.3	
4-11-10	V	22	S 120-16-120	114.6	14.3	128.9		7046	1030	0.1461	0.0907	0.0239	0.1700	0 533	105 5	0.835	4.8	
4-16-10		23	34	113.7	14.2	127 9	127.7	6688	1566	0 2341	0.1445	0.0240	0.2581	0.560	110.0	0.944	11.3	109.4
4-16-10		24	40	113.6	14.2	127 8		6466	1738	0.2687	0.1776	0.0227	0.2914	0.610	109.1	0.940	15.7	
4-29-10		25	62	112.0	14.2	126.2		6736	2664	0.3954	0.2708	0.0253	0.4207	0.644	113.0	1.004	27.3	
4-11-10	VI	27	S 140-16-120	131.9	14.3	146.2		7086	986	0.1391	0.0941	0.0240	0.1631	0.576	126.7	0.946	3.7	
4-16-10		28	34	133.2	14.2	147.4		6862	1466	0.2136	0.1426	0.0231	0.2367	0.602	130.8	0.950	8.3	
4-16-10		29	49	132.0	14.2	146.2	146.2	6306	2000	0.3172	0.2050	0.0220	0.3392	0.604	130.0	1.006	16.4	129.4
4-29-10		30	68	130.8	14.2	145.0		6840	2796	0.4087	0.2760	0.0256	0.4343	0.635	128.9	1.008	24.9	
5-10-10		31	77	132.0	14.4	146.4		6138	3276	0.5337	0.3762	0.0269	0.5606	0.671	130.6	1.010	35.3	
5-9-10	VII	32	S 120-14- 90	113.9	14.4	128.3		5258	1025	0.1949	0.0909	0.0251	0 2200	0.413	112.1	0.854	4.9	
5-9 10		33	30	113.9	14.4	128.3	128.4	5102	1585	0.3106	0.1641	0.0242	0.3348	0.490	113.2	0.932	12.8	113.7
5-2-10		34	41	114.1	14.4	128.5		4940	2090	0.4230	0.2402	0.0246	0.4476	0.537	115.7	0.930	23.2	
5-2-10	VIII	36	S 120-28-150	113.3	14.4	127.7		8772	1616	0.1842	0.1136	0.0260	0.2102	0.540	108.4	0.927	7.7	
5-2-10		37	55	114.2	14.4	128.6	127.9	8518	2554	0.2998	0.1990	0.0254	0.3252	0.612	110.3	0.964	19.7	110.3
5-2-10		38	78	112.8	14.4	127.2		8182.5	3520	0.4302	0.3080	0.0256	0.4558	0.676	110.1	1.045	34.4	
5-2-10		39	91	113.5	14.4	127.9		7625	4142.5	0.5433	0.3968	0.0259	0.5692	0.697	112.2	1.016	46.0	

TABLE 2. GENERAL LOG TESTS WITH SUPERHEATED STEAM

Date of Test	Series	No. of Test	Laboratory Designation	Pressure lb per sq. in. At Throttle by Gauge	Barometer	Absolute Pressure at Throttle	Average Absolute Pressure of Series at Throttle	Revolutions per Hour	Weight of Condensate per Hour, lb.	Weight of Condensate per Revolution, lb.	Lb. of Steam per Revolution Shown by Indicator at Cut-off	Lb. of Compression Saved per Revolution by Indicator	Lb. of Steam Mixture Actually Present per Revolution	Quality of Steam Mixture at Cut-off. Parts of Unity	Absolute Pressure at Cut-off, lb.	Average Value of n from Expansion Curves	Cut-off, Per Cent of Stroke	Average Absolute Pressure at Cut-off for Series, lb.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2-17-11	IX	40	500°-60-8-120	59.5	14.5	74.0	73.4	7040	815	0.1158	0.0838	0.0232	0.1390	0.602	61.2	0.904	11.7	61.1
2-17-11		41	16	60.5	14.5	75.0		6856	1058	0.1543	0.1235	0.0239	0.1782	0.693	61.6	1.053	21.6	
2-17-11		42	24	57.7	14.5	72.2		6522	1288	0.1975	0.1738	0.0228	0.2203	0.788	60.6	1.114	32.0	
2-17-11		43	28	57.8	14.5	72.3		6262	1396	0.2229	0.2110	0.0227	0.2456	0.859	60.8	1.144	41.3	
2-18-11	X	44	500°-80-10-120	76.1	14.5	90.6	91.0	6655	1201.7	0.1806	0.1106	0.0246	0.2052	0.539	79.2	0.951	12.4	78.5
2-18-11		45	21	77.4	14.5	91.9		6550	1294	0.1976	0.1342	0.0250	0.2226	0.603	79.2	0.957	16.5	
2-18-11		46	31	76.3	14.5	90.8		6290	1455	0.2313	0.1910	0.0230	0.2543	0.751	77.4	1.105	27.6	
2-18-11		47	40	76.0	14.5	90.5		5908	1822	0.3084	0.2720	0.0226	0.3310	0.822	78.0	1.112	41.5	
12-17-10	XI	48	500°-100-13-120	94.3	14.5	108.8	109.0	7112	822	0.1156	0.0877	0.0228	0.1384	0.634	92.8	0.924	6.1	94.1
12-17-10		49	26	93.0	14.5	107.5		6936	1270	0.1831	0.1311	0.0234	0.2065	0.635	90.7	0.982	13.3	
12-17-10		50	39	95.5	14.5	110.0		6682	1660	0.2484	0.2040	0.0226	0.2710	0.753	97.1	1.040	22.3	
12-20-10		51	45	94.7	14.4	109.1		6500	1780	0.2738	0.2479	0.0220	0.2958	0.838	96.4	1.116	28.8	
12-20-10		52	52	95.1	14.4	109.5		6240	2072	0.3321	0.2821	0.0226	0.3547	0.796	93.6	1.001	39.2	
12-20-10	XII	53	500°-120-16-120	115.2	14.4	129.6	128.4	7100	914	0.1287	0.0972	0.0224	0.1511	0.642	111.5	0.991	5.1	111.0
12-20-10		54	32	114.3	14.4	128.7		6932	1380	0.1991	0.1561	0.0225	0.2216	0.705	109.0	0.990	13.0	
12-20-10		55	48	114.1	14.4	128.5		6662	1906	0.2861	0.2419	0.0226	0.3087	0.783	111.7	1.032	22.9	
1-7-11		56	60	114.1	14.3	128.4		6326	2315	0.3660	0.3230	0.0219	0.3879	0.833	111.5	1.140	35.1	
2-18-11		57	62	112.3	14.5	126.8		5940	2350	0.3956	0.3709	0.0228	0.4184	0.886	111.5	1.209	40.3	
1-7-11	XIV	61	500°-140-17-120	132.5	14.3	146.8	145.4	7136	922	0.1292	0.0935	0.0228	0.1520	0.615	127.7	0.939	3.2	128.5
1-7-11		62	34	128.0	14.3	142.3		7010	1384	0.1974	0.1553	0.0229	0.2203	0.715	123.6	0.969	11.0	
1-7-11		63	51	131.3	14.3	145.6		6810	1789	0.2627	0.2331	0.0228	0.2855	0.817	131.0	1.074	18.1	
1-10-11		64	63	131.0	14.3	145.3		6602	2171	0.3288	0.3113	0.0222	0.3510	0.887	130.5	1.190	26.9	
2-18-11		65	73	132.6	14.5	147.1		6088	2640	0.4336	0.3988	0.0230	0.4566	0.873	129.5	1.165	37.1	
2-17-11	XV	66	500°-120-12-90	114.8	14.5	129.3	128.0	5322	808	0.1518	0.0950	0.0220	0.1738	0.546	112.3	0.948	4.6	111.7
2-17-11		67	24	114.2	14.5	128.7		5212	1218	0.2337	0.1540	0.0222	0.2559	0.602	111.1	0.941	12.4	
2-17-11		68	36	112.3	14.5	126.8		5018	1574	0.3137	0.2356	0.0226	0.3363	0.700	110.7	1.013	23.1	
2-17-11		69	43	112.7	14.5	127.2		4817	1809	0.3755	0.3078	0.0217	0.3972	0.774	112.5	1.148	32.0	
2-17-11	XVI	71	500°-120-36-150	112.8	14.5	127.3	126.9	8700	1512	0.1738	0.1447	0.0249	0.1987	0.729	108.0	1.093	12.3	108.3
2-17-11		72	54	111.8	14.5	126.3		8438	2018	0.2392	0.2080	0.0257	0.2649	0.786	109.0	1.091	20.5	
2-17-11		73	70	112.2	14.5	126.7		8084	2458	0.3041	0.2970	0.0236	0.3277	0.907	107.2	1.213	32.4	
2-17-11		74	82	112.6	14.5	127.1		7494	3110	0.4150	0.4135	0.0237	0.4387	0.943	108.8	1.234	46.9	

TABLE 3

Log of Group Averages

Group	Series	No. of Tests Inclusive	Laboratory Designation	Pressure lb. per sq. in.			Absolute Pressure from Indicator Diagram, lb. per sq. in.					Specific Volume of Steam at Pressure Adopted	Number of Tests in Series	Number of Tests in Group	Number of Diagrams Analyzed	Average Quality of Steam Mixture at Cut-off from Tests in Group Parts of Unity	Average Value of *n* from Tests in Group
				Average Absolute Pressure at Throttle by Gauge	Average Absolute Pressure of Group	Difference from Group Average of Next Lower Pressure	Average at Cut-off	Average of Group	Difference from Group Average of next Lower Pressure	Drop in Pressure Throttle to Cut-off	Pressure Adopted To Find Quality of Steam Mixture at Cut-off						
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	I IX	2-4 40-43	s-60- -120 500°-60- -120	71.3 73.4	72.4		60.6 61.1	60.9		11.5	61 0	7.06	3 4	7	14	0.6500	0.998
B	II X	6-9 44-47	s-80- -120 500°-80 -120	90.5 91.0	90.8	18.4	77.9 78.5	78.2	17.3	12.6	78.0	5.60	4 4	8	16	0.6235	0.981
C	IV XI	15-20 48-52	s-100- -120 500°-100- -120	109.1 109.0	109.1	18.3	95.3 94.1	94.7	16.5	14.4	95.0	4.65	6 5	11	22	0.6515	0.976
D E F D E F	V VII VIII XII XV XVI	22-25 32-34 36-39 53-57 66-69 71-74	s-120- -120 s-120- -90 s-120- -150 500°-120- -120 500°-120- -90 500°-120- -150	127.7 128.4 127.9 128.4 128.0 126 9	127.9	18.8	109.4 113.7 110.3 111.0 111.7 108 3	110.7	16.0	17.2	111.0	4.012	4 3 4 5 4 4	9 7 8	18 14 16	0.6885 0.5803 0.7363	1.010 0.967 1.073
G	VI XIV	27-31 61-65	s-140- -120 500°-140- -120	146.2 145.4	145.8	17.9	129.4 128.5	129.0	18.3	16.8	129.0	3.478	5 5	10	20	0.6995	1.026

general trend, it is seen that there is no exception to this general relation. No value of n below 1.00, for instance, is found for values of x_c above 0.80, and no value of n above 1.10 is found for values of x_c below 0.72.

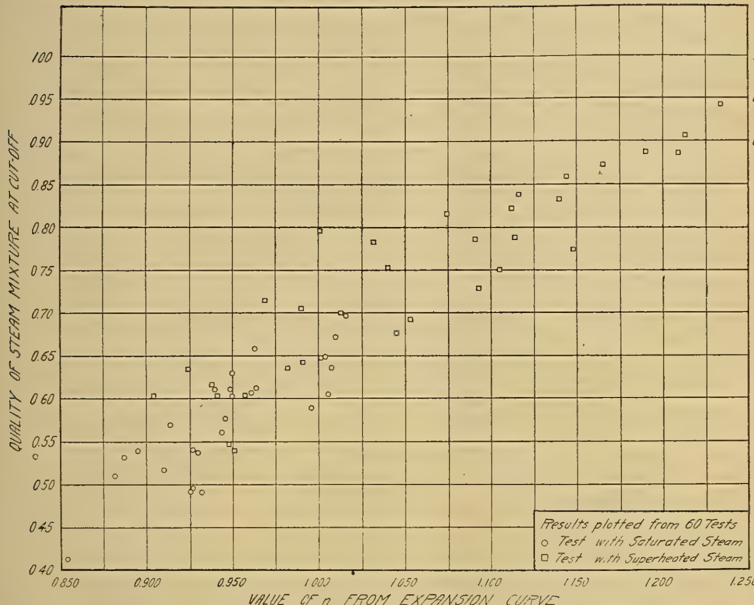

FIG. 1. GENERAL RELATIONS BETWEEN QUALITY AT CUT-OFF AND THE VALUE OF n FOR VARIOUS PRESSURES AND SPEEDS

It is also apparent that the points with long cut-off, obtained from saturated steam for given values of x_c and n, are in the same region occupied by the points with short cut-off obtained from superheated steam for the same given values of x_c and n. Examining the region of n=0.90–1.00 and of x_c =0.50 - 0.70, it may be seen that the points obtained with long cut-off with saturated steam, and with short cut-off with superheated steam, lie together indiscriminately. This shows conclusively, in a general way, that the value of n is practically independent of the length of cut-off, even though this length may vary from 5 per cent to 45 per cent, and that n depends solely on the value of x_c, the only other variable.

The points shown in Fig. 1 occupy a relatively wide region until they are separated into the various groups of similar pressures and speeds. The points for each group were plotted separately, and separate curves were determined for each condition. From preliminary plotting, the relations between n and x_c were found to be expressed closer by straight lines than by any other

family of curves. The method used to draw these lines will be given for group F, which includes series 8, with 4 tests, and series 16 with 4 tests. This group is shown in Fig. 2. All points

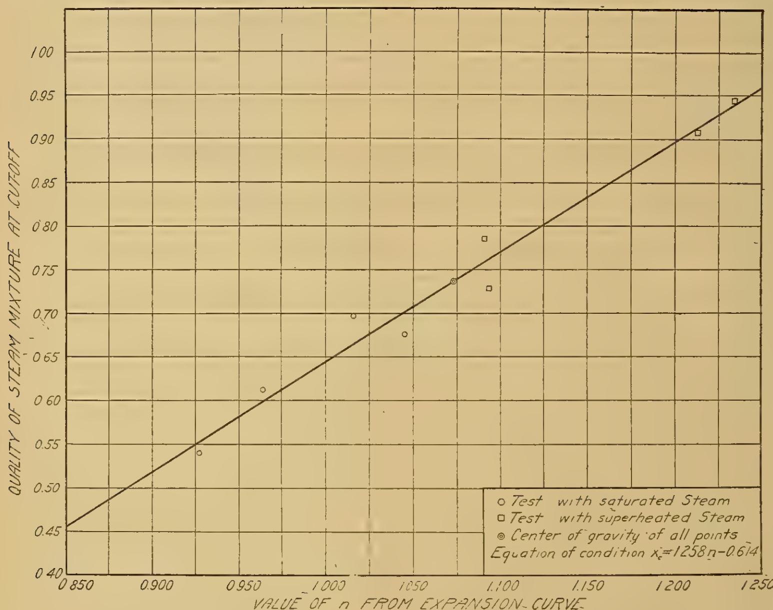

FIG. 2. RELATION OF QUALITY AND THE VALUE OF n FOR TESTS RUN AT 111 POUNDS ABSOLUTE PRESSURE OF CUT-OFF AND 150 R. P. M.

were given equal weight. The average of all the coordinates, or the "center of gravity" was found and the condition imposed that the line pass through this center as an axis, and that the slope be determined by the position of the points. The points in group F were divided into four logical pairs or groupings, and the center of gravity found for each grouping. The line was then drawn as shown. Where points were located so that a logical grouping was in doubt, various groupings were made, and each given weight in determining the slope.

The equation of the curve selected is $x_c = 1.258\, n - 0.614$. The average deviation of the points from this line is 2.6 per cent (measured from the zero of x_c) and the maximum deviation is 4.6 per cent. This average deviation, 2.6 per cent, is smaller than that for most of the groups. The straight line, in most cases, represents the points found as closely as any other curve that could be employed, and has the merit of simplifying greatly the subsequent use made of the relations for the different groups.

The values of n for group F were also plotted to the various accompanying cut-off positions at which each test was run. This is shown in Fig. 3. The points in Fig. 3 show that for each initial condition of steam, the value of n increased as the cut-off was lengthened, and that with any given cut-off, different values of n were obtained according as saturated or superheated steam was used. Thus, at a cut-off of 15 per cent, the value of n obtained is 0.950 with saturated steam, and 1.084 with superheated steam. The only variable present in these two cases is x_c, the value of which is higher with superheated steam than with saturated steam. A definite relation between n and cut-off occurs only when a given cut-off is accompanied by the same value of x_c, i. e., the value of n bears a direct relation to x_c but not to cut-off. Fig. 3 taken in conjunction with Fig. 2, proves that the value of n depends directly only upon the value of x_c, and that the relation of n

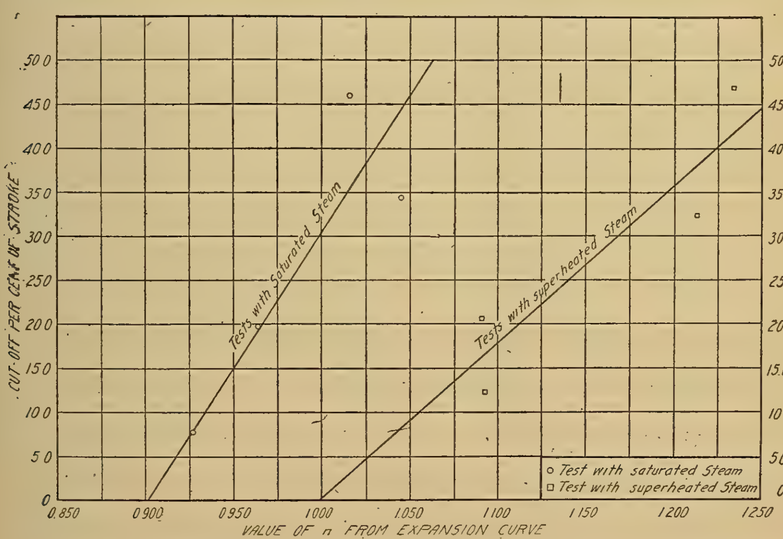

FIG. 3. RELATIONS OF PER CENT OF CUT-OFF AND THE VALUE OF n FOR TESTS RUN AT 111 POUNDS ABSOLUTE PRESSURE AT CUT-OFF AND 150 R. P. M.

and x_c is practically independent of the length of cut-off within the limits of the tests.

(a) *Effect of varying the steam pressure at constant speed.*—The relations of x_c and n were determined separately for all groups by the method outlined for Fig. 3. The lines for the five pressures used, comprising the results of groups A, B, C, D, and G, were replotted as shown in Fig. 4. This figure also contains other curves that are discussed in Appendix 1. The lines shown give

the relations of x_c and n for various absolute pressures at cut-off, all obtained with a speed of 120 r. p. m.

These curves were then examined to find the effect of varying the absolute pressure at cut-off (designated as p) on the relations of x_c and n. In Fig. 4, the constant pressure curves were intercepted at constant values of n, and the coordinates of x_c and p for the points of intersection plotted in Fig. 5. This process was repeated at intervals of 0.05, for the values of n from 0.850 to 1.250. The curves were adopted as shown. The method used was as follows: the maximum value, from the evidence of Fig. 4, was assumed to be at the value, $p = 95$; the values of $p = 61$ and $p = 78$ were combined, and the center of the line connecting each pair used as one point; with these assumptions the curves were drawn.

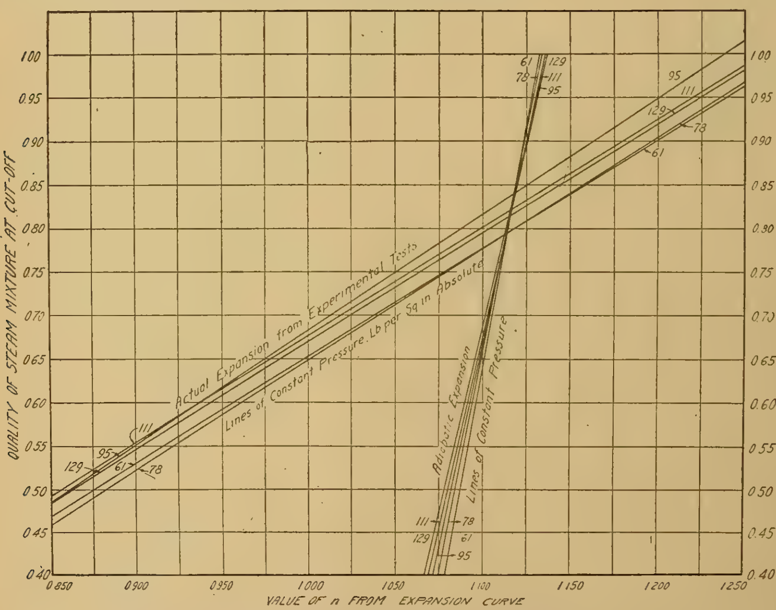

FIG. 4. RELATIONS AT CUT-OFF BETWEEN QUALITY AND THE VALUE OF n FOR VARIOUS PRESSURES AT CONSTANT SPEED

Since the relation of x_c and n at constant speed and pressure is a straight line, the curves of Fig. 5 were drawn by interpolation to increments in the value of n to 0.01.

This procedure gave a series of relations between x_c and p for constant values of n. Since, however, the independent variables, in any actual curve under examination, are n and p, the coordinates of the curves of Fig. 5 were changed so as to show the relations of n and p at constant values of x_c. These relations are

shown in Fig. 6.

The effect of a change of pressure on the relations of x_c and n is not great between the limits of 75 to 150 lb. An approximate equation has been worked out, therefore, which represents the

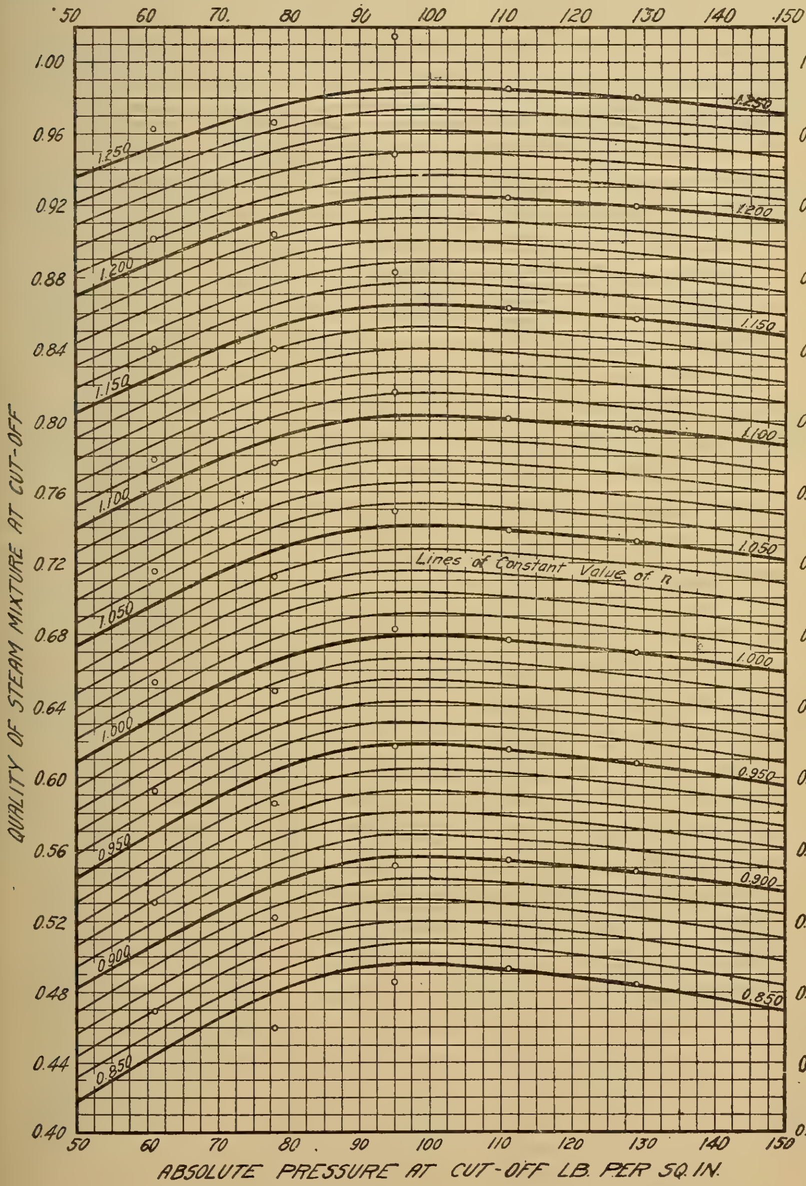

FIG. 5. CHART SHOWING THE RELATION AT CUT-OFF BETWEEN QUALITY AND PRESSURE FOR CONSTANT VALUES OF n FROM EXPANSION CURVE

relations of x_c and n at an average pressure between the limits mentioned. The equation corresponds to the relations at 129 lb. and is of the form

$$x_c = 1.245\,n - 0.576.$$

(*b*) *Effect of varying the speed at constant pressure.*—Groups D, E, and F, were run at speeds of 120, 90 and 150 r. p. m., respectively, with the average cut-off pressure on the diagrams constant at 111 lb. absolute. The regulation of the governor was very poor, there being about a 10 per cent drop in speed from no load to full load. For this reason, the relations of x_c and n with various speeds at constant pressure were affected by considerable variation of the speed itself for each group. The relations for each group were found as already described, and the curves plotted in Fig. 7. The relation of speed (designated as s) and x_c for constant values of n was derived from Fig. 7, and is given in Fig. 8.

The apparent relations of x_c, s, and n, obtained by drawing a smooth curve through the three points obtained for each value of n, are not satisfactory, owing to insufficient data and the change of the speed itself in the three groups due to poor regulation. The drop in the speed, for one group, does not seriously affect, however, the relations of x_c and n for the various pressures at constant speed.

2. *Relation of the value of n to the quality of the steam mixture at cut-off.*—From the evidence obtained from these tests, it may be stated that, for any one engine, running at a given pressure and speed, there is a definite relation between x_c and n which is practically independent of the cut-off position within the limits examined. This relation is apparently a linear one. It may also be stated that the relation of x_c and n is dependent, to some extent, on the absolute pressure at cut-off, and on the speed of the engine.

It remained to compare the relations of x_c and n for the engine tested with the relations for other engines. This comparison is made in the next section.

An investigation of the value of k for adiabatic expansion, (see page 83, Appendix 1) shows that there is a relation between the initial quality x_1 and the value of k which, like the experimentally determined relation, is also a linear one. The adiabatic relations of x and k are plotted in Fig. 4, for the pressures used in the tests.

Acknowledgment is made of the assistance rendered, in running these tests, by the following senior students, viz., Messrs. Jacobsen, Schuster, Parmely, Hodgson, Janda, Butzer and Wood, Class of 1910, and Messrs. Hasberg, Hagedorn, Allen, Herrcke, Cobb and Ponder, Class of 1911.

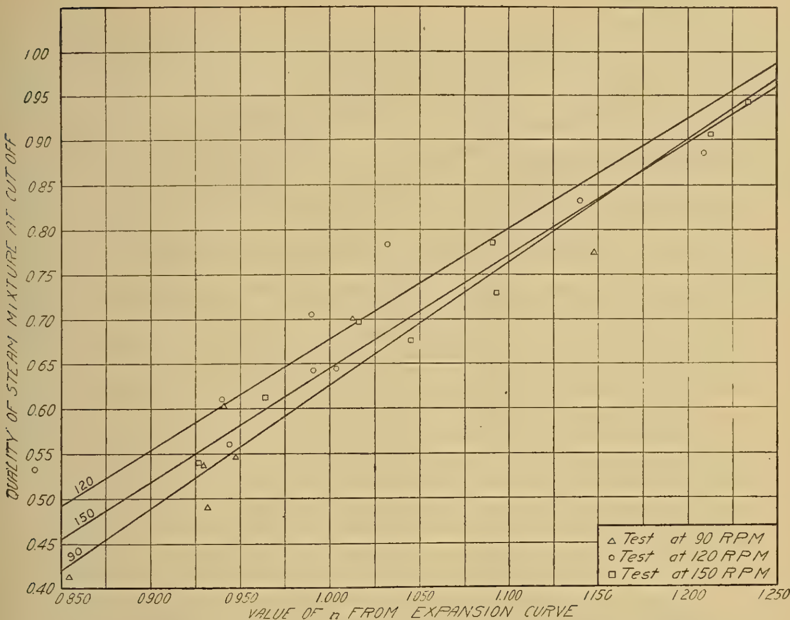

FIG. 7. RELATIONS OF QUALITY AT CUT-OFF AND THE VALUES OF n FOR VARIOUS SPEEDS AT THE CONSTANT CUT-OFF PRESSURE OF 111 POUNDS

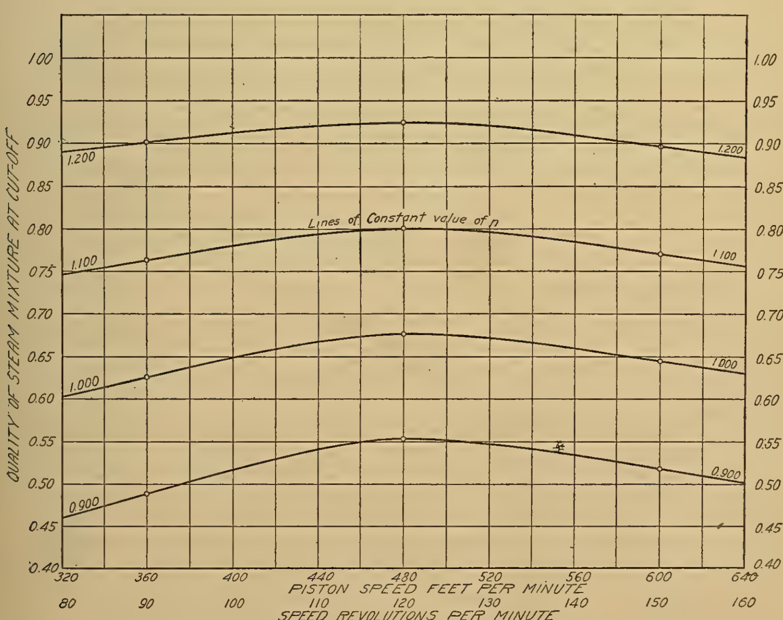

FIG. 8. RELATIONS AT CUT-OFF BETWEEN QUALITY AND SPEED FOR CONSTANT VALUES OF n AT CONSTANT PRESSURE

III. The Approximation of the Actual Steam Consumption from Indicator Diagrams

5. *Engine Tests.*—The most reliable method of determining the steam consumption of an engine is to measure or weigh the water directly, preferably by means of a surface condenser. This method necessitates an elaborate test, which disturbs the routine of the plant tested, and is very costly for long tests of large engines. The objections to tests of this kind are many, and a few of them will be stated. When the boiler feed is measured, both the engine and the boilers serving it have to be entirely disconnected from other units, sometimes necessitating shutting down the rest of the plant. Almost all tests, where there is more than one unit, necessitate changes in heavy and permanent piping. Boilers are apt to leak in service, and the measurements of the leaks are unsatisfactory. One serious objection to long time tests is that the different rates of steam consumption cannot be segregated. The ideal method of testing a steam engine would be a method analagous to that used with electrical machines, i. e., to measure instantaneous *rates* of consumption instead of the water consumed over a long time.

6. *The Missing Quantity.*—On account of the cost and difficulties of making a test, many engineers have devised methods of approximating the steam consumption without actually measuring it.

When indicator diagrams were first obtained, the loss from initial condensation, or the existence of the "missing quantity", was not suspected. The opinion, therefore, was that the consumption could be measured from the steam shown by the diagram at cut-off. After Clark[1] and Isherwood[2] made their tests the existence and amount of this initial condensation were revealed. The very great difference in the proportion that this initial condensation bears to the total weight of mixture present, either at cut-off or during the expansion, that is found in different types and sizes of engines has prevented any reliable determinations of the actual steam consumption by this method. The steam consumption computed from the diagram, when using saturated steam, is generally from 15 per cent to 50 per cent below the actual consumption.

The devising of an accurate method of measuring the actual weight of steam consumed from the diagram has therefore been

[1] Railway Machinery

[2] Engineering Researches.

regarded as impossible. Thurston[3] states that "the steam or water consumption of an engine cannot be exactly ascertained by the use of the indicator" for the reasons mentioned. Most other writers on the subject have expressed similar views.

7. *Approximation of the Initial Condensation.*—Methods of computing the weight or proportion of the "missing quantity", from the dimensions, type, and speed of the engine considered, as shown by the results of large numbers of tests, have been devised by Thurston[4], Escher[4], Marks[5], Fourier, English[6], Bodmer[7], Cotterill[8], Heck[9], and others. The results obtained from these methods have not been uniform, and do not agree closely enough with the test results to be used with confidence. Moreover, none of these methods is applicable when superheated steam is used. Professor Heck[10] states that the steam consumption computed by the use of his formula should ordinarily show not more than 10 per cent difference from the test results.

8. *The Relation of x_c and n.*—The relation of x_c and n, for the same engine, has been shown to be very definite under the same conditions of pressure and speed. This relation, or dependence, of n upon x_c, however, is not seriously affected by changes of pressure and speed, within the limits of the tests. The ordinary speeds of similar types of engines, 70 to 120 r. p. m., do not affect the relation seriously enough to be taken into account when examining such cases, because of the two-fold character of speed in its influence upon the action of the cylinder walls.

The engine experimented upon was operated at 120 r. p. m., and had a stroke of 2 ft. Other engines of this class run at speeds as low as 70 r. p. m., but have strokes of 5 or 6 ft. Cylinder condensation is not dependent upon rotative speed alone, but is also influenced by the piston speed, as determined by the length of the stroke. On account of different lengths of stroke, different engines cannot be compared on the basis of rotative speed. Thus while the small engine tested has a rotative speed of 120 r. p. m., its piston speed is only 480 ft. per minute. In large engines, while the rotative speed may be only 70 r. p. m., yet, with strokes of 6 ft., the piston speed is 840 ft. per minute. What the smaller engine gained by higher rotative speed, the large engine made up in a measure by higher piston speed.

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3 Engine and Boiler Trials, p. 237.

4 A Manual of the Steam Engine, p. 517.

4 Engineer (London), 1882. [p. 206.]

5 Relative Proportions of the Steam Engine, Proceedings of the Brit. Inst. of M. E., Oct, 1889.

7 Engineering (London), Mar. 4, 1892, p. 299.

8 The Steam Engine, p. 239.

9 The Steam Engine, p. 109.

10 The Steam Engine, p. 119.

After taking into account the two parts of which speed is composed, it is found that the speeds of stationary engines of the type tested are in substantially the same range. On account of this fact, only the results of the tests run at 120 r. p. m. have been used in the applications made at present.

The relation of x_c and n has been found to be practically independent of cylinder size. This statement is true for non-jacketed cylinders and for pressures in the range examined. This is shown in a general way in the following discussion for saturated steam.

9. *The Phenomena Occurring in the Cylinder.*—Cast-iron is universally used for steam cylinders. The greatest source of loss in the cylinder is due primarily to the use of a metallic structure, from practical considerations. The skin surface of this metal, which is a fairly good conductor of heat, must be heated once every cycle from the temperature acquired from contact with the exhaust steam, up to nearly the temperature of the admission steam, this heating being accomplished by the condensation of some of the incoming steam. The amount of this condensation, measured as the proportion of the mixture present, varies with the size, valve design, relative roughness of the interior surface, temperature range, length of cut-off, speed, location of ports and port passages, quality of the steam supplied, and the jacketing and lagging. It can easily be seen, from the number and relative magnitude of these variables, that the computation of the weight of condensation, by means of a formula which will take these variables into account, can never be an accurate operation.

After many examinations into the cases of different types and sizes of engines, with non-jacketed cylinders in good order, and with pressure limits similar to those used with the tests, it has been found that while the initial condensation is subject to the action of ten or more variables, yet the value of n resulting from a given value of x_c is almost always substantially the same. A few of the applications showing this point will be found in Table 4. Here the cylinder sizes vary from $10\frac{1}{2}$ in. x 12 in. to 34.2 in. x 60 in., the speeds from 48 to 263 r. p. m., and the types include slow-speed Corliss, high speed, and locomotive engines. The possibility of calculating accurately the weight of condensation in these different cases may be easily imagined.

The phenomena caused by the presence of the cylinder walls, in the class of engines discussed, have been found to be divided into two natural classes: those occurring before cut-off, and those

occurring after cut-off. The phenomena occurring before cut-off are controlled by the action of the ten or more variables already mentioned, and therefore are subject to all the variation that may occur in any individual case to be examined. For this reason, any method of computing the condensation accurately from the physical facts surrounding the case is open to objection. This method also cannot allow for the use of superheated steam, an increasingly important condition. The phenomena occurring after cut-off are practically independent of all variables except x_c, and initial pressure and speed. Of these variables, only the value of x_c and the initial pressure have proved to be of material importance in the applications made thus far. This may be summed up by stating that the value of x_c, in any particular case, is subject to the action of many important variables, but that the relation of x_c and n is practically independent of these variables within the limits examined in this investigation.

10. *The Phenomena of Condensation and Re-evaporation during Expansion.*—When adiabatic expansion of initially dry saturated steam takes place, a part of the steam is condensed as the pressure is lowered, the condensed steam giving up its latent heat which is converted into work. When superheated steam is expanded adiabatically, the steam loses its superheat until saturation is reached, after which condensation takes place as in the case of initially dry steam.

When, however, the steam is initially composed of a large proportion of water, both being at the same temperature, adiabatic expansion may take place without additional condensation and may even be accompanied by re-evaporation. This fact is due to the large amount of heat contained in the water, a part of which flashes into steam as the pressure is lowered, thus supplying and neutralizing the loss of steam volume by condensation which takes place with steam initially dry. Adiabatic expansion is accompanied by condensation when the initial quality is above the value 0.50 at an initial pressure of 240 lb. per sq. in. absolute, but below the value of 0.50, it is accompanied by re-evaporation. An examination of Table 20 will show the values of the initial quality which form the line of demarcation of condensation and re-evaporation during the adiabatic change of state.

In the actual engine using saturated steam, as has already been pointed out, some of the incoming steam is condensed in warming up the surface of the cylinder walls to approximately the temperature of the incoming steam. When the admission of

TABLE 5

CONDENSATION AND RE-EVAPORATION OF STEAM DURING EXPANSION

Example No	1	2
Initial Quality, Parts of Unity	0.540	0.950
PRESSURES, LB. PER SQ. IN. ABSOLUTE		
Initial	145.0	145.0
Final	20.0	20.0
VALUE OF n IN EQUATION $PV^n = C$		
Adiabatic expansion, from Table 20	1.082	1.133
Actual expansion in engine tested, from Fig. 6	0.900	1.230
VOLUME OF STEAM PRESENT, CU. FT. (VOLUME OF WATER NEGLECTED)		
Initial	1.00	1.00
Final, adiabatic expansion	6.24	5.75
Final, curve of constant steam weight	6.44	6.44
Final, actual expansion in engine tested	9.04	5.00
WEIGHT OF STEAM PRESENT, LB.		
Inital, plus water	0.594	0,338
Initial, steam only	0.321	0.321
Final, adiabatic expansion	0.311	0.287
Final, curve of constant steam weight	0.321	0.321
Final, actual expansion in engine tested	0.450	0.249
QUALITY OF STEAM AT FINAL PRESSURE, PARTS OF UNITY		
Adiabatic expansion	0.524	0.849
Curve of constant steam weight	0.540	0.950
Actual expansion in engine tested	0.757	0.736
CONDENSATION OR RE-EVAPORATION, PARTS OF UNITY		
Apparent re-evaporation	0.217	
Real re-evaporation	0.233	
Apparent condensation		0.214
Real condensation		0.113

steam is cut off and expansion commences, the condensation, due to the presence of the cylinder walls, continues in general until, at some point during expansion, the water on the cylinder walls begins to re-evaporate at such a rate that the weight of steam present at the end of expansion is greater than that which was present at cut-off.

To show the effects and extreme values of condensation and re-evaporation during expansion, there have been prepared in Table 5 two examples, using the average results of tests which have been run on the engine tested (see Appendix 2).

11. *Examples.*—Example 1 is a condition which obtains in the engine tested when using saturated steam at about 140 lb. gauge pressure with a length of cut-off of about 3 per cent. All values of the qualities mentioned are portions of the total weight of mixture in parts of unity. The value of x_c is 0.540, a low value, yet one which often obtains in small engines. If this steam were expanded adiabatically to the back pressure, 20 lb. absolute, the resulting quality would be 0.524, giving a condensation of 0.016. The expansion which actually takes place in the engine tested under these conditions results in a final quality of 0.757, showing that the apparent re-evaporation from the value of x_c has been 0.217. However, the steam mixture in expanding did expand adiabatically in order to give up heat to work, but the actual or what might be called the gross expansion, was changed in character by the re-evaporation of a large proportion of the water present, due to the return of heat from the cylinder walls and the consequent flashing into steam of part of the water when the pressure and the temperature were lowered. The real re-evaporation, measured by its effect upon adiabatic expansion, has been the difference between 0.757 and 0.524 or 0.233.

The actual expansion in this case has been the result of two factors which worked simultaneously; adiabatic expansion, and the return of heat from the cylinder walls to the mixture. The first factor, adiabatic expansion, as already explained, is itself the result of two neutralizing or opposing conditions, i. e., the condensation of initially dry steam during expansion, and the relatively smaller amount of re-evaporation of water initially in the mixture due to the liberation of its excess of heat when the pressure and temperature were lowered. The net result of the two conditions of this adiabatic expansion, however, was a condensation. The second factor is the large amount of re-evaporation due to the return of heat from the surface of the cylinder walls to the condensed steam, amounting in example 1 to 0.233, or, roughly, there has been re-evaporated during expansion $\frac{1}{4}$ of the entire weight of mixture present.

Example 2 shows conditions which obtain in the engine tested when served with steam at about 140 lb. gauge pressure, superheated about 125° F. with a length of cut-off of about 45 per cent. The value of x_c is 0.950, a very high value for this class of engine. The quality after adiabatic expansion would be 0.849, a condensation of 0.101. Where values of x_c are as high as 0.950, however, no re-evaporation takes place in practice, but condensa-

tion continues throughout expansion. After expansion in the engine tested, the quality would be 0.736, showing much greater condensation than that due to adiabatic expansion alone. The apparent condensation has been 0.214, but the real condensation, measured by its effect upon adiabatic expansion, has been 0.113.

The actual expansion in example 2, as in example 1, has been the result of two factors; adiabatic expansion, and the further abstraction of heat during the whole expansion by the cylinder walls. Heat is abstracted during expansion by the cylinder in the engine tested at all values of x_c above 0.85, thus giving values of n higher than the adiabatic value k.

The phenomena of condensation and re-evaporation during expansion are the causes of the relations existing between x_c and n in the cylinders of steam engines. The two examples given show values obtained in extreme cases which illustrate very well the effect of x_c upon the character of the expansion, and therefore upon the value of n, and show the range of values that n assumes in one engine due to a change in the value of x_c.

12. *The New Method of Approximating the Value of x_c.*—The fact that the value of n depends upon the value of x_c and that the values have definite relations under definite conditions, makes it possible to reverse the order of procedure followed in obtaining the relation and to approximate the value of x_c (and, therefore, the actual steam consumed) from the value of n.

It appears, therefore, that this method of approximating the value of x_c at cut-off from experimentally determined relations, and thus accounting for the "missing quantity", is upon much surer ground than any method of computing condensation from the physical facts surrounding the case. It approaches the problem from the side where the phenomena occurring are practically independent of all the variables mentioned.

13. *Advantages of the Method.*—This method is free from several objections to which tests are open. It measures the consumption in one revolution, and is, therefore, practically measuring a rate instead of a quantity. The only data needed for an approximation are one set of indicator diagrams, taken simultaneously, the constants of size and clearance, and the speed of the engine tested. No interruption of any kind in the routine of a plant is caused, and the expense incurred is not to be compared with that of an equally accurate test. The method is accurate enough for almost all purposes except guarantee tests subject to bonus and forfeit contracts. In the case of locomotives on the road, it is

the only possible method of approximating the steam consumption of the main engines, due to the use of steam by the air-pump, train-heating system, blower, generator sets, safety valves, whistle, blow-off valves, and leaks. The same is true of marine engines, where many auxiliaries are supplied with steam from the same boilers, and exhaust into the same surface condensers. The method is especially useful for non-condensing engines, where the boiler-feed measurement method is the only practicable one. Steam consumption may be obtained as often as is desired instead of probably once in an engine's life.

14. *Limitations*—The relations of x_c and n given in chapter II are applicable, however, only to non-jacketed cylinders exhausting at very close to atmospheric pressure. When the back pressure is raised to 30 lb. absolute, for instance, there is a new series of relations existing for the same initial pressure, due to a different temperature range in the cylinder and the consequent alteration of the phenomena occurring after cut-off. Steam jackets also alter the phenomena occurring after cut-off, and therefore have to be examined separately for the relations of x_c and n. The steam used in the jackets and in reheaters has of course to be collected and weighed as heretofore.

Since this method rests entirely on the indicator diagram, great care must be observed in taking these diagrams. The indicator itself must be an accurate instrument in the best possible condition. The indicator connections must be short and direct. An extensive investigation by W. F. M. Goss[11] shows that the long and indirect pipe connections materially alter the form and character of the expansion curves. A correct reducing motion, free from lost motion, must be used so as to reproduce the actual expansion. The arrangement of having one indicator at each end of the cylinder is always to be preferred.

The applications of this method must be made with judgment and care. If large leakage exists, only an approximate solution can be obtained, as certain assumptions have to be made. The various steps involved in the use of the method must be thoroughly comprehended to give satisfaction.

15. *Application of the Method.*—The relations of x_c and n, as determined for various pressures at constant speed from the engine described (see Appendix 2,) were plotted in the form of the chart shown in Fig. 6.

The next step was to examine, with certain restrictions, the

[11] Trans. A. S. M. E. XVII, p. 398.

tests of other engines, and to compare the relations of x_c and n with those given in Fig. 6. The restricting conditions imposed were: (1) that the tests should come from reliable sources; (2) that the data supplied should be complete enough to be able to compute the quantities needed for comparison; (3) that the cylinders should be non-jacketed; (4) that the diagrams furnished should be representative of average conditions; (5) that the back pressure in the cylinder examined should be practically atmospheric; (6) that no large leaks should exist.

The values of x_c, n and p were first found from the set of diagrams to be examined. Next, the values of n and p were located in Fig. 6 and the corresponding value of x_c found, as obtained in the tests given in chapter II. The value of x_c obtained from the chart and that obtained from the test examined were compared, and the steam consumption, as computed by the value of x_c taken from the chart, was obtained.

The results of tests which fulfilled the conditions imposed are given in Table 4. Four distinct classes of engines were examined. These include simple Corliss, two-valve and four-valve types, the high pressure cylinders of compound engines, the intermediate pressure cylinders of triple expansion engines, high speed, and simple locomotive engines. The sizes range from $10\frac{1}{2}$ in. x 12 in. to 34.2 in. x 60 in., and the speeds from 263 to 47.98 r. p. m.

The final results, given in columns 24, 29, 31 and 36 of Table 4 were averaged (analysis 201 excepted) and the averages are given in the following table:

APPROXIMATION FROM CHART—FIG. 6

Average Difference from Test Results	Value of X_c per cent	Actual Steam Consumption per cent
Irrespective of Sign....................................	3.06	3.72
Higher (+) or Lower (−) than Tests Results.............	−1.93	+2.50

APPROXIMATION FROM EQUATION $X_c = 1.245\,n - 0.576$

Irrespective of Sign...	3.32	3.98
Higher (+) or Lower (−) than Test Results................	−1.2	+1.91

TABLE 4

Approximation of the Actual Steam Consumption From Indicator Diagrams Taken From Non-Jacketed Steam Engines

No. Analysis	Class of Engine	Makers	Where Tested	By Whom Tested	Date of Test	Source of Diagrams and Test Results	Cylinder Dimensions inches	Piston Rod Diameter inches	Clearance, Per Cent of Piston Displacement	Actual Weight of Steam Passing through cylinder, lb. per hr	Total i. h. p. Developed	Actual Steam per i. h. p. hr., lb.	Revolutions per Minute	Revolutions per Hour	Weight of Steam: Passing through Cylinder per Revolution, lb.	Weight of Steam: Shown by Indicator at cut-off, lb.	Weight of Steam: Retained in Compression per Revolution by Indicator, lb.	Weight of Steam: Mixture Actually Present per Revolution, lb.	Quality of Steam Mixture at Cut-off (Xe) Parts of Unity	Average Value of n from Expansion Curves	Absolute Pressure at Cut-off lb. per sq. in	Approximation from Chart: Value of Xe from Chart, Parts of Unity	Approximation from Chart: Per cent that the Value of Xe from Chart is Higher (+) or Lower (−) than Test Results	Approximation from Chart: Total Weight of Steam Mixture Present per Revolution lb.	Approximation from Chart, Weight of Steam: Passing through Cylinder per Revolution lb.	Approximation from Chart, Weight of Steam: Passing through Cylinder per hour lb.	Approximation from Chart, Weight of Steam: Per i. h. p. hr. lb	Approximation from Chart: Percent That Total Weight of Steam per Hour is Higher (+) or Lower (−) than Test Results	Approximation from Equation $Xe = 1.245\,n - 0.576$: Value of Xe from Equation $Xe = 1.245\,n - 0.576$	Approximation from Equation: Per cent That the Value of Xe from Equation is Higher (+) or Lower (−) than Test Results	Approximation from Equation: Total Weight of Steam Mixture Present per Revolution lb	Approximation from Equation, Weight of Steam: Passing through a Cylinder per Revolution lb	Approximation from Equation, Weight of Steam: Passing through Cylinder per Hour lb	Approximation from Equation, Weight of Steam: Per i. h. p. hr. lb.	Approximation from Equation: Percent That Total Weight of Steam per hour is Higher (+) or Lower (−) than Test Results	Method Giving Closest Approximation to Test Results, Chart (C) Equation (E)	REMARKS
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
	SIMPLE CORLISS, 2-VALVE, AND 4 VALVE NON-CONDENSING ENGINES																																				
101	Corliss, Single Cylinder	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 1	23 x 60	$3\frac{1}{4}$	2.5	8477	305.2	27.77	74.7	4482	1.890	1.666	0.130	2.020	0.825	1.098	65	0.767	−7.0	2.172	2.042	9160	30.00	+8.1	0.790	−4.2	2.110	1.980	8880	29.18	+4.8	E	Reproductions of diagrams used, said to be representative
102	Corliss, Single Cylinder	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 2	$28\frac{1}{2}$ x $59\frac{1}{2}$	4	3.0	13670	506.5	25.80	64.8	3888	3.362	2.822	0.167	3.529	0.801	1.108	75	0.735	0.7	3.551	3.384	13150	25.96	+0.7	0.804	+0.4	3.542	3.315	13000	25.57	0.5	E	Reproductions of diagrams used, said to be representative
103	Corliss, Single Cylinder	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 7	$28\frac{1}{4}$ x 48	$3\frac{3}{4}$	3.0	6742	232.3	29.03	64.7	3882	1.736	1.402	0.116	1.852	0.757	1.151	75	0.732	−3.3	1.915	1.799	6985	30.06	+3.6	0.733	3.2	1.913	1.797	6980	30.02	+3.[illegible]	E	Reproductions of diagrams used, said to be representative
104	Double-Valve, 2 Cylinders	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 10b	17 x 21 2	$2\frac{1}{4}$	2.0	7952	310.1	25.64	152.9	9174	0.8670	0.7290	0.0635	0.9305	0.784	1.131	70	0.816	+4.1	0.8935	0.8300	7615	24.53	−4.2	0.833	+6.3	0.8752	0.8117	7445	24.00	−6.4	C	Reproductions of diagrams used, said to be representative
105	Four-Valve, Single Cylinder	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 11	$16\frac{3}{4}$ x 32	$2\frac{3}{4}$	4.0	1879	50.2	37.43	79.8	4788	0.3925	0.2732	0.0607	0.4532	0.603	0.964	60	0.586	−2.8	0.4665	0.4058	1942	38.69	+3.3	0.624	+3.5	0.4381	0.3771	1807	36.00	3.8	C	Reproductions of diagrams used, said to be representative
106a	Four-Valve, Single Cylinder	Ridgway Dynamo and Engine Co	Makers Works Ridgway, Pa.	A. R. Mees	10-10-10	Makers	$10\frac{1}{2}$ x 12	2	5.0	2453	97.9	24.04	263	15780	0.1491	0.1377	0.0242	0.1733	0.794	1.076	120	0.769	3.1	0.1791	0.1549	2440	24.92	+3.7	0.764	−3.8	0.1802	0.1560	2461	25.15	+4.6	C	Blue-print of tracings furnished, said to be representative
106b	Same Engine as 106a									1878	79.2	23.71	258	15480	0.1213	0.0959	0.0160	0.1373	0.698	1.031	115	0.717	+2.7	0.1337	0.1177	1822	23.01	−3.0	0.711	+1.9	0.1349	0.1189	1840	23.22	2.0	E	Blue-print of tracings furnished, said to be representative
106c	Same Engine as 106a									997	41.7	23.90	260	15600	0.0639	0.0537	0.0260	0.0899	0.598	0.931	125	0.587	1.8	0.0915	0.0855	1021	24.50	+2.4	0.583	2.5	0.0921	0.0661	1031	24.72	+3.4	C	Blue-print of tracings furnished, said to be representative
106d	Same Engine as 106a									693	25.8	26.87	263	15780	0.0439	0.0402	0.0351	0.0790	0.509	0.868	120	0.511	+0.4	0.0787	0.0436	688	26.68	0.7	0.504	−1.0	0.0748	0.0447	[illegible]	27.32	+1.2	C	Blue-print of tracings furnished, said to be representative
107	Corliss, 2 Cylinders	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 31b	16 x 12	$2\frac{1}{4}$	2.5	8873	342.43	25.91	84.9	5094	1.742	1.420	0.041	1.783	0.767	1.077	95	0.771	2.0	1.835	1.794	9140	26.69	+3.0	0.765	−4.0	1.856	1.815	9245	27.00	+4.2	C	Reproductions of diagrams used, said to be representative
108a	Four-Valve, Single Cylinder	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 17a	18 x 30	2	5.0	4707	213.2	22.08	165.6	9936	0.4735	0.3850	0.0330	0.5085	0.768	1.035	63	0.759	1.2	0.5125	0.4795	4762	22.32	+1.2	0.788	+2.6	0.4937	0.4607	4577	21.45	2.8	C	Reproductions of diagrams used, said to be representative
108b	Same Engine as 108a (Condensing)					"Engine Tests", No. 17b				6050.?	209.1	28.93	164.4	9864	0.6137	0.5657	0.0712	0.6849	0.826	1.116	63	0.785	−5.0	0.7202	0.6490	6402	30.60	+5.8	0.814	1.5	0.6947	0.6235	6150	29.40	+1.7	E	Reproductions of diagrams used, said to be representative
109	Corliss, 2 Cylinders (1 Condensing)	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 15	23 x 60	$3\frac{3}{8}$	3.0	13187	615.1	21.44	61	3660	3.602	3.140	0.080	3.682	0.853	1.149	80	0.853	+0.0	3.682	3.602	13187	21.44	+0.0	0.854	+0.1	3.677	3.597	13160	21.40	0.2	C	Reproductions of diagrams used, said to be representative
110	Corliss, 1 Cylinder (1 end Condensing)	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 4	34 2 x 60	$4\frac{7}{8}$	3.0	11613	523.13	22.19	53.3	3198	3.633	2.835	0.093	3.726	0.761	1.071	74	0.749	−1.6	3.785	3.692	11810	22.57	+1.7	0.758	−0.1	3.740	3.647	11655	22.28	+0.4	E	Reproductions of diagrams used, said to be representative
111	Corliss, 2 Cylinders (1 Condensing)	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 29	28 x 60	4	2.5	14295.7	562.1	25.43	60.27	3616.2	3.953	2.860	0.250	4.203	0.680	1.048	68	0.709	+4.3	4.033	3.783	13685	24.33	−4.3	0.729	+7.2	3.922	3.672	13280	23.62	7.1	C	Reproductions of diagrams used, said to be representative
	COMPOUND CONDENSING ENGINES—H. P. CYLINDERS																																				
151	Corliss Cross-Compound	Watts Campbell Co	Thos. Oakes & Co., Bloomfield, N. J.	G. A. Oakes, W. M. White	1-20-03	Makers	20 x 36 x 30	3	2.5	6710	433.5	15.00	75.83	4550	1.481	1.244	0.1012	1.582	0.786	1.079	100	0.777	−1.1	1.600	1.4988	6820	15.73	+1.2	0.767	2.4	1.621	1.5198	6915	15.95	+2.6	C	Blue-print of tracing furnished and was representative
152	Buckeye Cross-Compound	Buckeye Engine Co	Dayton & Troy El. Ry. Co., Tippecanoe City, Ohio	Sargent & Lundy	5-1-02	Makers	$18\frac{3}{4}$ x 36 x 36	$3\frac{1}{2}$	4.3	8675	639.2	14.20	123.3	7398	1.173	1.023	0.1490	1.322	0.774	1.060	120	0.749	3.2	1.366	1.2170	9005	11.09	+3.8	0.744	3.9	1.375	1.226	9070	14.19	+4.6	C	Blue-print of tracing furnished and was representative
153	Corliss Cross Compound	Geo. H. Corliss	Pawtucket Water Works, Pawtucket, R. I.	Wm. Kent, D. S. Jacobus	5-15-91	Trans. A. S. M. E. XIII, 176	15 x $30\frac{1}{8}$ x 30	$2\frac{1}{4}$	4.0	2007	140.78	14.26	47.98	2879	0.6970	0.5492	0.0218	0.7188	0.764	1.047	130	0.728	1.7	0.7542	0.7324	2109	14.97	+5.1	0.728	−1.7	0.7542	0.7324	2109	14.97	+5.1	C & E	Reproductions of diagrams used, said to be representative
	TRIPLE EXPANSION CONDENSING ENGINES I. P. CYLINDERS																																				
201	Corliss (slow speed pumping engine)	Not ascertained	National Transit Co., Lake-ton, Ind.	J. I. Denton	1-5-93	Trans. A. S. M. E. XIV, 1316	$24\frac{1}{8}$ x 34 x 54 x 36	$1\frac{1}{2}$	2.10	1622	328.05	14.09	27.66	1659.6	2.787	1.432	0.031	2.818	0.508	0.914	38.5	0.470	7.5	3.048	3.017	5003	15.25	+8.2	0.502	+10.6	2.550	2.519	4180	12.71	9.6	C	Reproductions of diagrams, said to be representative. Speed and Cut-off pressure, outside the limits of investigation.
	HIGH SPEED SINGLE-VALVE, NON CONDENSING ENGINES																																				
401	Piston valve, Single Cylinder	A. L. Ide & Sons	Not ascertained	Not ascertained	Not ascertained	Makers	16 x 16	$2\frac{5}{8}$	13.15	4820	184.6	26.35	253.4	15204	0.3196	0.3060	0.0948	0.4144	0.733	1.027	110	0.710	3.4	0.4310	0.3362	5115	27.70	+6.1	0.763	1.9	0.4352	0.3404	5178	28.02	+7.4	C	Blue-prints of tracings furnished, said to be representative
402	Single valve, Single Cylinder	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 13	$14\frac{1}{8}$ x 14	$2\frac{1}{4}$	10.0	1740	53.26	32.67	246	14760	0.1179	0.1107	0.0613	0.1792	0.668	0.970	85	0.637	4.6	0.18[illegible]	0.1206	1870	35.10	+7.5	0.631	5.5	0.1898	0.1285	1892	35.61	+9.0	C	Reproductions of diagrams used, said to be representative
403	Single valve, Single Cylinder	Not ascertained	Not ascertained	Geo. H. Barrus	Not ascertained	"Engine Tests", No. 21	$14\frac{1}{2}$ x 13	$2\frac{1}{4}$	12.0	2018.6	61.7	32.71	248.4	14904	0.1353	0.1314	0.0594	0.1947	0.675	0.985	74	0.641	−5.0	[illegible]	0.1150	2170	35.19	+7.5	0.650	3.7	0.2021	0.1427	2158	34.48	+5.1	E	Reproductions of diagrams used, said to be representative
	SIMPLE LOCOMOTIVE ENGINES																																				
451	Two Cylinder Slide Valve	Schenectady Loco. Works	Purdue Univ. Laboratory, Lafayette, Ind.	W. F. M. Goss	7-3-05	"High Steam Pressures in Locomotive Service" No. 20-8-120	16 x 24	$2\frac{1}{4}$	H C; R 7.44 7.98; L 7.34 7.63	7180	252.79	28.40	97.08	5824.8	1.232	1.0164	0.1947	1.4267	0.711	1.003	100	0.683	3.9	1.188	1.2963	7540	29.83	+5.0	0.673	5.3	1.5110	1.3163	7675	30.38	+6.9	C	Original diagrams furnished as representative

Analysis 201 shows an application the conditions of speed and cut-off pressure of which are far outside of the limits examined. The values given were obtained by extrapolating as straight lines the lower portions of the curves of constant value of x_c in the chart of Fig. 6. Although the speed is only 27.66 r. p. m. and the cut-off pressure only 38.5 lb. absolute, the value of x_c by chart was determined as 0.470, while the value by test is 0.508, a difference of 7.5 per cent based on the test value of x_c. This application is given to show that an extrapolation of the method to unusual conditions of speed and cut-off pressure does not lead to absurd results, although it is not nearly as accurate as the applications to speeds higher than 50 r. p. m.

The results of the applications made up to the present time, with the restricting conditions imposed, tend to show that the steam consumption of engines may be approximated from the indicator diagram to within an average difference of less than 4 per cent from the test results. Individual examples, however, may show as much as 8 per cent difference in rare cases.

16. *Directions for Applying the Method to Engine Testing.*—The manner of applying the method to the class of engines tested is best illustrated by taking an actual case and tracing the steps necessary to a determination of the steam consumption. It is to be remembered that the method accounts for the actual weight of steam and water present in one revolution only as represented by the set of diagrams analyzed.

The set of diagrams to be analyzed is selected in different ways, according to the test conditions. If the load on the engine to be tested is fairly uniform, and if the average steam consumption over a period of time be desired, one set of diagrams taken simultaneously is selected after the manner described in detail in Appendix 3, p. 97. This method briefly is as follows: all diagrams taken over a period of time at equal intervals are integrated and the mean effective pressure of each diagram obtained. The combination of one set of diagrams is sought nearest to the average mean effective pressure and taken at the average steam pressure. This set is taken to represent the mean condition of power and is the set to be analyzed. If the load is extremely variable, the diagrams must be separated into groups of similar cut-off values, and one set from each group analyzed as outlined for the uniform load condition.

If it be desired to obtain a water-rate curve, the engine is operated under various loads, ranging from no-load to full-load,

and one set of diagrams, taken simultaneously is obtained at each load. Each set is analyzed and the result used to obtain the usual water-rate curve. In this case, no average diagrams are necessary as only the steam consumed at each load is desired.

It is now assumed that the set of diagrams to be analyzed is selected, that they are taken from a single-cylinder non-jacketed non-condensing Corliss engine, 28½in. x 59½ in., the engine selected as an example being analysis 102 in Table 4.

Logarithmic diagrams of both indicator diagrams are constructed as described in detail in Appendix 1. The average value of n from the expansion curves of the head-end and crank-end diagrams is then found to be 1.108, and the average cut-off pressure 75 lb. per sq. in. absolute.

The next step is to examine Fig. 6 and locate the intersection of the vertical line for the value of $n=1.108$ with the horizontal line for the value of the absolute cut-off pressure of 75 lb. This intersection is seen to be half-way between the "lines of constant quality" of 0.79 and 0.80, giving a value of 0.795. This means that the steam accounted for by the indicator as being present in the clearance and displacement spaces up to cut-off is present at a quality of 0.795, or that it contains water to the amount of 0.205.

From the logarithmic diagram, it is found that the head-end of the cylinder contains 8.57 cu. ft. of steam at the average cut-off pressure of 75 lb. absolute, while the crank-end contains 7.83 cu. ft., making a total of 16.40 cu. ft. From the Marks and Davis steam tables, the specific value of steam at 75 lb. pressure is 5.81 cu. ft. per lb., thus accounting for 2.822 lb. of dry steam. This weight at the quality of 0.795 equals 3.551 lb., the total weight of steam and water present per revolution. In a similar manner, from the logarithmic diagram, as described in Part II, chapter X, it is found that the total weight of dry steam retained in the compression in both ends equals 0.167 lb. This compression steam is present in the amount accounted for at cut-off so that the net weight of steam passing through the cylinder per revolution is 3.551—0.137, or 3.384 lb. As the engine is running at 64.8 revolutions per minute, or 3888 revolutions per hour, the total steam consumed per hour equals 3888 x 3.384 = 13150 lb. From the diagrams, the indicated horse-power is found to be 506.5, thus the steam consumption is determined at 25.96 lb. per i. h. p. hour, while the condenser test of this engine from which these diagrams were taken shows a steam consumption of 25.80 lb. per i. h. p. hour, a difference of 0.6 per cent.

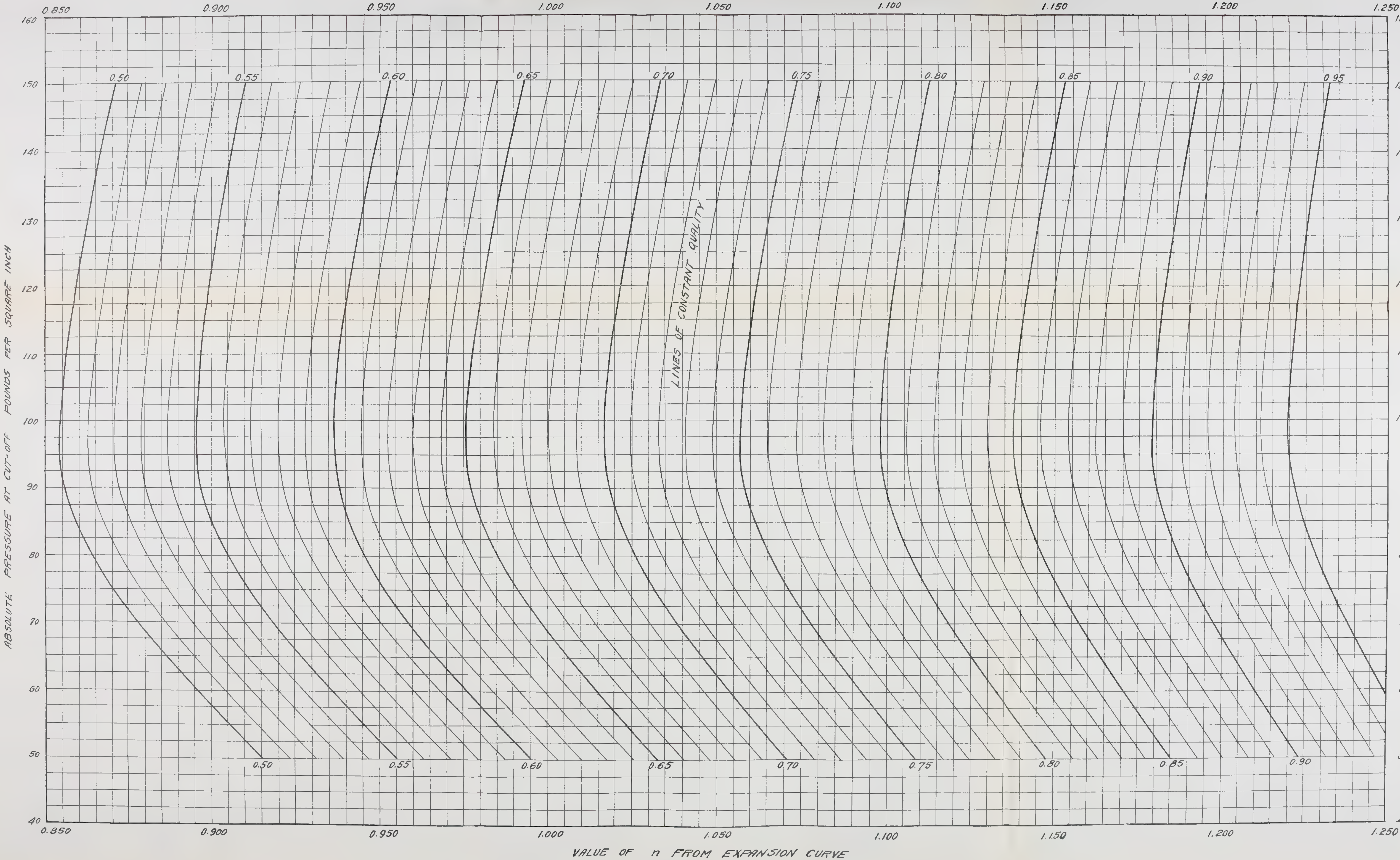


Fig. 6 Chart Showing the Relation between Pressure at Cut-off and Values of n for Constant Quality at Cut-off

IV. CONCLUSIONS

The following conclusions have been drawn from the results of this investigation, as applicable to non-jacketed steam cylinders in good condition exhausting at or near atmospheric pressure, and with the limitations imposed as given in Part I.

1. At a given initial pressure and speed of engine, there is a definite relation existing between x_c and n, in any one cylinder, which is practically independent of the cut-off position.

2. This relation is practically independent of cylinder size and of engine speed; it is therefore applicable to other cylinders of the same type.

3 By means of the experimentally determined relations of x_c and n, the value of x_c may be approximated from the average value of n obtained from the expansion curves of one set of indicator diagrams, taken simultaneously; therefore the actual weight of steam present in one revolution may be approximated.

4. The actual steam consumption may be obtained by this method from the indicator diagram to within an average of 4 per cent of the amount consumed as measured by test.

5. This method has the following advantages not possessed by tests: it is more accurate than the average test, and is the most accurate method available for testing certain classes of engines; it virtually measures an instantaneous rate instead of an average quantity over a long time, and thus enables a large number of points to be obtained for a water-rate curve; it permits of making tests at frequent intervals instead of once in the engine's life; the expense is not to be compared with that of an equally accurate test; it involves no change in the routine of the plant tested.

PART II. THE LOGARITHMIC DIAGRAM APPLIED TO ALL ELASTIC MEDIA

V. THE LOGARITHMIC DIAGRAM

17. *The Indicator Diagram Plotted on Logarithmic Cross-section Paper.*—The logarithmic diagram is obtained by transferring the indicator or PV-diagram to logarithmic cross-section paper. The method of transfer is as follows. The coordinates of the PV-diagram are proportional to pressure and stroke, the latter being proportional to the volume displaced by the piston. The coordinates of from 10 to 30 points on the PV-diagram are found in terms of

absolute pressures, preferably in lb. per sq. in., and of absolute volumes, preferably in cu. ft. These points are plotted on logarithmic cross section paper and are connected by a smooth curve, forming a figure which will be called the logarithmic diagram. Fig. 9 shows the logarithmic diagrams derived from the *PV*-diagrams of Fig. 15.

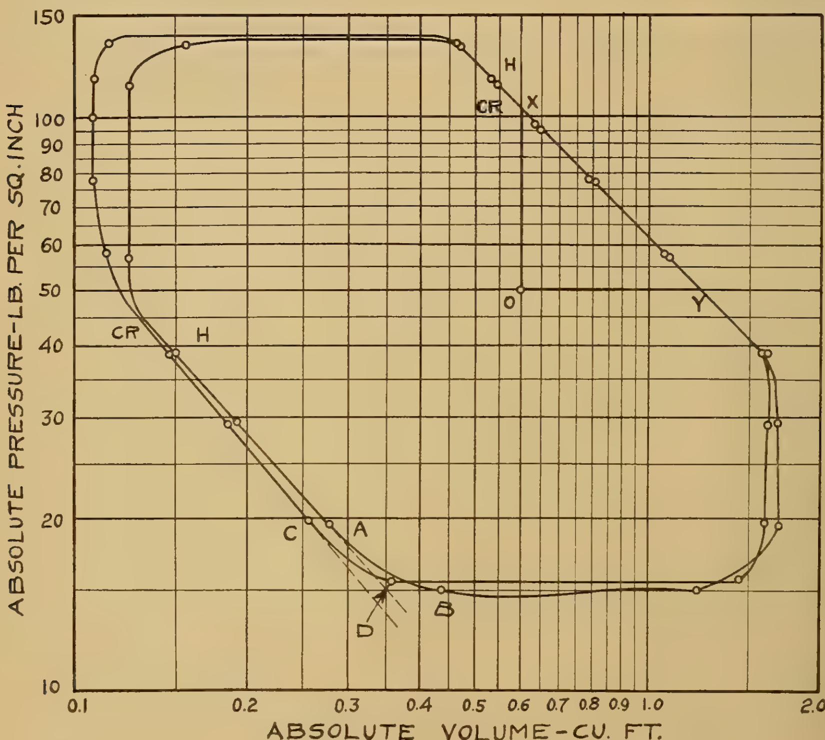


FIG. 9. LOGARITHMIC DIAGRAMS PLOTTED FROM FIG. 15

18. *The Form of Expansion and Compression Curves from Practice.*—About 300 *PV* diagrams from engines using steam, gas, air, and ammonia have been examined to investigate the form and character of the expansion and compression curves. As a result, it may be stated that, free from certain abnormal influences, expansion or compression of an elastic medium takes place in the cylinders of reciprocating engines substantially according to the law, $PV^n = C$.

19. *Mathematical Relations of the Law,* $PV^n = C$.—The equation of the polytropic curve, $PV^n = C$, when plotted on rectangular cross-section paper, gives a curve depending for its form and

position on the values of P, V, n, and C. When this curve is plotted on logarithmic paper, it becomes a straight line, depending for its slope on the value of n and for its position upon the value of C. The relations for such curves follow.

Given

$$PV^n = C$$

Taking the logarithm of both terms

$$\log P + n \log V = \log C$$

Transposing

$$\log P = -n \log V + \log C \ldots\ldots\ldots (1)$$

This equation is of the form of the straight line

$$y = mx + b$$

where

$$y = \log P$$
$$m = -n$$
$$x = \log V$$
$$b = \log C$$

Thus $m = -n$, the slope; or measure of inclination of the line to the axis, $\log V$. In Fig. 9, for example, at a point X on the line

$$\log P = -n \log V + \log C,$$

draw OX parallel to the axis $\log P$, and draw OY parallel to the axis $\log V$. The slope of the line will be the value of the ratio $\frac{OX}{-OY} = -n$. OY is negative, being measured to the left, giving n its negative sign.

20. *Use of the Logarithmic Diagram.*—The logarithmic diagram forms the basis of the methods of analyzing the cylinder performance of reciprocating engines which are developed in detail in the following pages. These methods apply only, however, to the logarithmic diagrams derived from the cylinders of reciprocating engines using an elastic fluid for the working medium, and having, as a part of the cycle of operation, an expansion, a compression, or both. The figures of one set of indicator diagrams and the corresponding set of logarithmic diagrams are numbered the same, but the letters a and b are used in addition to the figure number to denote the indicator and logarithmic diagrams, respectively.

VI. Rational Method of Approximating Clearance

21. In the cases of the great majority of the PV–diagrams which were examined, the expansion and compression curves became straight lines in the logarithmic diagram, showing that the law $PV^n = C$ was applicable, or in other words, that n was a constant for one curve. The clearances furnished with the diagrams examined had been carefully found by the displacement method.

It was desired to see what forms the lines assumed when the clearance was taken larger or smaller than the measured quantity. The diagram shown in Fig. 10*a*, taken from a 42 in. x 60 in. gas engine, was used for this purpose. The true clearance, measured as 18.0 per cent, was used in the full logarithmic diagram of Fig. 10*b*. Trials were made with clearances assumed as 14.0, 16.0, 20.0 and 22.0 per cent of the piston displacement. With the true clearance of 18.0 per cent, the curves became almost perfectly straight lines, while with the values of clearance less than 18.0 per cent, it is seen that the lines become bent to the left, and with values of over 18.0 per cent, the lines become bent to the right. Hence the straight line for the value of 18.0 per cent is the transition between the family of curves bending to the left, representing a clearance smaller than the real value, and the family of curves bending to the right, representing a clearance larger than the real value.

The practical significance of this fact is that there is now available a rational method of approximating the clearance of any cylinder using an elastic medium, which has, as a part of the cycle of operation, an expansion or a compression. This method is based on the fact, already mentioned, that, in practice, all elastic media, except under certain exceptional conditions, obey substantially the law $PV^n = C$, when subject to change of state, and therefore become straight lines in the logarithmic diagram.

22. *Graphical Method of Approximating Clearance.*—The graphical method of approximating clearance requires only the scale of the indicator spring to be known, and the atmospheric line to be drawn, in order to locate the zero line of pressure. The exact order of procedure necessary to make a trial, and the degree of accuracy obtained in any given case, is shown in detail in page 35 for a 25⅝ in. x 37⅜ in. gas engine. All that is necessary is to assume different values of clearance, and to plot the logarithmic diagram for each assumed value. The straight-line position of the curves is found by trial and error, to lie between the two di-

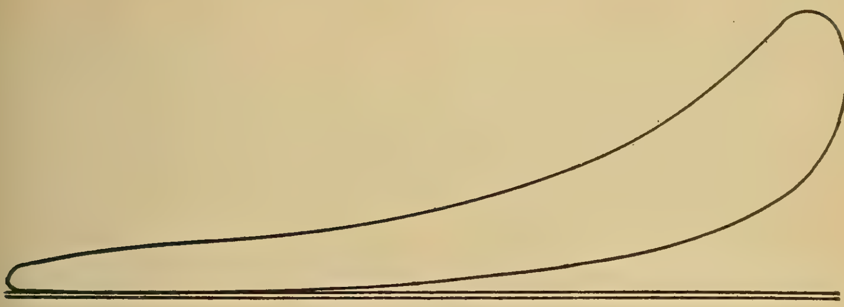


FIG. 10*a* (SCALE—160 LB.)

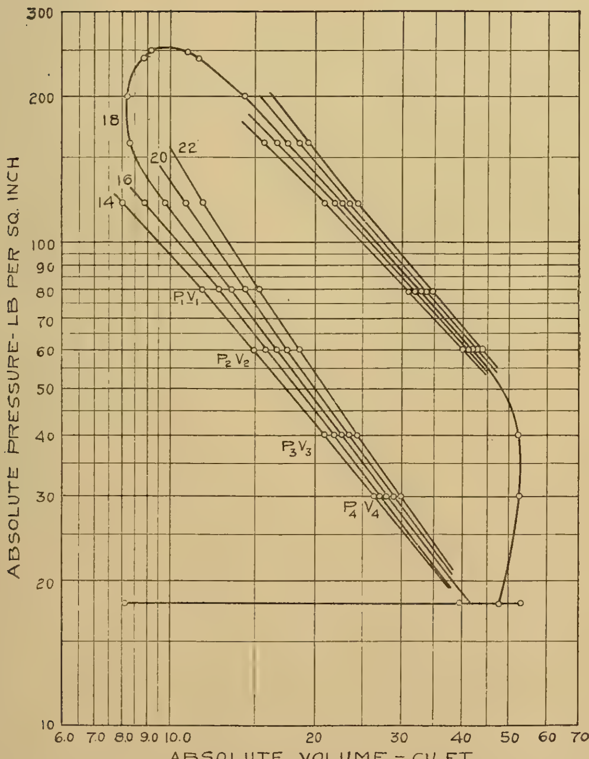


FIG. 10*b*. TOD 42-IN. X 60-IN. GAS ENGINE, BLAST FURNACE GAS

verging families of curves representing too small and too large clearance.

It follows also from the curves shown by Fig. 10*b* that the clearance being known, the scale of the spring used may be obtained in the same manner if the atmospheric line is given.

23. *Mathematical Method of Approximating Clearance.*—The results obtained from the graphical method of trial and error may also be accomplished by the use of the purely mathematical process upon which the method depends.

When the law $PV^n = C$ holds, n is a constant for any part of the curve. When the wrong clearance is used, the law $PV^n = C$ does not hold, and n varies from point to point. In the graphical method, trials of various values of clearance are made, until the curve becomes approximately a straight line; this resultant straight line is, therefore, the law $PV^n = C$, in which n is a contant for all parts of the curve. The one condition necessary to be fulfilled, therefore, is that n be constant for all parts of the curve, but not of any particular value.

24. *Application of the Mathematical Method.*—To illustrate the use of the mathematical method in Fig. 10*b*, let us assume several points, P_1V_1, P_2V_2, P_3V_3, and P_4V_4 at various intervals on one of the curves, as on the compression curve at the clearance value 14.0 per cent. It is desirable for convenience to locate the points at about equal intervals as shown. The law $PV^n = C$ is assumed to hold. Then, for two points, P_1V_1 and P_2V_2, called group *a*, we have

$$P_1V_1^n = C$$
$$P_2V_2^n = C$$

Equating these, we obtain

$$P_2V_2^n = P_1V_1^n$$

Transposing and dividing

$$\left(\frac{V_2}{V_1}\right)^n = \frac{P_1}{P_2}$$

Taking logarithm of both sides

$$n(\log V_2 - \log V_1) = \log P_1 - \log P_2$$

whence

$$n_a = \frac{\log P_1 - \log P_2}{\log V_2 - \log V_1} \quad \text{.........................} (1)$$

In the same manner for the points P_3V_3 and P_4V_4 called group *b*, we obtain

$$n_b = \frac{\log P_3 - \log P_4}{\log V_4 - \log V_3} \quad \text{.........................} (2)$$

For the correct value of clearance, the following condition must be fulfilled by trial and error

$$n_a = n_b \qquad (3)$$

The values of the logarithms of the coordinates of all points are then found, and the values of n_a and n_b computed. If the points are located in the order shown, then with too small a clearance, n_a is lower in value than n_b. A larger value of clearance is then assumed, the operation being merely to add a constant number to the values of V_1, V_2, V_3, and V_4. The process is repeated until the value of n_a becomes practically equal to n_b. When the clearance is assumed too large, n_a becomes higher in value than n_b, indicating that the true value has been passed.

25. *Comparison of the Two Methods.*—The trial by the mathematical method is neither as accurate nor as short as the graphical method. It is not as accurate because the points assumed may not be representative. When this is the case, the graphical method allows judgment to be exercised in selecting the straight-line position, thereby eliminating irregularity of points.

The question arises as to whether the form of the lines due to wrong clearance can be distinguished from the form due to leakage or to "hooks", on the logarithmic diagram. This case is treated in chapter IX.

The curve of the *PV*-diagram nearest the clearance space, or the compression curve in Fig. 10*b*, is generally the better guide in the graphical trials. This is well shown in Fig. 10*b*. A given difference in the values of clearance used for trial causes more horizontal variation in the position of the compression curve than in the expansion curve. This fact allows closer locations of the straight-line transition region to be made from the compression curve than from the expansion curve.

26. *Examples.*—It was desired to determine the clearance of the diagram shown in Fig. 11*a*. From general knowledge of this class of engines, a trial by the graphical method was made in Fig. 11*b* with the clearance assumed as 12.4 per cent, a value purposely assumed as being too small. This value is seen, by the bending of both curves to the left, to be much too small. Trials were, therefore, made with the clearance assumed as 13.8, 15.1, and 16.0 per cent of the piston displacement. The values of 15.1 per cent gave practically perfect straight lines for both the expansion and compression curves, while the value of 16.0 per cent shows that the lines have begun to bend to the right, indicating too large a clearance. By inspection, it will be seen that the re-

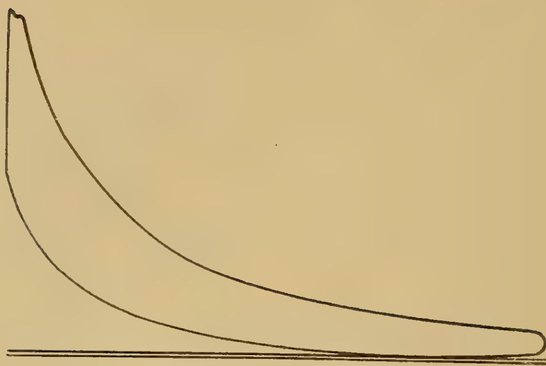


FIG. 11*a*. (SCALE—180 LB.)

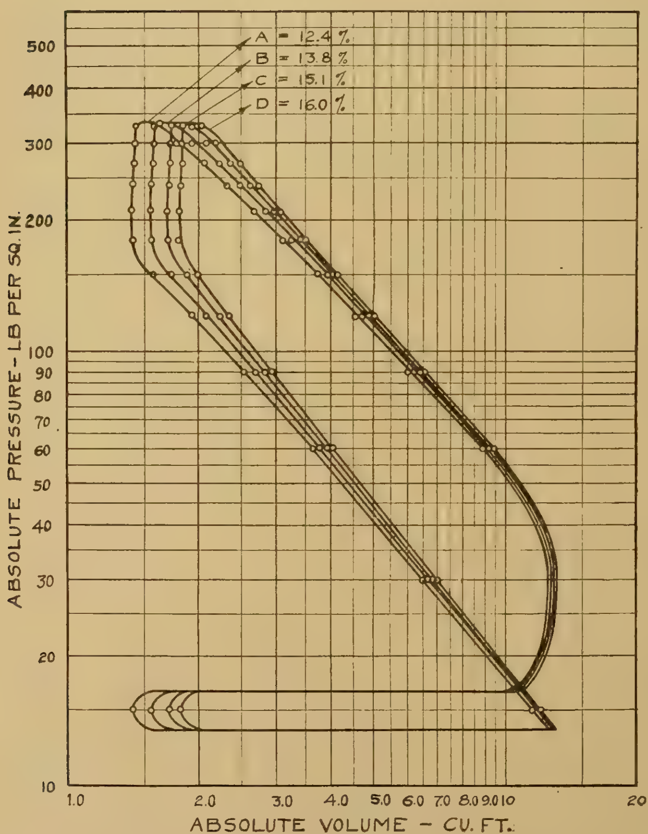


FIG. 11*b*. KOERTING FOUR-CYCLE 25⅝-IN. X 37⅝-IN. PRODUCER GAS ENGINE

gion of fairly straight lines may be located as lying between the values of about 14.5 per cent to 15.5 per cent. The clearance is, therefore, selected as 15.0 per cent, a value which may be high or low by not more than 4 per cent in this case. This clearance value, 15.0 per cent, is a common value for engines of this class.

The graphical method is more accurate for large clearances, measured in per cent of the piston displacement, than for small ones. The closeness of location of the straight-line region, lying between the two families of diverging curves, will be found to be within 5 per cent to 10 per cent of the clearance volume, for values of clearance between 20 per cent and 2 per cent, respectively, of the piston displacement.

VII. Rational Method of Locating the Stroke Position of Cyclic Events

27. It is often desirable to know at what part of the stroke the cyclic events occur. This knowledge can be best obtained from the *PV*-diagram. For ordinary purposes, these events can be closely located in most cases by inspection on the *PV*-diagrams themselves; thus, on a diagram from a Corliss engine, cut-off may generally be located to within 1-16 in., measured along the length of the diagram.

The actual beginning of true compression, however, can never be accurately located on the *PV*-diagram. True compression, unaffected by leakage, begins after the exhaust valve, in closing, has acquired enough seal to prevent leakage. The point of the beginning of true compression is generally at least 5 lb. above the back pressure. The point at which leakage ceases cannot be located on the *PV*-diagram because the curve of true compression, and the curve during the time the valve has insufficient seal, are of the same direction of curvature, and are not reverse curves as in the general case of admission and expansion.

The fact that expansion and compression of a constant weight of medium take place according to the law, $PV^n = C$, thus becoming straight lines in the logarithmic diagram, enables us to locate cyclic events very closely, even in cases where they cannot be detected at all in the *PV*-diagram.

28. *Application.*—An example is shown in Fig. 23a, containing locomotive *PV*-diagrams taken at short cut-off and high speed. The events of cut-off, release, compression, and lead are very difficult to locate on such diagrams. These events

are located on the logarithmic diagram in Fig. 23*b* by noting when the expansion and compression curves become straight, indicating a constant weight of steam mixture.

A sufficient number of points are plotted to show clearly the direction of the diagram near the events desired. Thus these events, even though obscure in the *PV*-diagram, may be located to well within about 1–16 in. in the logarithmic diagram, this length being equivalent to about 1-32 in. when re-transferred to the *PV*-diagram itself.

The use of this method has one great advantage in that it largely eliminates the variable element of personal judgment. It is a common occurrence to see *PV*-diagrams where two persons have located an event such as cut-off, $\frac{1}{8}$ in. apart, each location being the best judgment of the person doing the work. The logarithmic diagram will at all times give closer locations of events for these reasons than will the *PV*-diagrams.

The method also allows the point of true compression to be located, the location of which is practically impossible in the *PV*-diagram.

VIII. Rational Method of Detecting Leakage

The law $PV^n = C$ is applicable only to cases where the weight of the working medium remains practically constant during any expansion or compression. When this weight changes materially, either by leakage into, or out from, the cylinder containing the medium, the resulting expansion or compression no longer obeys the law, and it becomes a curve on logarithmic cross-section paper. This fact is very clearly shown in the curves of the logarithmic diagram derived from cylinders in which large leaks were known to exist.

29. *Examples of Known Leakage. (a) Gas Engine.*—The first case, shown in Fig. 12*b*, occurred in a 10 in. x 19 in. gas engine, intended for producer gas, but using illuminating gas at high compression. The piston, a single-acting trunk type, allowed a large leak, clearly detected by the noise of escaping gas, at the beginning of the stroke. Both the compression and expansion curves show the effect of this leak in a clear manner when transferred to the logarithmic form. After that portion of the stroke was reached where no sound of leakage was heard, the two curves became straight lines. This indicated very clearly that the effect of leakage, if appreciable, may be detected in the

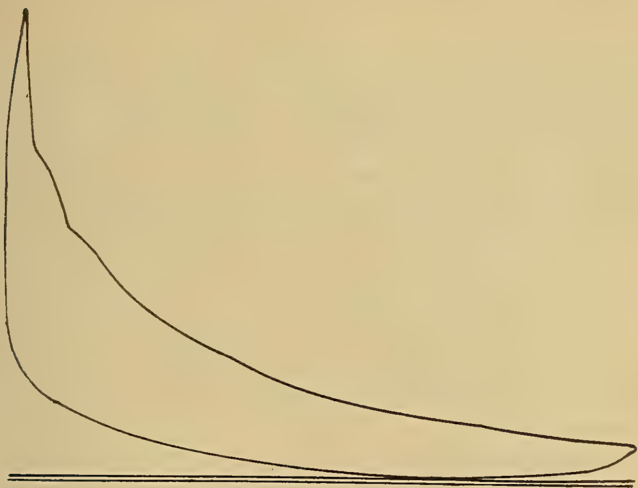


FIG. 12*a*. (SCALE—190 LB.)

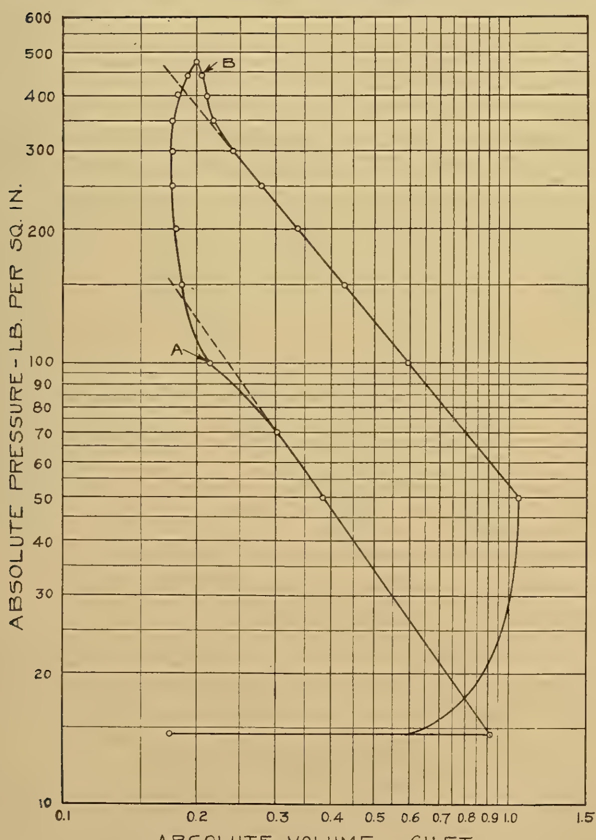


FIG. 12*b*. 10-IN. X 19-IN. GAS ENGINE (ILLUMINATING GAS)

form of the curves of the logarithmic diagram.

(*b*) *Steam Engine.*—The second case, shown in Fig. 13*b*, is from a 14 in. x 35-in. Corliss engine. The knowledge of the leaky condition of the piston and valves came from the engineer in charge.

The expansion and compression lines indicate by their form at the upper ends, a large leak from the cylinder, or through the exhaust valve. The lines also show, by the rising of the curves at the lower ends, a considerable addition to the steam in the cylinder during expansion and compression. This steam could come only from a large leak in the steam valve. The seven other diagrams taken from this same engine all show the effect of leakage in a similar manner.

(*c*) *Ammonia Compressor.*—The third case of known leakage, shown in Fig. 14*b*, is from an 11 ¼ in. x 22-in. double-acting ammonia compressor. This cylinder was known to be in very bad condition as regards wear and leakage of piston and valves. The re-expansion curves, by the enormous amount of re-expansion shown, indicate large leakage into the cylinder during this operation. The lower part of the compression curves, by rising, indicates leakage into the cylinder, either past the piston or through the discharge valves. The upper part of these curves indicates leakage from the cylinder, either past the piston or through the suction valves. These three examples show abnormal conditions which are comparatively rare.

Very smooth curves may be obtained in the *PV*-diagram even if there is large leakage taking place. This is seen by referring to Fig. 13*a*, both the expansion and compression curves being fairly regular. The logarithmic diagram, however, shows clearly, in connection with the discussion and the examples shown, that large leakage of two kinds was taking place during expansion and compression. Leakage which occurs during admission or during exhaust has no effect upon the lines of the diagram as the weight of the medium is continually changing.

30. *Method of Detecting Leakage.*—It is seldom found, when leakage occurs in a cylinder, that only one source of leakage exists. Leakage is usually the result of wear, which affects most of the possible sources of leakage in about an equal proportion. As a result, several leaks are generally affecting the curves. This is the case in Fig. 13*b* and 14*b*. In Fig. 13*b*, leakage was taking place both into and out from the cylinder.

In discussing leakage, it must be kept in mind that difference

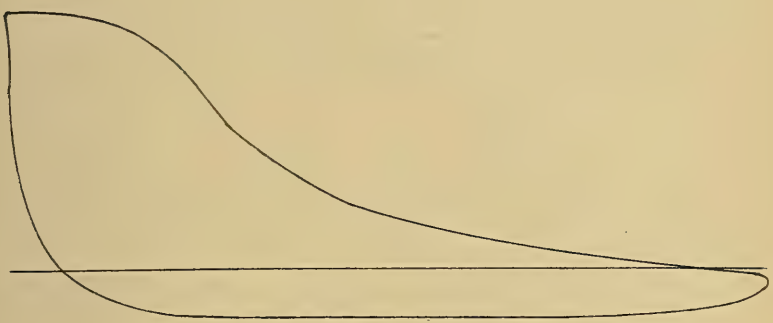


FIG. 13a. (SCALE—40 LB.)

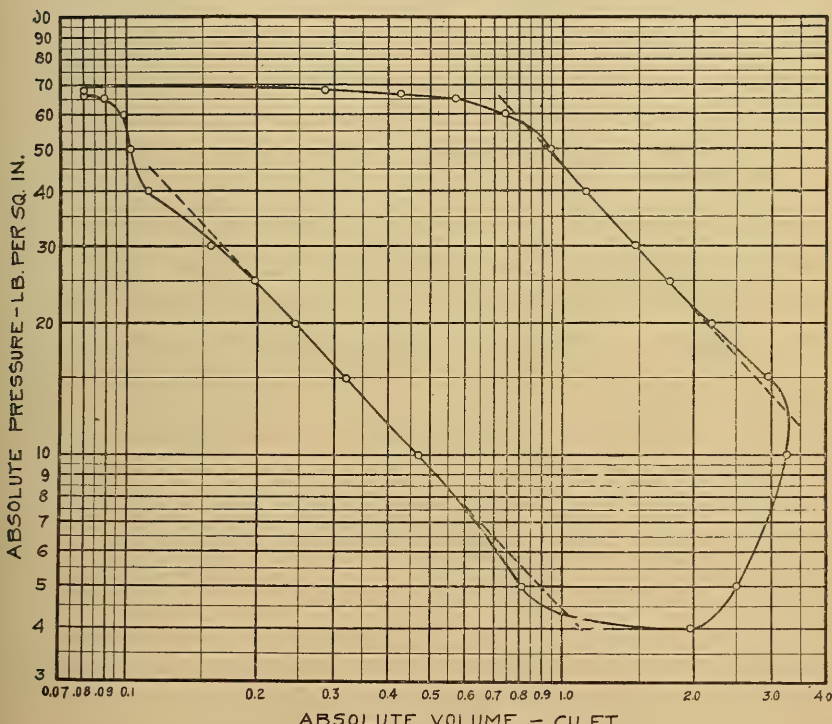


FIG. 13b. 14-IN. x 35-IN. CORLISS ENGINE

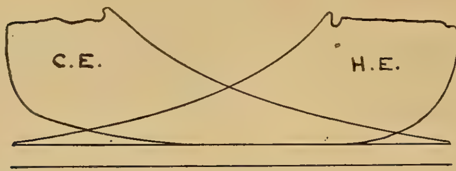


FIG 14a (SCALE—168 LB.)

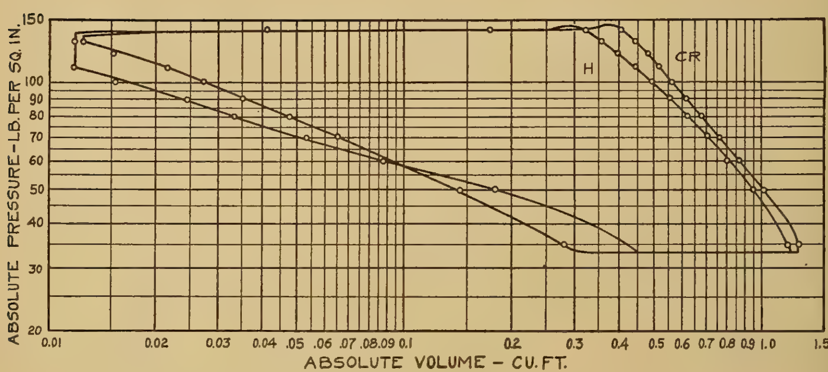


FIG. 14b. 14½-IN. x 2 -IN. AMMONIA COMPRESSOR

in pressure between two regions is the cause of this phenomenon. In the steam engine, there are three pressures which must be considered, i. e., the pressure in the steam chest, in the cylinder at the point discussed, and in the exhaust passage. Leakage, being due to difference of pressure, becomes material only when this difference becomes considerable. Thus leakage into, or out from a steam cylinder has been found to occur, in most cases, only when the pressure difference is over about 20 lb. In Fig. 13b, the leakage into the cylinder, shown by the lower parts of the lines, begins to occur at about 25 lb. absolute, or 35 lb. lower than the pressure at admission. The leakage out from the cylinder, shown by the upper parts of the lines, ceases to occur at a pressure of about 40 lb. absolute for the expansion curve, and begins to occur at about 25 lb. absolute for the compression curve. The difference of pressure between the steam in the cylinder and that in the exhaust passage is about 35 lb. in the first case, and about 20 lb. in the second case.

31. *Division of the Lines of Expansion and Compression.*—This fact, found on many diagrams analyzed, enables us to divide

the expansion and compression lines roughly into three equal parts on the logarithmic diagram (when these lines extend from the initial pressure to nearly the back pressure): (1) the upper third, influenced by leaks *out from* the cylinder; (2) the middle third, practically *uninfluenced* by leakage; (3) the lower third, influenced by leakage *into* the cylinder. Thus fairly reliable values of n, free from the effect of leakage, may be obtained from the middle third of the lines.

Returning to Fig. 13*b*, both the lines indicate leaks out from the cylinder. This can occur either past the piston or through the exhaust valves. The piston generally becomes leaky sooner than Corliss exhaust valves, and, in this particular engine, one of the piston rings was found to be broken upon examination. When diagrams from both ends of the cylinder are available, piston leakage causes nearly an equal effect on the expansion-curves of both ends. The leakage into the cylinder can come from only one source which can influence the curves, i. e., the steam chest. The effect of this leak is seen in both lines in the lower thirds.

In Fig. 14*b*, both forms of leakage are shown in the ammonia compression curves. The lower thirds show leakage into the cylinder, either through the discharge valves or past the piston. The upper thirds of the lines show large leakage from the cylinder, caused by the condition of either the suction valves or the piston. Ordinarily, it is not possible to distinguish between two leaks occurring in the same third of the curves.

32. *Approximation of the Volume That Leaked.*—Fig. 12*b* is an example where only one kind of leakage is present. Here, the piston alone leaked badly at the commencement of the stroke. The effect of this leak is seen in the upper third of both lines.

When only one kind of leakage exists, it is possible to compute with fair accuracy the volume of leakage taking place during expansion or compression. The lines are extended, as shown in Fig. 12*b*, giving the lines of constant weights of the medium. The volume of gas that had leaked during compression, up to 100 lb. absolute pressure, is then seen to be 0.014 cu. ft., or 6.3 per cent of the volume remaining. The volume of gas, measured at the pressure of 450 lb. absolute, that leaked after combustion during expansion, is seen to be 0.032 cu. ft., or 18.7 per cent of the volume remaining after the leakage stopped.

The leakage that took place during combustion at the end of

the stroke cannot be computed, but it can be estimated by making the assumption that this leakage was proportional to the mean rate of leakage shown by the two curves, and that its duration was the time interval occurring between the point *A* and the point *B*.

The important result that is attained by this method is not, however, the approximation of leakage, but the knowledge that it is taking place, so that it can be located and stopped.

33. *The Use of the Method in Testing for Maximum Economy.*—Many engines are sold and their prices fixed on the basis of their test performances. The importance to the manufacturer of being able to eliminate leaks during this test does not have to be emphasized. The engineer in charge of the test should know whether or not the engine is tight under regular operating conditions. All of our present knowledge of leakage is an inference drawn from the leakage "standing". Nobody knows whether an engine that is tight "standing" leaks when in operation, or vice-versa. This method should be applied to all engines about to undergo any test where maximum economy is the object desired.

34. *Knowledge as to When General Repairs of the Cylinder and Valves Are Necessary.*—Leaks are caused by wear, poor design, and accidents. The accidents include scoring of cylinders and valves, cutting of valves, and cracks in the cylinder. Most leakage is the result of wear and tear due to long and hard use. After the wear and tear has become marked, it is the custom to rebore the cylinder and to resurface the valves and valve seats.

Several methods are in use for determining when general repairs are necessary for steam cylinders. One of these methods is to judge the time from the general appearance of the parts on inspection. Cylinders are rebored by some engineers when they have worn "out of round" by a given amount. In small plants, the most general method seems to be to wait until the leakage is so large as to become clearly noticeable either by the reduced capacity of the unit, or by the effect upon the coal pile. Some railroad companies overhaul the cylinders and valves of locomotive cylinders at regular intervals of, for instance, 150 000 miles of travel. Some cylinders in stationary plants are rebored at equal time intervals of some four or five years each.

As a pure question of economy, other things not considered, general repairs of cylinder and valves should take place when the extra annual cost of fuel and water due to leakage equals the

annual interest on the money necessary for general repairs. The total cost of general repairs is composed of several items: the actual cost of the repairs, the cost of losing the unit from service, the interest on the cost of the extra capacity that may have to be installed to take the load when units are out of service, and the interest on the money invested in the unit out of service.

The existing methods of determining when general repairs are necessary are not standardized as regards economy, and the personal judgment of the man making the decision may be liable to great variation. The method of detecting leakage from the logarithmic diagram offers a more rational solution of this important question.

35. *Leakage of Engines.*—The results of an analysis made in connection with this investigation by means of logarithmic diagrams of 296 *PV*-diagrams taken from the cylinders of 47 engines indicate that the majority of engines, in good condition, are practically tight as regards leakage into or out from the cylinder.

IX. Interpretation of the Logarithmic Diagram

In the discussions upon the effect of wrong clearance, leakage, and excessive condensation upon the lines of the logarithmic diagram, it was assumed, for the sake of clearness, that only one of these effects existed at one time. The examples were selected so as to illustrate only one of these effects in each case.

Cases occur where several of these effects exist at one time in the same diagram. The separation of one effect from another is not an exact process. However, the character of the curves showing excessive condensation, wrong clearance, and leakage is quite different. For instance, wrong clearance affects the lines throughout their length. Excessive condensation, in the cases of the steam diagrams examined, always affects only the upper parts of the curves. Leakage, as has been mentioned, affects materially only the upper and lower thirds of the lines, where these lines extend from the initial pressure to nearly the back pressure. When excessive condensation and large leakage exist together, no close approximation of the clearance can be made.

An adequate treatment of the segregation of these various effects, when found together, is beyond the scope of this bulletin. The treatment is long and complicated. It has been found, however, that experience in the use of logarithmic diagrams enables one to separate these effects qualitatively, in some cases, from the form of the expansion and compression curves.

The logarithmic diagram is more useful for analysis than any other form of diagram because of the natural limitations of the human mind. We do not possess the power to distinguish between curves. We are, however, able to see clearly the difference in these curves after they have been transformed into straight lines, which fact alone makes these new methods of analysis possible. We are now enabled in their straight-line form, to comprehend curves which we have always seen, but could not distinguish one from another in their original form.

X. Common Errors Made In Analyzing Steam Indicator Diagrams

36. *The Use of the Equilateral Hyperbola as a Standard of Comparison.*—The values of n for the expansion curves of steam indicator diagrams are not closely constant but are subject to a very wide range of variation. The range of variation found in the present investigation is from 0.70 to 1.34.

The range of values in the engine tested was from 0.835 to 1.234. The average values were 0.947 for the tests run with saturated steam, 1.056 for the tests run with superheated steam, and 1.004 for all tests.

The values of n for most engines of ordinary size using saturated steam at normal cut-off is between 0.95 and 1.05, while for superheated steam, the range is usually from 1.00 to 1.30. For saturated steam, the value of $n = 1.0$ is about an average value.

The explanation of the value $n = 1.0$ can be seen from the results of the tests given on pp. 11 and 20. The only meaning that the average value of $n = 1$ ever possessed is that the average value of x_c, in the class of engines examined, lies in the range between 0.60 and 0.70.

The law of Boyle or Mariotte, or the law of isothermal expansion of a perfect gas, has no bearing of any kind whatsoever on the expansion of steam in a cylinder. The equilateral hyperbola sometimes occurring in steam cylinders is only a special case of expansion according to the polytropic law $PV^n = C$, while Boyle's law is another special case which never occurs in steam engine practice.

Because of the agreement in form between Boyle's law and the equilaterial hyperbola (the special case of the law, $PV^n = C$, where $n = 1.0$), this latter curve has been called the ideal or theoretical curve of expansion to which curves in practice are

supposed to approach as a measure of practical perfection in the use of steam. The equilateral hyperbola is in no sense whatsoever an ideal or theoretical curve, and its use for the purposes of comparison is an empirical or arbitrary convention only. It should be called the conventional expansion. It has ever been contended that because an expansion curve did not coincide with the equilateral hyperbola, some grave fault exists in the engine. A value of n may be as low as 0.60 with no graver fault than very excessive initial condensation, while a value of 1.35 may be found from no graver fault than that of using steam superheated about 250° F.

The only rational use of applying the equilateral hyperbola to steam *PV*–diagrams is to act as a guide to see whether n is greater or less than 1.0. If the actual curve is not close to this hyperbola, if no faults exist, and if the cylinder is nonjacketed, then this fact means that the value of x_c for the case examined is less than about 0.60 or greater than about 0.70. The assumption that $n = 1.0$ as a standard of expansion is equivalent to assuming that the value of x_c is standard at about 0.65; however, no engineer would seriously propose that the value of x_c of 0.65 should be selected as a standard of economy. The elaborate theory of analysis built on the assumption that $n = 1.0$ is the natural result of the use of averages in any art where the actual facts have never been investigated.

The use of the equilateral hyperbola to predict the form of *PV*–diagrams for the purposes of design is satisfactory in the case of ordinary sizes of engines using saturated steam. When steam superheated over 100° F. is used, the value of n should be assumed at between 1.10 and 1.25. The high values of n obtained with highly superheated steam in large engines alter materially the division of work and the tangential forces acting from those obtained when n is assumed to be 1.0. This fact should be considered in the design of engines to use superheated steam.

The use of the equilateral hyperbola to obtain the ratio known as the "diagram factor" has no rational basis, but its use for this purpose gives results which are valuable for the purpose of design.

37 *The Graphical Method of Approximating the Clearance.* If n has the value 1.0 on a *PV*–diagram, the clearance may be found by locating the zero of volume on the zero line of pressure. This process is performed graphically by reversing the method used

in constructing the equilateral hyperbola.

In actual expansions, however, n is almost never exactly equal to 1.0, but is greater or less as already explained. The accuracy of the result by this method is dependent on how close the value of n approximated 1.0. The clearance obtained by this method may be as much as 100 per cent larger or smaller than the actual clearance volume in ordinary cases, while errors of 25 per cent and 50 per cent are very common. Where errors of this size are possible, the method is of no use for important work.

A rational method of approximating the clearance cannot be based upon the assumption that $n = 1.0$, but only upon the fact that it be of a constant value, the value itself being immaterial.

38. *Combined Steam Indicator Diagrams.*—Very little of value is obtained from the combined *PV*-diagrams of steam engines, except the measure of the diagram factor for the purpose of design.

One of the uses that has been made of the combined diagram is to see whether continuity of expansion exists. It has been assumed by various writers that continuity of expansion should exist.

A study of the relations of x_c and n shows that continuity of expansion does not and should not exist except under very special conditions.

39. *Division of Feed for Applying Hirn's Analysis.*—One of the requirements of Hirn's analysis is that we know exactly how much steam was admitted to each end of a cylinder. These amounts are not usually equal in practice, so an assumption must be made to cover the needs of the case.

The usual assumption is to divide the feed between each end of a cylinder in the ratio of the values of the mean effective pressure shown by the *PV*-diagrams from the two ends. This assumption is probably not far from the actual division in most cases, and is the best that can be done under the circumstances.

In the light of the facts presented in this investigation, this feed may now be divided on a more rational basis. It has been found that the presence of the piston rod in only one end of the cylinder has no appreciable effect upon the value of n. This fact enables us to divide the feed according to the volumes filled and the values of x_c as determined by the resulting values of n. This method is believed to be the closest solution obtainable in the case where the supply for each end of the cylinder cannot be separately measured.

40. *Computing the Weight of Steam Retained in Compression.*—A carefully made investigation indicates,[1] when saturated steam is used, that the steam in the cylinder is dry or even very slightly superheated at the closure of the exhaust valve. As shown by the logarithmic diagrams, leakage of the steam in compression continues until the exhaust valve, in closing, has acquired considerable seal.

The point which is selected to compute the volume of steam retained in compression generally lies between the points A to B, and C to D of Fig. 9. As the weight of steam retained is yet decreasing, the point selected nearly always accounts for more steam than was actually retained. In other words, we do not find the value of x_c to be as high as it actually is.

The following method shown in Fig. 9 has been adopted in the present investigation. The straight line of the compression curve on the logarithmic diagram, or the line of constant weight of steam mixture, is prolonged dotted as shown to the back pressure. The intersection of this prolonged line with the back pressure line extended is taken as the volume of dry steam retained in compression.

This method almost always gives less steam retained in compression than the ordinary method, and is believed to be rational in the present state of knowledge of this subject.

XI. Conclusions

1. The indicator diagram, taken by means of a correct reducing motion and with a reliable indicator, contains the evidence necessary for a complete and useful analysis of cylinder performance.

2. The logarithmic diagram, derived from the indicator diagram, discloses a new and complete analysis of cylinder performance.

3. Free from the influences of leakage, wrong clearance, wrong location of the line of zero pressure, and excessively low speed (principally in steam engines), expansion or compression of an elastic medium takes place substantially according to the law, $PV^n = C$.

4. The clearance of cylinders may be found by graphical trial on logarithmic cross-section paper to within 5 per cent to 10 per cent of the clearance volume, depending as the clearance it-

[1] George Duchesne, Revue de Mecanique, July, 1899, quoted in Power, (Jan. 10, 1911, P. 71).

self varies from 20 per cent to 2 per cent, respectively, of the piston displacement.

5. The cyclic events, even though entirely obscure in the indicator diagram, may be located on the logarithmic diagram (when plotted on logarithmic paper of 5 in. per square) to within $\frac{1}{16}$ in., this quantity being the equivalent of about $\frac{1}{32}$ in. when retransferred to the indicator-diagram.

6. Leakage (if appreciable) may be reliably detected from the logarithmic diagram, and may, in some cases, be approximated in volume.

7. The weight of steam retained in compression should be obtained from the logarithmic diagram by prolonging the line of constant weight of steam mixture to the back pressure line extended; the intersection of these two extended lines is the volume of steam which is retained.

APPENDIX 1

THE LOGARITHMIC DIAGRAM

APPENDIX 1

Method of Constructing the Logarithmic Diagram

1. *Description of Logarithmic Cross-section Paper.*—Logarithmic cross-section paper differs from rectangular cross-section paper in that the distances from the origin are proportional to the logarithms of the numbers to be plotted instead of to the numbers themselves. This system of coordinates gives an uneven scale similar to that on a slide rule. The numbers of the divisions on logarithmic paper are placed opposite the lines corresponding to their logarithms, as on a slide rule, instead of to the values of the logarithms of the numbers. This fact aids in plotting, as the logarithms are employed without having to ascertain their values.

The logarithmic cross-section paper used in this investigation consists of four squares arranged two each way. These squares are five inches each way, making the four squares together ten inches each way. The use of four squares enables values to be plotted ranging from 0.1 to 10.0, 1.0 to 100.0, etc., thus giving a range of ten times the values obtainable if only one square were used.

2. *Construction of the Logarithmic Diagram.*—The coordinates of the *PV*–diagram are proportional to pressure and stroke, the latter being proportional to the volume displaced by the piston. The coordinates of several points on the *PV*–diagram are found in terms of absolute pressures, preferably in pounds per square inch, and absolute volumes, preferably in cubic feet. The scale of units employed is not material as long as it starts at the line of zero pressure, or of zero volume. However, the units are more easily manipulated afterwards if they are the same as those in the steam tables.

The method of transferring the *PV*–diagram to the logarithmic form is described in detail for the diagrams of test 30, given in Fig. 15. The method of drawing the pressure ordinates is shown on Fig. 15, crank end. The diagram is shown in outline by *AB YX*. Perpendiculars *QR* and *EX* are drawn to the atmospheric line *EQ*, and pass through the extreme stroke positions of the diagram. The distance *EQ* is then the length of the diagram. *OM* is laid off perpendicular to the atmospheric line *EQ* (extended) which was drawn by the indicator pencil. *OM* is the line of zero volume, and is drawn at a distance *FE* from the admission end *EX* of the di-

agram, the distance FE being the same length in per cent of the line EQ, or length of the diagram, as the proportion that the per cent of clearance, or waste space, of the cylinder bears to the piston displacement. In this case, the length of the diagram is 3.99 in., and the clearance is 7.04 per cent. The length FE is therefore 0.0704 x 3.99, or 0.281 in. ON is the line of zero pressure and is drawn a distance FO below the atmospheric line, to the scale of the spring used in obtaining the PV–diagram. This distance is proportional to the barometer reading, corrected for temperature, prevailing at the day and place of the engine test. In this case, the

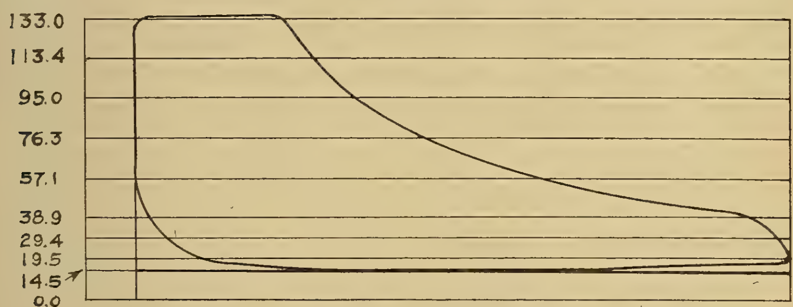


FIG. 15. HEAD END (SCALE 77.0 LB.)

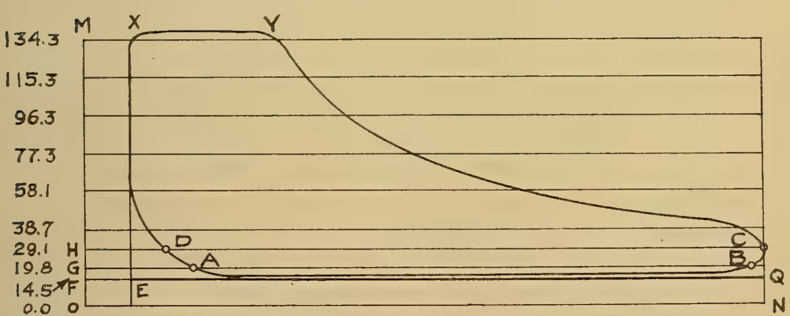


FIG. 15. CRANK END (SCALE—79.0 LB.)

corrected barometer reading is 14.2 lb. per sq. in. absolute, hence, the distance FO is $\frac{14.2}{79.0}$, or 0.180 in.

From ON, points are laid off on QR and EX corresponding to the absolute pressures at the intervals where it is desired to read off the corresponding volumes. Fine lines are drawn connecting similar pressure points, as 19.8–19.8, 29.1–29.1, etc. The volumes G–A, G–B, H–D, H–C, etc., are read off in hundredths of an inch to the nearest half-hundredth. The tabular form used in this investigation is given in Table 6 for the diagrams of Fig. 15 taken

in test 30. Thus the length *G–A* is read off as 0.66 in., and is given under the column for 19.8 lb. pressure headed Comp., meaning the compression curve, for the crank end diagram. The volumes in inches are then multiplied by the constant ratio which one inch of length of the diagram bears to the displacement of the piston. From Table 6, it is seen that the piston displacement of the crank end is 1.523 cu. ft., and the length of diagram 3.99 in.; hence, the ratio is $\frac{1.523}{3.99}$, or 0.382 cu. ft. of piston displacement per inch of diagram length. The length *G–A* in cu. ft. of displacement now becomes 0.66 in. × 0.382 cu. ft. per in., or 0.252 cu. ft., the volume of steam present at this point. This process is repeated at intervals until the coordinates of from ten to thirty points are determined. In the diagram shown in Fig. 15, the coordinates of 18 points were found in each diagram.

The coordinates of *P* and *V* are then plotted, on logarithmic cross-section paper, as shown in Fig. 9, which are the logarithmic diagrams derived from the *PV*-diagrams of Fig.15. The points plotted in Fig. 15 are taken from the columns headed cu. ft. at the pressures shown. A smooth curve is drawn through the points thus plotted, and the diagram is in shape to be studied.

TABLE 6

Construction of the Logarithmic Diagrams of Test 30[1]

	Head End						Crank End				
No.	Absolute Pressures lb. per sq. in.	Volumes					Absolute Pressures lb. per sq. in.	Volumes			
		inches		cu. ft.				inches		cu. ft.	
		Comp.[2] X to A	Exp.[2] Y to B	Comp.	Exp.			Comp.	Exp.	Comp.	Exp.
1	133.0	0.39	1.18	0.156	0.470		134.3	0.30	1.205	0.115	0.4605
2	113.4	0.31	1.36	0.124	0.5425		115.3	0.28	1.39	0.107	0.531
3	95.0	0.31	1.625	0.124	0.649		96.3	0.28	1.65	0.107	0.6305
4	76.3	0.31	2.02	0.124	0.8055		77.3	0.28	2.06	0.107	0.7875
5	57.1	0 31	2 705	0.124	1.080		58.1	0.295	2.76	0.113	1.055
6	38.9	0.375	4.03	0.150	1.610		38.7	0.385	4.12	0.147	1.572
7	29.4	0.48	4.22	0.192	1.685		29.1	0.485	4.26	0.185	1.626
8	19.5	0.70	4.26	0.279	1.699		19.8	0.66	4.20	0.252	1.600
9	15.0	1.10	3.06	0.439	1.220		15.0	0.93	3.77	0.355	1.440

	H E.	CR. E.
Length of indicator diagram	3 95 in.	3.99 in.
Ratio of clearance to piston displacement, same end	0.0789	0.0704
Length on diagram proportional to clearance ratio	0.312 in.	0.281 in.
Length of diagram plus clearance	4.26 in.	4.27 in.
Piston displacement (cylinder 12.02"×24")	1.575 cu. ft.	1.523 cu. ft.
Clearance volume	0.124 cu. ft.	0.107 cu ft.
Displacement plus clearance, total volume	1.699 cu. ft.	1.630 cu. ft.
Ratio, cu. ft. per inch of length on diagram	0.399 cu. ft.	0.382 cu. ft.
Scale of indicator spring per inch of ordinate	77.0 lb.	79.0 lb.

[1]Letters refer to Fig. 15, crank end.

[2]Final results given in Table 1.

APPLICATION OF THE LAW $PV^n = C$ TO CURVES FROM PRACTICE

3. *Does the Law $PV^n = C$ Hold for Curves from Practice?* It has long been known that adiabatic expansion or compression of any elastic medium takes place substantially according to the law $PV^n = C$. The values of n are different for different media, and sometimes vary for the same medium with different conditions of the initial state. These values, for media commonly used in reciprocating engines, are given on page 38 et seq.

In practice, however, expansion is never adiabatic, but is changed in character by the presence of the metal surrounding the working medium, and by imperfection of mechanism. Since the actual change of state is not adiabatic, the question arises how it has been changed in character, and whether this modified expansion still obeys the law $PV^n = C$.

Many investigators, Zeuner,[1] Leloutre,[2] Luders,[3] and Perry[4], have examined the curves from actual diagrams to clear up this point for steam. They find that expansion in steam cylinders takes place substantially according to the law $PV^n = C$, but that n varies in value between wide limits in different cases.

An examination was made of the curves of 296 diagrams from the cylinders of 47 different engines using steam, gas, air and ammonia to investigate this point. As a result, it may be stated that, in the cases of the great majority of engines using elastic media, expansion and compression take place substantially according to the law $PV^n = C$. Certain exceptions, however, have been found, and the causes studied. These causes are treated in the next section.

4. *Examples of Logarithmic Diagrams from Various Types of Engines.*—The logarithmic diagrams, obtained by plotting the indicator diagrams as described on p. 52, are of a distinctly different form from either the PV–diagrams or the temperature–entropy diagram. While the various typical forms of PV–diagrams assume rather different forms after plotting, yet the resultant figures retain in a general way the peculiar characteristics of each PV–type, except that these peculiarities are exaggerated.

Various typical PV–diagrams are given in Fig. 16–29. They include examples from many types of engines, using steam,

[1] Technical Thermodynamics II, p. 111.
[2] Recherches experimentals, Bulletin de la Societe industrielle du Nord de la France, 1874.
[3] Zur Theorie des Indikator diagrammes, Zivilingenieur, 1881, Vol. XXVII, p. 225.
[4] The Steam Engine, p. 106.

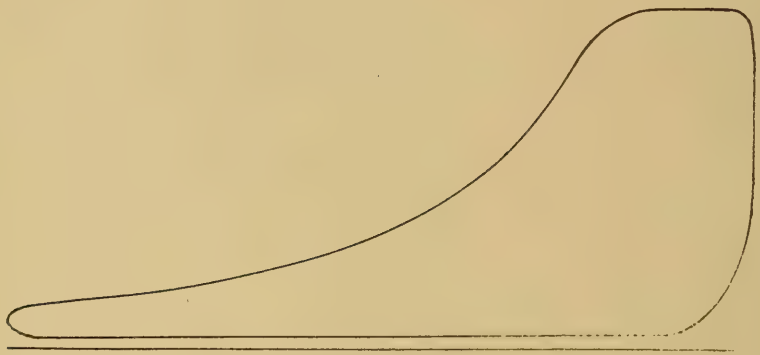


FIG. 16*a*. CRANK END (SCALE—50 LB.)

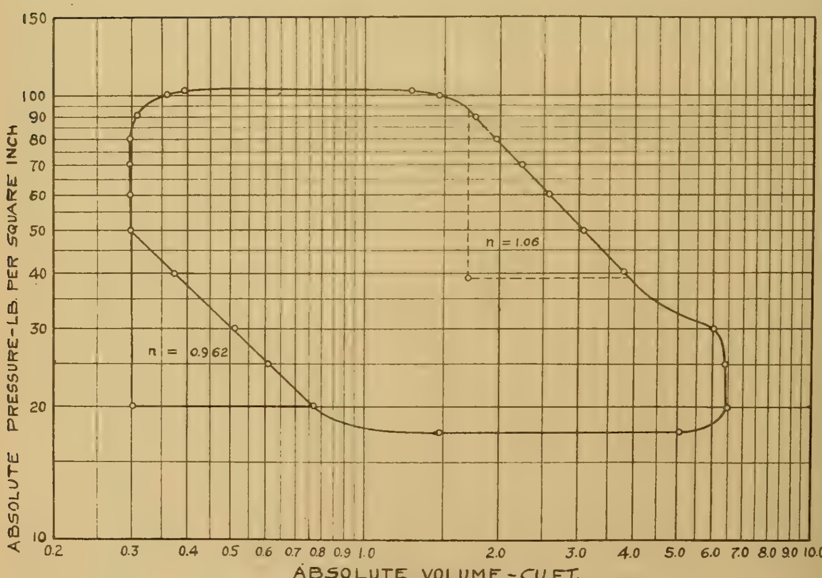


FIG. 16*b*. GREENE 18¼-IN. X 43-IN. STEAM ENGINE

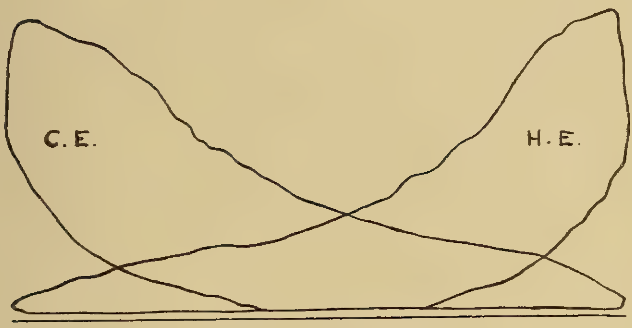


FIG. 17*a*. (SCALE—80 LB.)

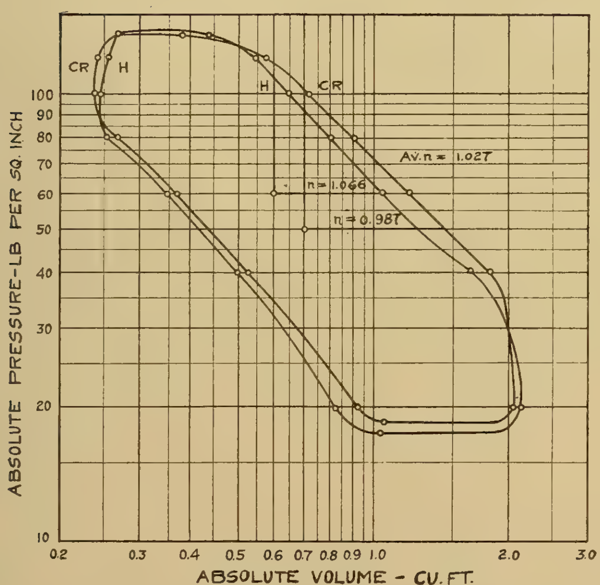


FIG. 17*b*. IDE 16-IN. X 16-IN. HIGH SPEED STEAM ENGINE

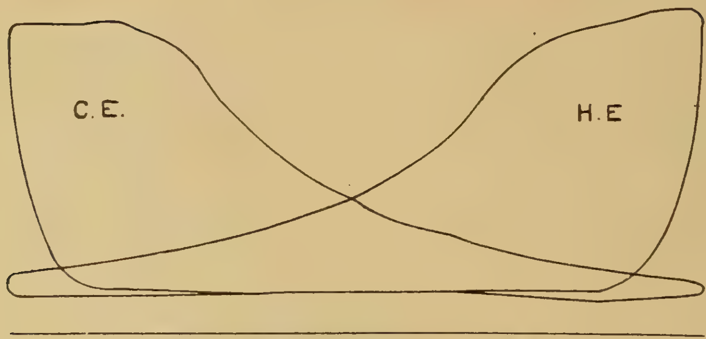


FIG. 18a. HIGH PRESSURE (SCALE—80 LB.)

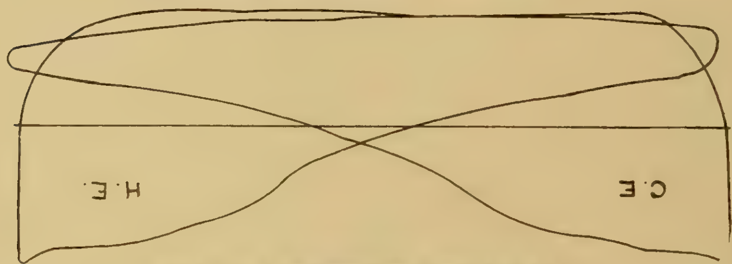


FIG. 18a. LOW PRESSURE (SCALE—20 LB.)

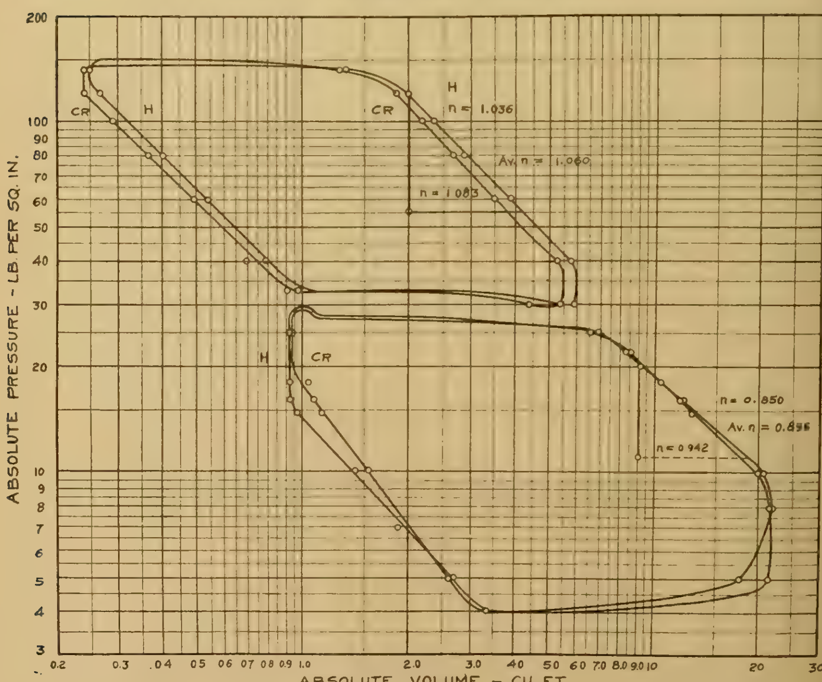


FIG. 18b. BUCKEYE 18¾-IN. x 36-IN. x 36-IN. STEAM ENGINE

FIG. 19*a*. RICE AND SARGENT 16-IN. x 28-IN. x 42-IN. SUPERHEATED STEAM ENGINE (SCALE—64.6 LB.)

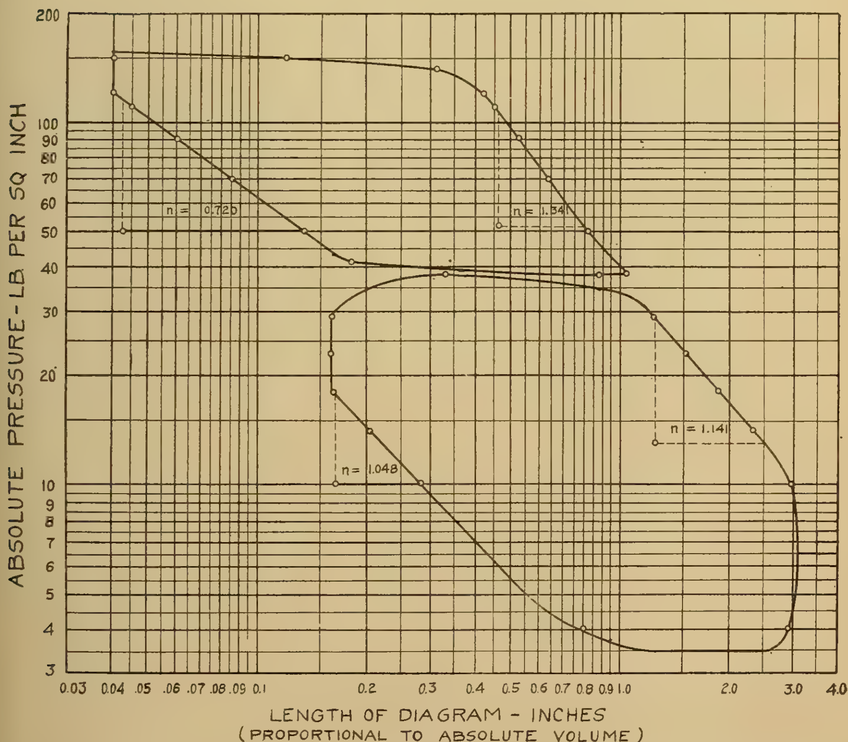


FIG. 19*b*. RICE AND SARGENT 16-IN. x 28-IN. x 42-IN. STEAM ENGINE USING HIGHLY SUPERHEATED STEAM

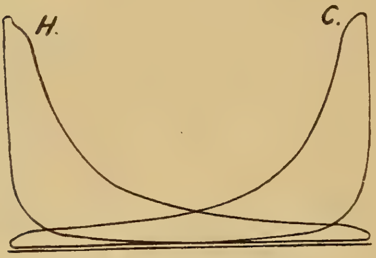


FIG. 20a. (SCALE—162 LB.)

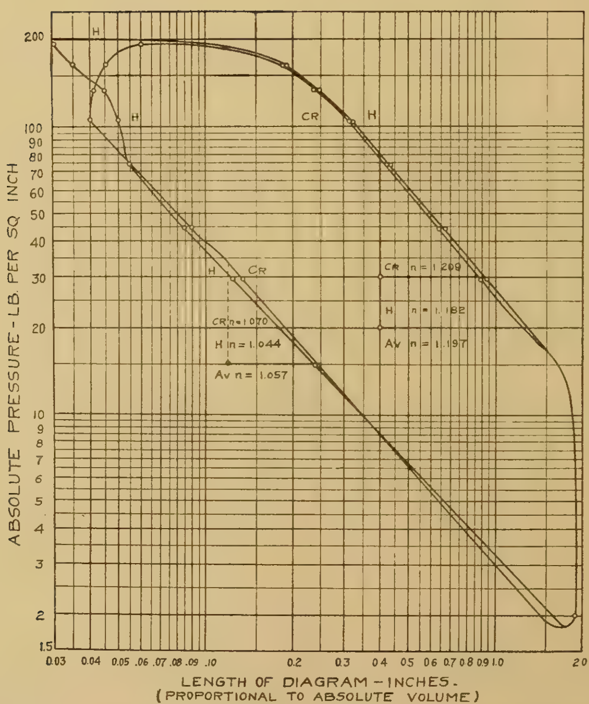


FIG. 20b. STUMPF 23½-IN. X 31½-IN. UNI-DIRECTIONAL-FLOW STEAM ENGINE USING SUPERHEATED STEAM

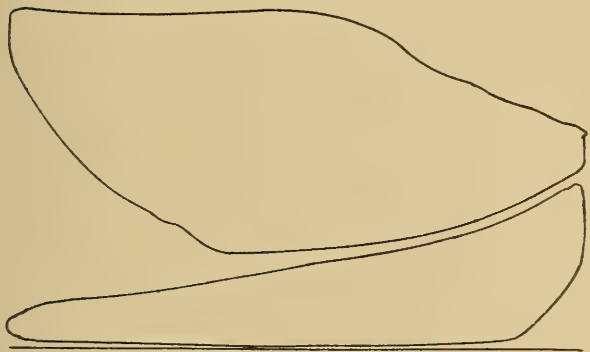


FIG. 21a. (SCALE—80 LB.)

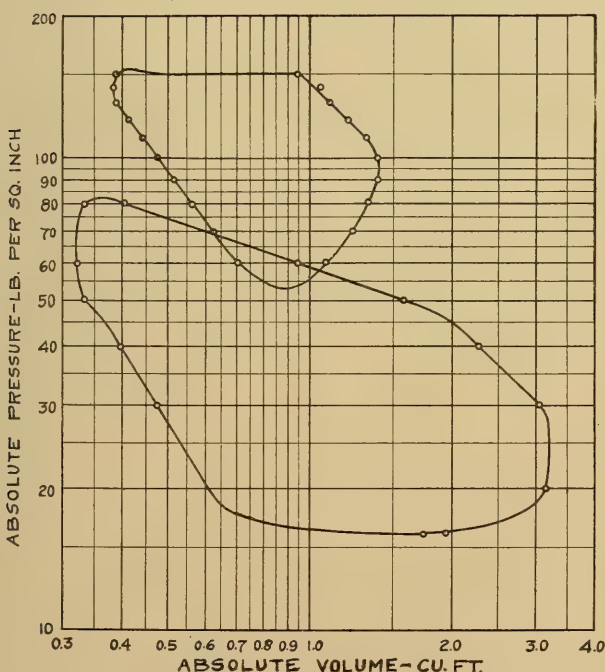


FIG. 21b. WESTINGHOUSE SINGLE-ACTING 13-IN. X 22-IN. X 13-IN. STEAM ENGINE

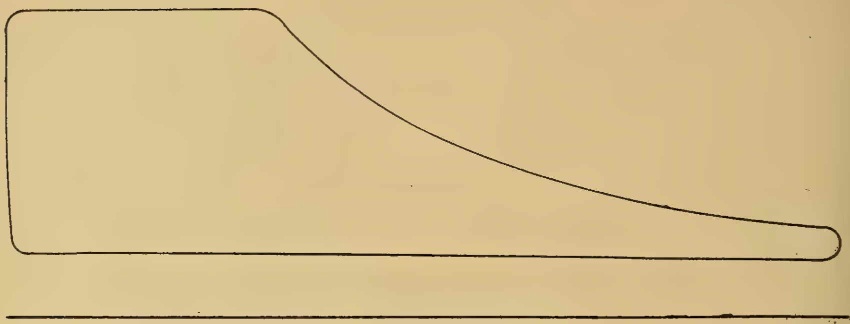


FIG. 22a. H. P. TOP (SCALE—100 LB.)

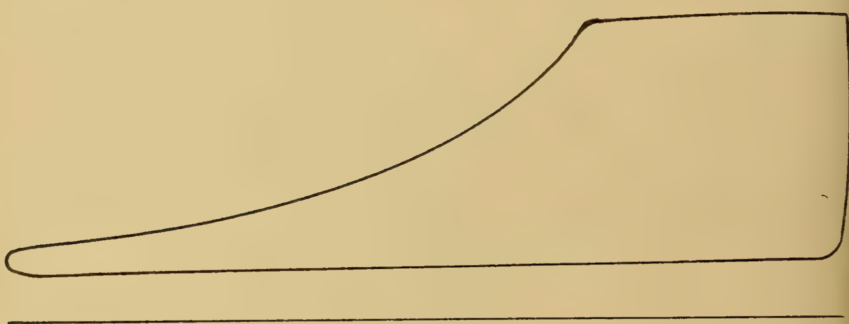


FIG. 22a. H. P. BOTTOM (SCALE—100 LB.)

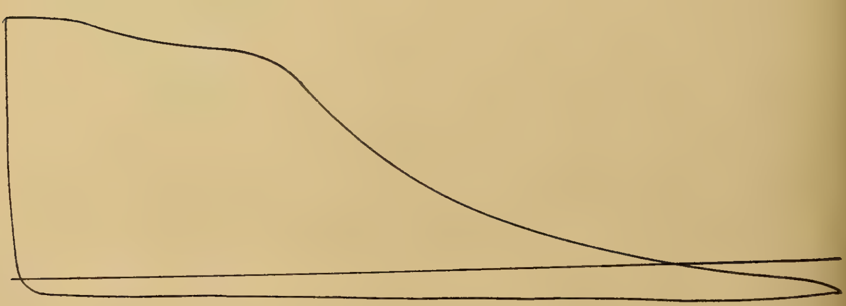


FIG. 22a. I. P. TOP (SCALE—20 LB.)

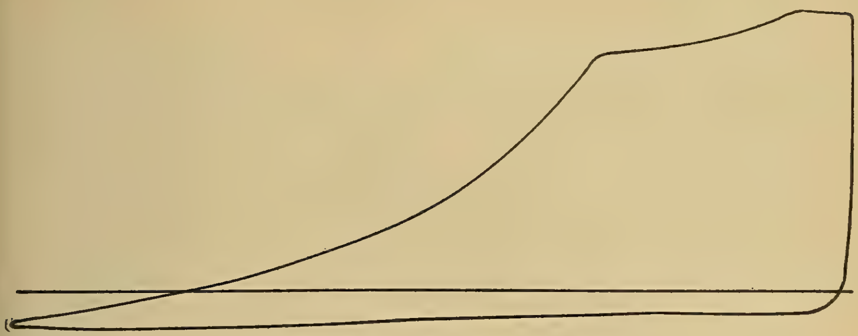


FIG. 22a. I. P. BOTTOM (SCALE—20 BL.)

FIG. 22a. L. P. TOP (SCALE—10 LB.)

FIG. 22a. L. P, BOTTOM (SCALE—10 LB.)

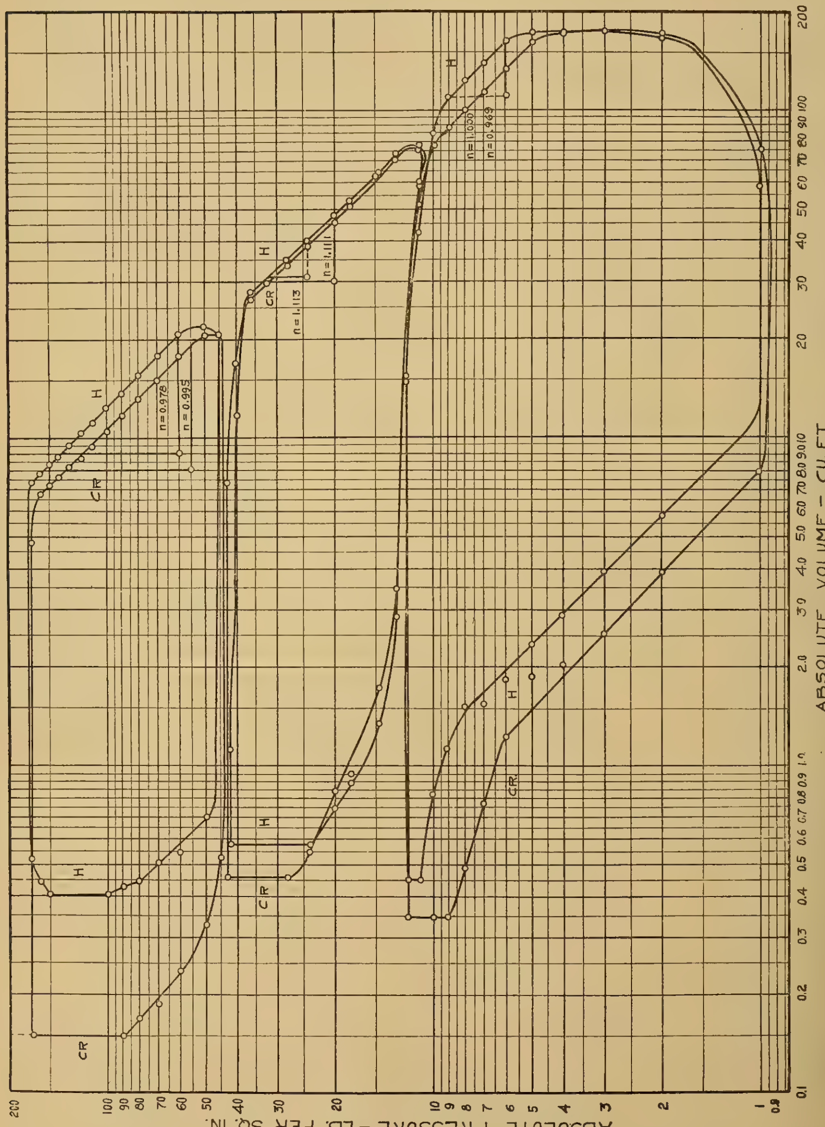


FIG. 22b. ALLIS-CHALMERS PUMPING ENGINE 28-IN. x 54-IN. x 80-IN. x 60-IN. USING SATURATED STEAM

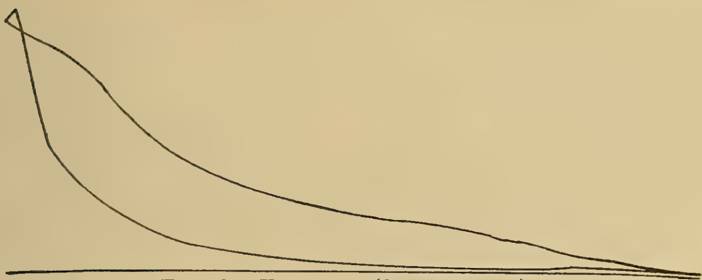


FIG. 23*a*. HEAD END (SCALE—150 LB.)

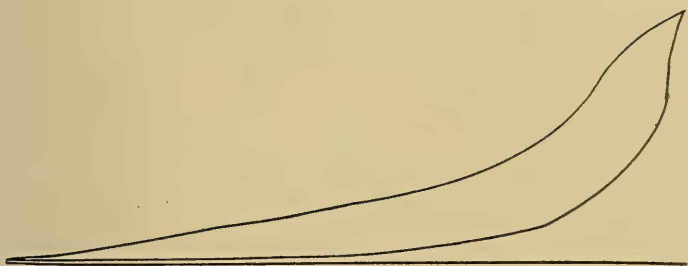


FIG. 23*a*. CRANK END (SCALE—150 LB.)

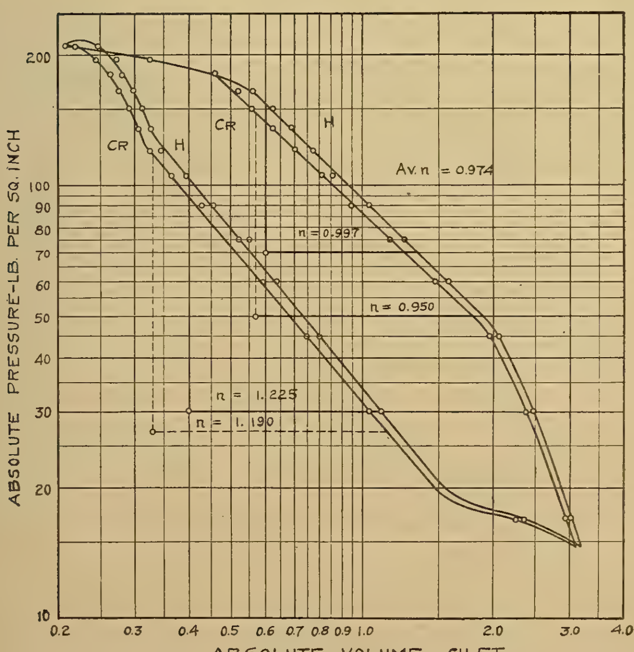


FIG. 23*b*. PURDUE 16-IN. x 24-IN. SUPERHEATED STEAM LOCOMOTIVE

FIG. 24*a*. HEAD END (SCALE—100 LB.)

FIG. 24*a*. CRANK END (SCALE—100 LB.)

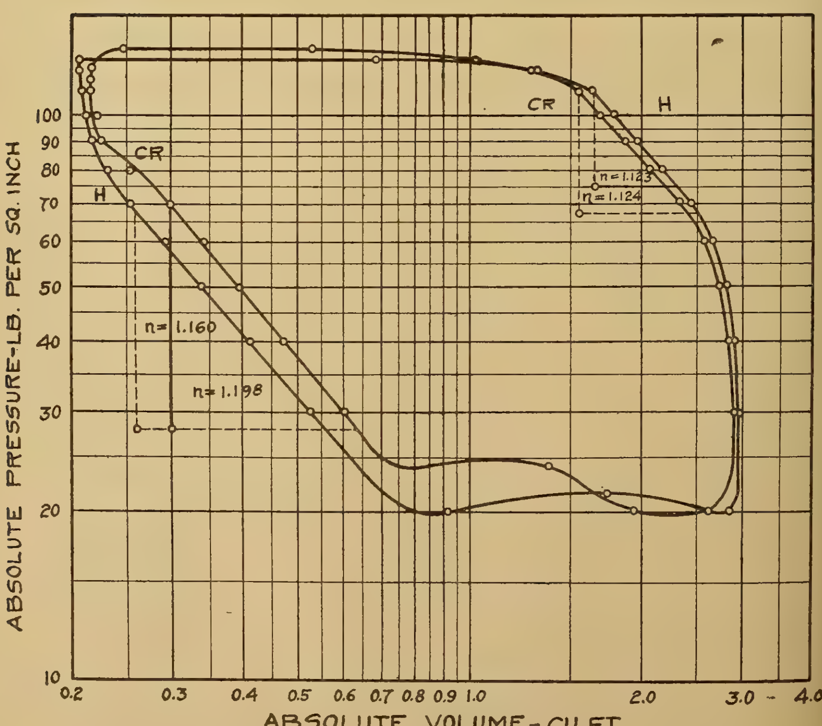


FIG. 24*b*. PURDUE 18-IN. x 24-IN. SUPERHEATED STEAM LOCOMOTIVE

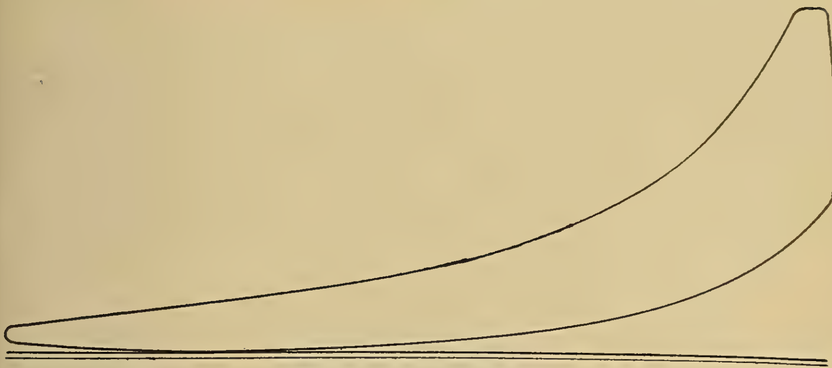


FIG. 25a. (SCALE—160 LB.)

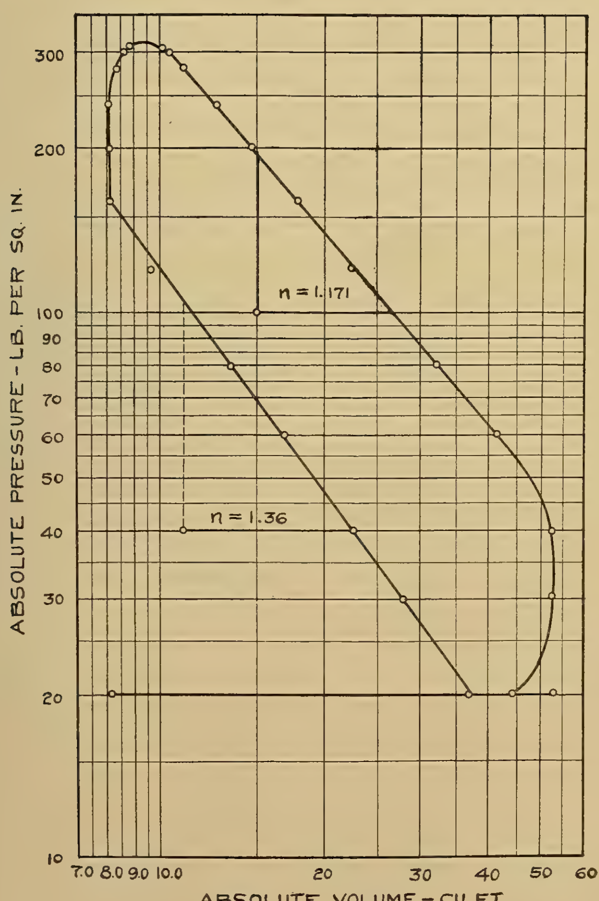


FIG. 25b. TOD 42-IN x 60-IN. GAS ENGINE BLAST FURNACE

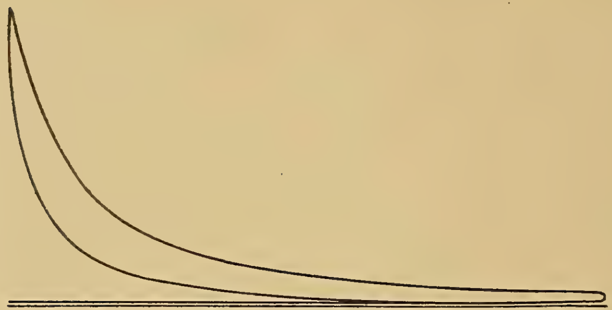


FIG. 26a. (SCALE 400 LB.)

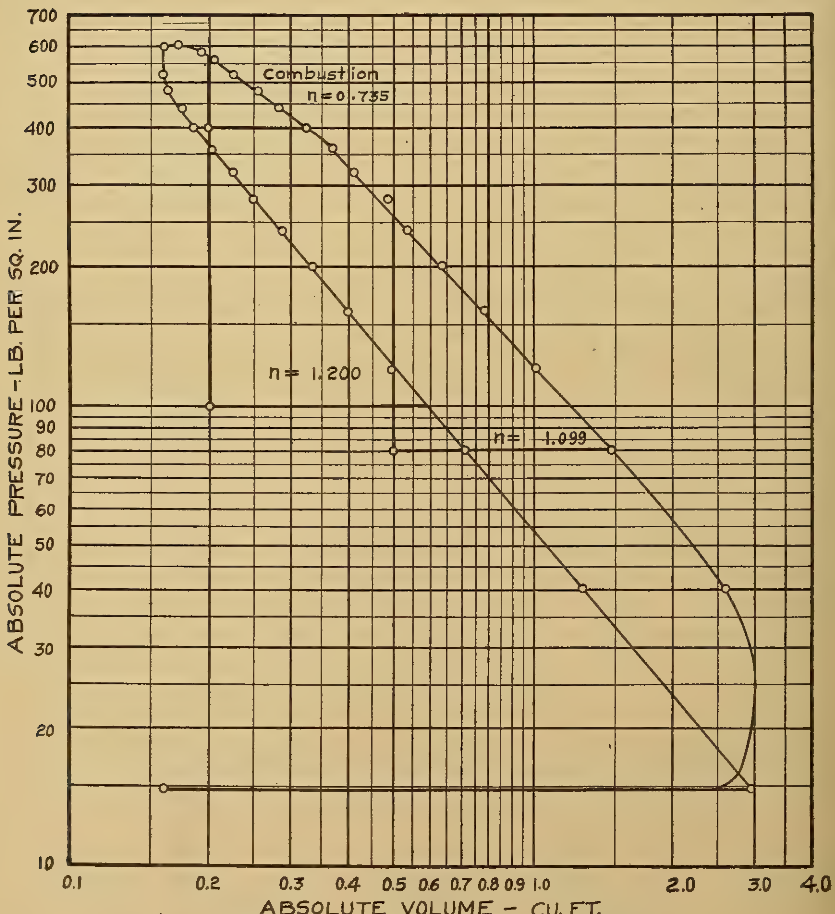


FIG. 26b DIESEL 16-IN. x 24-IN. OIL ENGINE USING CRUDE OIL

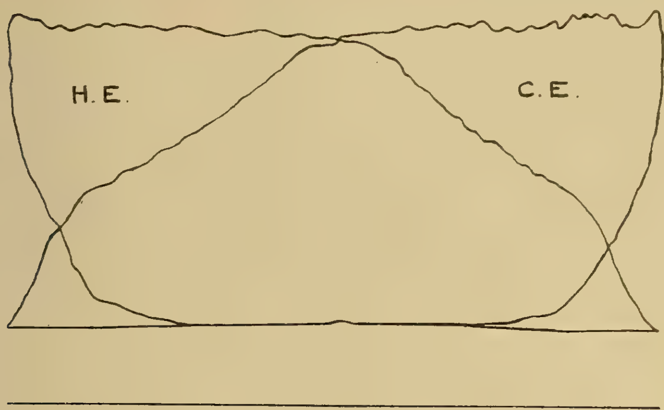


FIG. 27*a* HIGH PRESSURE (SCALE—120 LB.)

FIG. 27*a* LOW PRESSURE (SCALE—40 LB.)

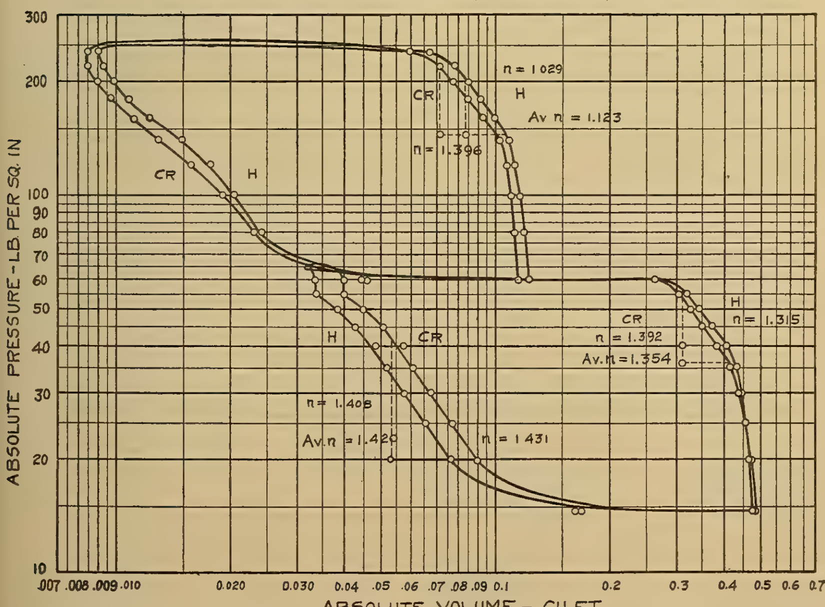


FIG. 27*b* PORTER AIR LOCOMOTIVE—COMPOUND 5-IN. x 10-IN. x 10-IN

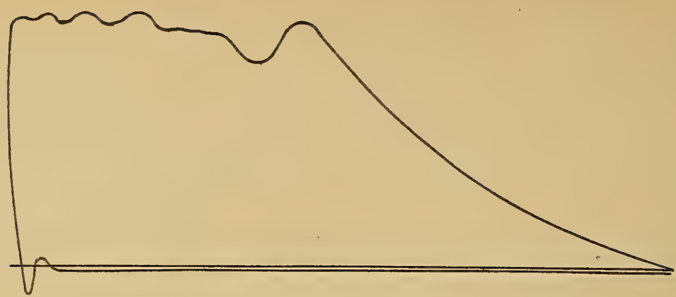


FIG. 28*a*. HEAD END—LOW PRESSURE (SCALE 17.5 LB.)

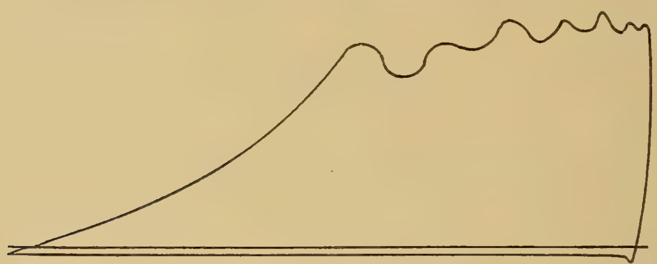


FIG. 28*a*. CRANK END--LOW PRESSURE (SCALE—21.6 LB.)

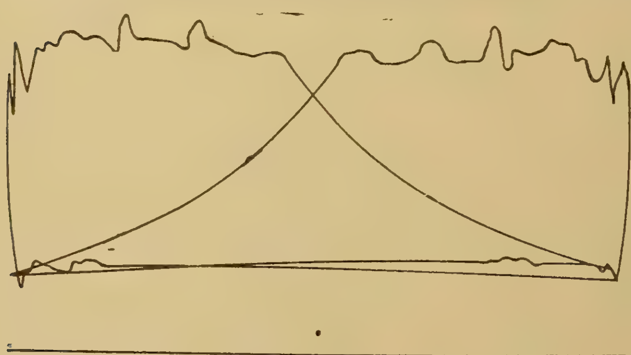
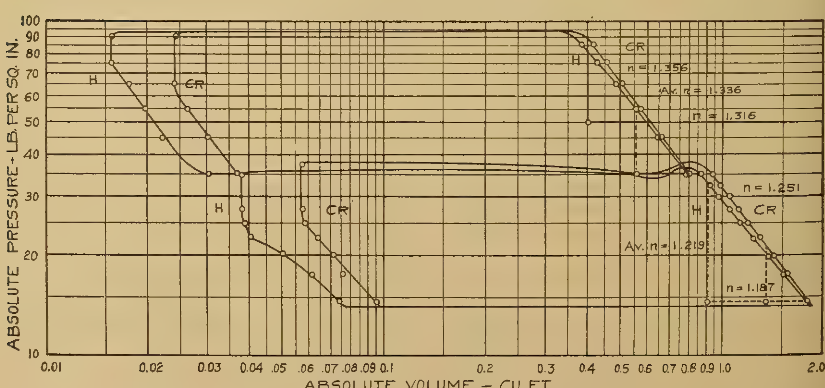


FIG .28*a*. HIGH PRESSURE (SCALE—50 LB.)



.28*b* FIG. INGERSOLL —SERGEANT 12¼-IN. X 18¼-IN. 12-IN TWO —STAGE AIR COMPRESSOR

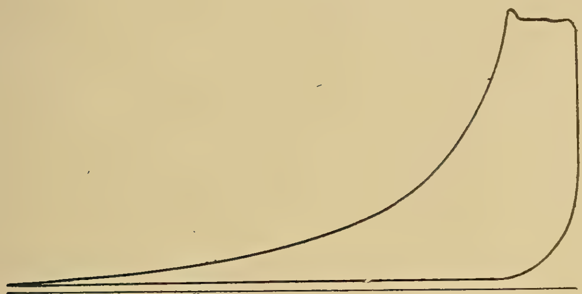


FIG. 29a. (SCALE—136 LB.)

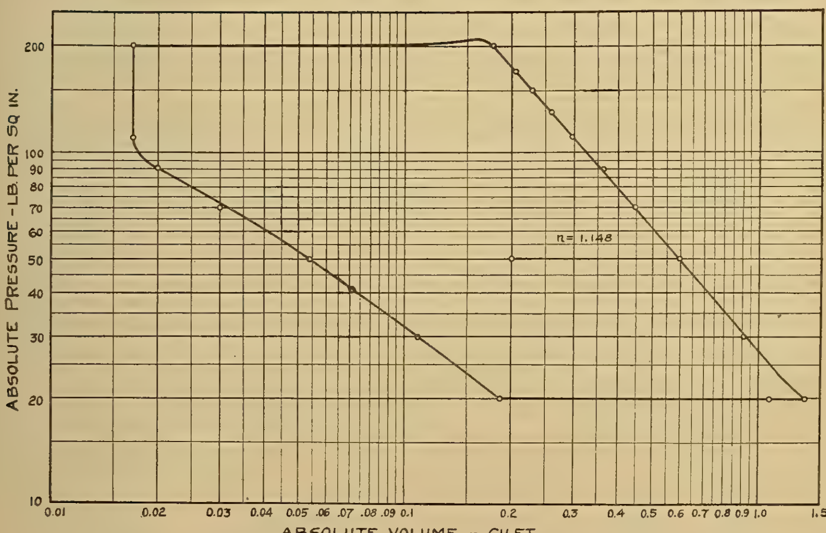


FIG. 29b. YORK SINGLE-ACTING 12½-IN. X 18-IN. AMMONIA COMPRESSOR

gas, air and ammonia as the active media. The values of n for the curves are given in each figure. The types of steam engines represented include Corliss, high speed, auxiliary cut-off, poppet valve, single acting, pumping and locomotive engines. The gas-engine diagrams include two four cycle types, one using blast furnace gas, and the other a Diesel engine using crude petroleum. The air diagrams contain one set from a compound air locomotive, and one set from a two-stage air compressor.

These figures show most of the typical forms of diagrams that are obtained in practice. At first sight, the logarithmic diagrams look distorted, but after the meanings of the different lines become clear, they begin to seem as natural as the PV-diagram. These logarithmic diagrams show how closely the law $PV^n = C$ holds in actual curves from a great variety of engines using diferent media.

5. *Cases Where the Law $PV^n = C$ Does Not Hold.*—The curves of expansion and compression, obtained from PV-diagrams, do not always follow the law $PV^n = C$. This fact is due to several causes, some of which have been definitely determined. These causes will be treated separately.

a. Wrong Clearance, or Wrong Location of the Zero Line of Pressure.—The law $PV^n = C$, is true only where P and V are measured in absolute units. The clearance must be accurately determined in order to give absolute values of V. The scale of the spring, for the PV-diagram analyzed, must be known, and the atmospheric line drawn by the indicator, in order to locate the zero line of pressure. The units used for P or V may be of any denomination, but they must be measured from the zero of P and V.

When a curve, $PV^n = C$, is plotted on logarithmic paper, the resultant curve is a straight line. The value of n, as already explained, is the slope of this line, measured from any two points. When the values of P and V are not absolute values, this curve is no longer a straight line, but becomes a curve of the second degree. The value of n being the slope obtained from two points on this curve, is no longer constant for all parts of the curve, but varies from point to point. Therefore, when PV-diagrams are transformed to logarithmic diagrams, the values of both P and V must be measured in absolute units. When these values are not in absolute units, the resulting curve is not of the form, $PV^n = C$, and therefore is not a straight line on logarithmic paper. The form of the curve obtained, when the values of V alone are not in absolute units, is given in Fig. 10*b*.

b. Leakage.—The law $PV^n = C$, is applicable only to cases where the weight of the working medium remains practically constant during any expansion or compression. When this weight changes materially either by leakage into, or out from, the cylinder containing the medium, the resulting curve no longer obeys the law, and it becomes a curve in the logarithmic diagram. This fact is very clearly shown in the curves of the logarithmic diagram derived from cylinders in which large leaks were known to exist. The examples showing this condition are shown in pp. 39, 41, 42.

c. Low Speed in Steam Engines.—Very low speeds of rotation, together with very low piston speeds, will cause the compression curve to deviate from the law $PV^n = C$. The most common cases of this effect are seen in the "hook", or excessive condensation, near the upper end of compression curves. These "hooks" are found almost altogether in small engines having very low piston speeds, and in larger engines with small clearance, having very low rotational speeds, as in pumping engines.

Diagrams containing "hooks" in the compression curves have been published by Professor Dwelshauvers-Dery[1] from the experimental engine at Liege. This is an example of a very low speed in a small engine. The size was 12 in. x 24 in. and the speed from 30 to 60 r. p. m. That this hook was caused by low speed and consequent excessive condensation was proved in a measure in the case of the engine described on page 88. This engine was also 12 in. x 24 in., but was operated at from 90 to 150 r. p. m. In no case was a hook in the compression curve obtained during the tests, although some 1600 diagrams were taken. A set of diagrams from these tests is shown in Fig. 15 and shows no sign of a hook. On one occasion, however, the diagram shown in Fig. 30 was obtained. This was taken just after the engine was started from cold, and had been brought up to a speed of 120 r. p. m. In conjunction with the other diagrams obtained from this engine in regular operation, this hook is believed to be due to excessive

FIG. 30. TAKEN WHILE STARTING FROM COLD

[1] Power, June 28, 1910, p. 1165.

condensation while the cylinder was comparatively cold.

Diagrams with the hook present in the compression curves have been obtained from two other engines; one was an $8\frac{1}{2}$ in. x 12 in., running 110 r. p. m., and the other, 5 in. x 6 in., running 140 r. p. m.

This excessive condensation at the end of compression, or near the dead center, seems to be due to the fact that the deleterious surface effect of the cylinder walls is so enormous, compared with the weight and volume of steam present at this point.

The three causes treated above are believed to be the important conditions that cause the curves in practice to depart materially from the law $PV^n = C$. Sometimes only one of these conditions is present, while in the other cases, a combination of these may influence the resulting curves. The separation of these conditions by their effect on the curves is treated in page 45.

VALUES OF n FROM PRACTICE.

6. *Steam Engines.*—The values of n for the expansion and compression curves of indicator diagrams are subject to a wide variation. Zeuner[1] gives the values of n found by several early investigators. These values were taken mostly from the diagrams of small and slow-speed engines. Leloutre found that the value of n was practically constant for any one case, but varied greatly in different engines, according to the initial pressure and ratio of expansion. Lüders found values ranging from 0.903 to 0.535. Zeuner found values of from 0.900 to 0.436 from diagrams taken by Hallauer from a Corliss engine. In none of these cases was n found to be as high as 1.0. Zeuner concludes that the value is generally close to 1.0, and does not vary much either way.

Heck[2] shows diagrams from the cylinders of compound Corliss engines, locomotives, pumping engines, and marine engines, all using saturated steam. The values of n for saturated steam from the expansion curves of the h. p. cylinders of these engines are all close to 1.0. The values for the l. p. expansion curves are generally less than 1.0, ranging from about 0.95 to 0.90. The values for the compression curves shown do not depart far from 1.0.

A large number of curves have been examined by the graphical method described on page 52. These examples, shown in Table 7–14, may be classed as follows:

[1] Technical Thermodynamics, II. p. 111.
[2] The Steam Engine, II. p. 476.

Class A. Corliss, four-valve, and riding-cut-off types, all simple expansion

B. Same, compound.

C. Same, triple expansion.

D. Gridiron valve type, compound, steam jacketed.

E. Poppet valve type.

F. High speed engines, single valve types.

G. Single-acting engines.

H. Locomotive engines.

These classes cover the values derived from 138 diagrams taken from 36 separate engines. On page 8, are given the values obtained from 60 sets of diagrams taken from a 12 in. × 24 in. Corliss engine.

The values given in Tables 7–14 are the average values from both ends of one cylinder, and, in the case of two-cylinder simple engines, from both ends of both cylinders. In many cases only the expansion curves were examined.

These diagrams were taken from cylinders ranging in size from 11½ in. x 13 in. up to 80 in. x 60 in., and in speed from 300 r. p. m. down to 23.6 r. p. m. Most of the cylinders were unjacketed. Examples of both saturated and superheated steam are shown. It will be seen, therefore, that the range of the exhibit is very broad.

It is evident that the values of n are subject to extremely wide variations, the range being from 0.436, found by Zeuner, up to 1.341, found in this investigation in the case of an engine using highly superheated steam. It is true, however, that the average of all the values cited is not far from 1.0.

7. *Gas Engines.*—The values of n for the curves of gas engine diagrams have a smaller range of variation than that found in steam diagrams.

Güldner[1] finds that the value of n for the compression curves varies from 1.30 to 1.38, with an average of about 1.35, but he mentions rare values higher than the adiabatic value, due to high temperature of cylinder walls, and the consequent addition of heat to the gas during compression. He finds that n for expansion[2] varies normally from 1.35 to 1.50, but cites lower values than 1.35 due to leakage, and higher values than 1.50 due to excessive temperature of cylinder walls from poor cooling.

[1] Internal Combustion Engines, p. 34.

[2] Internal Combustion Engines, p. 38.

Wimperis[3] gives the values found by Professor Burstall from diagrams taken during 10 tests on the same engine, presumably. Professor Burstall finds n for expansion to vary from 1.199 to 1.344, with an average of 1.288. The values of n for compression vary from 1.345 to 1.364, with an average of 1.352.

The examination of 17 diagrams from 5 separate engines gave values for expansion and compression as given in Table 15. These values given show that the variation of n for expansion is ordinarily from 1.10 to 1.37, while for compression it is from

TABLE 7

VALUES OF n FROM CLASS A STEAM ENGINES

Size inches	Value of n		No. of Diagrams Examined	Make	Remarks
	Expansion	Compression			
18¼ x 43	0.952	1.007	2	Greene	Saturated steam
"	0.998	1.115	2	"	
15¼ x 24	1.049	1.170	2	Buckeye	
26½ x 36	0.928	1.063	2	"	
"	0.823	1.036	2	"	
"	0.996	1.121	2	"	
20 x 30	0.624	0.985	2	"	
"	1.024	0.992	2	"	
18 x 42	1.047	0.994	2	Unknown	
16 x 32	1.065	1.154	2	Buckeye	
"	1.111	1.341	2	"	
16⅝ x 32	0.964		2	Unknown	
26 3/16 x 48	1.051		2	"	
17 x 24.2	1.131		4	"	
23 x 60	1.098		2	"	
28½ x 59½	1.108		2	"	
11 Engines			Total—34 Diagrams		

TABLE 8

VALUES OF n FROM CLASS B STEAM ENGINES

Size inches	Expansion		Compression		No. of Diagrams Examined	Make	Remarks
	h. p.	l. p.	h. p.	l. p.			
18¾ x 36 x 36	1.060	0.896			4	Buckeye	Saturated steam
15 x 40½ x 27	0.955				2	Fleming	
20 x 36 x 48	1.055	0.973	0.973	0.742	4	Watts Campbell	
21 x 42 x 36	1.068		0.987		4	Gaskell	
25 x 50 x 37	0.841	0.879	0.850		8	Worthington	
22 x 40 x 60	1.070	1.009	1.018	1.272	4	Harris-Corliss	Cyls. jacketed
20 x 36 x 48	1.079	0.977			4	Watts Campbell	" non-jacketed
16 x 40 x 48	1.090	0.950			4	Cooper Corliss	
"	1.048	1.116			4	" "	Cyls. jacketed
8 Engines					Total—38 Diagrams		

[3] The Internal Combustion Engine, p. 73.

TABLE 9

Values of n From Class C Steam Engines

Size inches	Expansion h. p.	Expansion i. p.	Expansion l. p.	No. of Diagrams Examined	Remarks
28 x 54 x 80 x 60	0.987	1.112	0.984	6	Saturated steam Cylinders jacketed
	Compression				
28 x 54 x 80 x 60	0.964	0.416	0.972		
1 Engine				Total—6 Diagrams	

TABLE 10

Values of n From Class D Steam Engines

Size inches	Expansion h. p.	Expansion l. p.	No. of Diagrams Examined	Make	Remarks
28 x 58 x 48	1.046	1.070	2	McIntosh	Superheated steam
23 x 48 x 48	0.955	0.969	2	Seymour & Co.	Jackets not used
"		1.108	1		
"	0.905	0.925	2		
"	1.119	1.075	2		
29 x 60 x 56		1.231	1		
29 x 60 x 56		1.172	1		
18 x 38 x 42	1.170	0.925	2		
18 x 38 x 42	1.118	1.087	2		
31 x 64 x 48	1.024	0.973	2		
7 Engines			Total—17 Diagrams		

TABLE 11

Values of n From Class E Steam Engines

Size inches	Expansion h. p	Expansion l. p.	Compression h. p.	Compression l. p.	No. of Diagrams Examined	Make	Remarks
16 x 28 x 42	1.341	1.141	0.720	1.048	2	Rice and Sargent	Highly superheated steam
"	1.260	1.180	1.262	1.210	2	"	
"	1.293	1.152	0.980	0.989	2	"	
"	1.033	1.011	0.710	1.060	2	"	Saturated steam
23½ x 31½	1.197		1.057		2	Stumpf Straight Flow	Highly superheated steam
2 Engines			Total—10 Diagrams				

TABLE 12

Values of n From Class F Steam Engines

Size inches	Expansion h. p.	No. of Diagrams Examined	Make	Remarks
14½ x 13	0.970	2	Unknown	Saturated steam
16 x 16	1.027	2	Ide	
11½ x 18½	0.706	2	Unknown	
3 Engines		Total—6 Diagrams		

TABLE 13

Values of n From Class G Steam Engines

Size inches	Expansion h. p.	Expansion l. p.	No. of Diagrams Examined	Make	Remarks
13 x 22 x 13	1.073	0.874	2	Westinghouse	Saturated steam
"	1.054	0.863	2		
"	1.062		1		
1 Engine			Total—5 Diagrams		

TABLE 14

Values of n From Class H Steam Engines

Size inches	Expansion	Compression	No. of Diagrams Examined	Make	Remarks
16 x 24	1.003		4	Schenectady No. 2	Saturated steam
22 x 30	0.985	0.987	2	I. C. No. 940	
"	0.981		2	"	
22 x 30	0.975	0.970	2	I. C. No. 920	
"	1.006	1.125	2	"	
16 x 24	0.974	1.208	2	Schenectady No. 3	Superheated steam
"	1.124	1.179	2	"	
"	1.167	1.195	2	"	
"	1.046	1.188	2	"	
"	1.149		2	"	
3 Engines			Total—22 Diagrams		

TABLE 15

Values of n From 4-Cycle Gas Engines

Size inches	Expansion	Compression	No. of Diagrams Examined	Make	Gas Used
10 x 19	1.36	1.19	1	Otto	Illuminating
"	1.37	1.09	1	"	
"	1.27	1.26	1	"	
"	1.21	1.35	1	"	
"	1.26	1.35	1	"	
"	1.25	1.74	1	"	
"	1.30	1.43	1	"	
"	1.16	1.27	1	"	
"	1.21	1.43	1	"	
42 x 60	1.16	1.32	1	Tod	Blast furnace
"	1.16	1.30	1	"	
"	1.09	1.32	1	"	
32 x 42	1.18	1.34	1	Allis-Chalmers	Producer
25⅝ x 37⅝	1.12	1.24	1	Koerting	
16 x 24	1.10	1.20	1	Diesel	Petroleum
"	1.11	1.22	1	"	
"	1.02	1.22	1	"	
5 Engines			Total—17 Diagrams		

TABLE 16

Values of n From Compressed Air Locomotives

Size inches	Expansion h. p.	Expansion l. p	No. of Diagrams Examined	Make
5 x 10 x 10	1.123	1.354	4	Porter
6 x 10	1.369		4	"
2 Engines	Total— 8 Diagrams			

TABLE 17

Values of n From Air Compressors

Size inches	Compression h. p.	Compression l. p.	No. of Diagrams Examined	Make	Remarks
12¼ x 18¼ x 12	1.336	1.219	4	Ingersoll-Sargent	Small amt. of cooling water
"	1.266	1.254	4	"	Large " " " "
1 Compressor	Total— 8 Diagrams				

TABLE 18

Values of n From AmmoniaCompressors

Size inches	Compression	No of Diagrams Examined	Make	Remarks
12½ x 18	1.148	1	York	Dry comp. sing. acting
12½ x 18	1.186	2	"	" " Doub. "
"	1.235	2	"	Wet compression
2 Compressors	Total— 5 Diagrams			

TABLE 19

Values of n From Gas Compressors

Size inches	Compression	No. of Diagrams Examined	Make	Remarks
28¼ x 24	1.140	2	Ingersoll-Sargent	Illuminating gas
"	1.145	2	"	
1 Compressor	Total— 4 Diagrams			

about 1.20 to 1.40, leaving out of account very high values due to imperfect cooling and very low values due to leakage and to cool walls during the period of "starting up." The values of n are low for expansion in large cylinders with lean gases, being about the same as those found in steam cylinders using superheated steam.

8. *Compressed Air Engines.*—Very few data have been found concerning the values of n from diagrams taken from compressed air engines. Locomotives comprise the large part of this class which use air expansively.

Curves were examined from eight diagrams taken from the four cylinders of two locomotives, one a compound, and the other a simple expansion type. The results are given in Table 16.

The expansion curves of these diagrams were very satisfactory, but the compression curves were all irregular, and no values were obtained from most of them. This irregularity was probably due to the vibration that is present in most locomotives under running conditions. The values for expansion range from about 1.12 to 1.37.

9. *Air Compressors.*—Only 8 diagrams from one two-stage compressor were available for examination. The values of n for compression are given in Table 17. No values for re-expansion were obtained.

The values for compression in air compressors do not vary much, and will generally fall between 1.20 and 1.35.

10. *Ammonia Compressors.*—The examples available of this type of compressor were limited to 5 diagrams from 2 compressors. The compression curves were very satisfactory, but the re-expansion curves were quite irregular. These values will be found in Table 18. The values found fall between about 1.15 and 1.24, very little variation being observed.

11. *Gas Compressors.*—This class refers to compressors used to raise the pressure of illuminating gas in order to send it to distant points in small pipes. The analysis of the gas compressed in the single case examined is given on page 86. Values of n were obtained for the compression curves only. These values are given in Table 19. Although taken at different conditions of the speed and the discharge pressure, these values show substantial agreement, but are considerably lower than the values obtained for air compression.

Theoretical Values of k for Adiabatic Expansion

The relations of P and V during adiabatic expansion of steam and gases may be closely represented by the equation for the polytropic curve $PV^k = C$.

The value of k for steam depends upon the initial state, while for gases k is equal to the ratio $\frac{C_p}{C_v}$.

12. *Values of k for Steam.*

(a) Saturated Steam. Rankine[1] gives the value of k as $\frac{10}{9}$, or 1.111, applicable to all initial states.

Grashof[2] examined the condition of initially dry saturated steam and gives the value of k as 1.140.

Zeuner[3] examined the condition generally occurring in engineering practice, where the steam is initially composed of a mixture of vapor and water, and gives the relation

$$k = 1.035 + 0.1\,x.$$

He found the influence of initial pressure to be negligible.

Mr. E. H. Stone[4] examined the condition of various initial states of pressure and quality, using the tables of Marks and Davis, and gives the relation

$$k = 1.059 - 0.000315P + (0.0706 + 0.000376P)\,x.$$

Table 20 was computed from this equation and expresses the relations of P and V with an average error of less than 0.2%.

(b) Superheated Steam. Zeuner[5] gives the value of k as constant at $\frac{4}{3}$, or 1.333. Several conditions have been examined for the initial state of 200 lb. absolute pressure with the superheat varying from 80° to 530°. The relations of P and V were computed from Professor G. A. Goodenough's characteristic equations[6] for superheated steam.

It has been found, after many trials, that these relations are closely expressed by the equation

$$P(V + 0.088)^{1.305} = C.$$

This equation expresses the relations of P and V with an average error of about 0.5 per cent. The initial pressure and the degree

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1 The Steam Engine, p 385.

2 Zeitschrift des Vereins deutscher Ingenieure, Vol. VIII, p 151.

3 Technical Thermodynamics II, p.83.

4 Thesis, University of Illinois, 1910.

5 Technical Thermodynamics, II, p. 223.

6 Principles of Thermodynamics, p. 203.

of superheat have very little effect upon the value of k for the ranges commonly used in practice.

13. *Values for Gases.*—The values of k for gaseous mixtures commonly used in gas engines exhibit considerable variation due to the variation in the relative proportion of the constituents. The value of k for any gas is the value of the ratio $\frac{C_p}{C_v}$, and its constancy depends on the relative constancy of the value of C_p and C_v at different temperatures. It seems certain, from experiments of Mallard and Le Chatelier, that the values of C_p and C_v increase with increase of temperatures, but that the ratio $\frac{C_p}{C_v}$, at temperatures common to actual gas engine cycles, is very closely constant.

Most of the following data on the value of k for gases has been taken from Guldner[1]. The value of k depends, in any particular mixture, upon the relative proportion of the constituents. This may be seen from the values of k for different gases given in Table 21, from Guldner. This table shows that by different proportions of these gases in a mixture, values of k may be obtained between the extreme limits of 1.210 and 1.418. Thus k for illuminating gas alone, for example, may have a value of 1.332, while for a mixture of this gas with air in the proportion of 1 to 12, the value rises to 1.402, or very nearly the value of pure air. After combustion, the analysis of the gas changes, causing a change of k, the effect being a lowering of its value because of the increase in proportion of CO_2 and H_2O. Thus for the mixture cited above the value of k after combustion is 1.387, a drop of 1.1 per cent. This, in a hypothetical case of an ideal gas engine cycle, would cause an appreciable difference in the form of the expansion curve.

Table 22 gives all the data necessary to compute the value of k for the case of one sample of illuminating gas without air. The case of this same illuminating gas with varying ratios of $\frac{\text{air}}{\text{gas}}$ is given in Table 23. This table gives the data for the same gas with the varying combustible ratios of $\frac{\text{air}}{\text{gas}}$ such as would obtain in gas engine practice.

Table 24 gives the data and values after combustion of the different mixtures of Table 23.

The value of k for any particular gas or gaseous mixture is found as in Table 22. In general, rich gases, such as illuminat-

[1]Internal Combustion Engines.

TABLE 20

Adiabatic Values* of k for Various Initial States of Pressure and Quality

Initial Quality	Initial Pressure lb. per sq. in. abs. 20	40	60	80	100	120	140	160	180	200	220	240
1.00	1.131	1.132	1.133	1.134	1.136	1.137	1.138	1.139	1.141	1.142	1.143	1.145
0.95	1.127	1.128	1.128	1.129	1.131	1.131	1.133	1.132	1.134	1.135	1.135	1.137
0.90	1.123	1.123	1.124	1.124	1.126	1.125	1.127	1.126	1.127	1.127	1.128	1.129
0.85	1.119	1.119	1.119	1.119	1.120	1.120	1.119	1.119	1.120	1.120	1.120	1.121
0.80	1.115	1.115	1.114	1.114	1.114	1.114	1.113	1.113	1.113	1.113	1.112	1.112
0.75	1.111	1.110	1.110	1.109	1.109	1.108	1.107	1.106	1.106	1.105	1.104	1.104
0.70	1.108	1.106	1.105	1.104	1.104	1.102	1.101	1.100	1.099	1.098	1.097	1.096
0.65	1.104	1.102	1.101	1.099	1.098	1.096	1.095	1.093	1.092	1.091	1.089	1.088
0.60	1.100	1.098	1.096	1.094	1.093	1.091	1.089	1.087	1.085	1.084	1.081	1.080
0.55	1.096	1.093	1.092	1.089	1.087	1.085	1.083	1.080	1.078	1.076	1.074	1.072
0.50	1.092	1.089	1.087	1.084	1.082	1.079	1.077	1.074	1.071	1.069	1.066	1.064

*Values calculated by equation $k = 1.059 - 0.000315\ P + (0.0706 + 0.000376\ P)\ x$

TABLE 21

Adiabatic Values of k for Various Gases

Gas	Value of k
H	1.412
CH_4	1.270
CO	1.408
C_2H_4	1.210
CO_2	1.293
O	1.418
N	1.408
H_2O	1.305
Air	1.410

TABLE 22

Adiabatic Values of k for an Average German Illuminating Gas

Gases	Composition by Volume	Composition by Weight	Cp	Cv	$G \times Cp$	$G \times Cv$
H	0.485	0.0845	3.430	2.430	0.2890	0.2050
CH_4	0.350	0.4855	0.593	0.468	0.2880	0.2270
CO	0.070	0.1703	0.245	0.174	0.0416	0.0296
C_2H_4	0.045	0.1093	0.400	0.330	0.0437	0.0360
CO_2	0.020	0.0765	0.200	0.155	0.0153	0.0119
O	0.0025	0.0070	0.217	0.153	0.0015	0.0019
N	0.0275	0.0670	0.245	0.174	0.0164	0.0117
				Total	0.6955	0.5231

$$k = \frac{0.6955}{0.5231} = 1.330$$

TABLE 23

Adiabatic Values of k for Illuminating Gas of Table 22 with Various Ratios $\frac{\text{AIR}}{\text{GAS}}$

Ratio $\frac{\text{air}}{\text{gas}}$ by vol.	V	6	8	10	12
Ratio $\frac{\text{air}}{\text{gas}}$ by wt.	G	15	20	25	30
Wt. per cu. ft. of Mixture	lb.	0.0732	0.0748	0.0757	0.0769
	C_p	0.2667	0.2595	0.2556	0.2527
	C_v	0.1914	0.1858	0.1828	0.1803
	k	1.393	1.397	1.399	1.402

TABLE 24

Adiabatic Values of k for Gas Given in Table 23, After Combustion

Volume Ratio $\frac{\text{air}}{\text{gas}}$		6	8	10	12
Combustion of Burned Gases, volumes	CO_2	0.530	0.530	0.530	0.530
	H_2O	1.275	1.275	1.275	1.275
	O	0.150	0.570	1.000	1.410
	N	4.768	6.348	7.928	9.508
Constants for Burned Gases from 1 lb.	C_p	0.2787	0.2674	0.2623	0.2566
	C_v	0.2035	0.1940	0.1894	0.1851
	k	1.370	1.380	1.385	1.387

ing gas or natural gas, require very large ratios of $\frac{\text{air}}{\text{gas}}$ and therefore the resulting mixtures have values of k very near the value for pure air, 1.405. In the case of lean gases, as blast furnace gas, or producer gas, requiring small ratios of $\frac{\text{air}}{\text{gas}}$ the value of k is lower or nearer the value k for the gas alone. The limits for ordinary gases and mixtures are between the values of 1.320 and 1.405, while after combustion these values are lowered somewhat, in general about 1 to 2 per cent. The range of values for gaseous mixtures is not nearly so large as with steam.

14. *Values for Air Compressors.*—The proportion of the constituents of air is almost always constant, so that the values C_p and C_v are always constant if we are discussing them in temperatures in the range of compressor practice. This makes the ratio of k constant. The value of k for air has been determined many times by different investigators in different ways. The degree of variation in determining this constant will be seen from Table 25. Zeuner[1] after discussing all the values and the experiments supporting them, concludes that the value 1.410 most clearly fits the case of pure dry air. Zeuner also shows that in cases of water injection in compressors, used to keep down the temperature, the value of k does not change materially from 1.410. The true value seems to lie between 1.405 and 1.410.

15. *Values for Compressed Air Engines.*—The temperatures existing in the engines using compressed air for the active medium are not far below that existing in compressors. Within the range covered by both, the value of k is, as far as can be ascertained, practically constant.

16. *Values for Ammonia.*—The properties of ammonia have not been so thoroughly investigated experimentally as those of air and the vapor of water. The value of k, depending on the value of the ratios $\frac{C_p}{C_v}$, therefore is not so certain as the values for the other working media given. Various determinations place the value of k as between 1.30 and 1.333, these values being somewhat similar to those of superheated steam.

17. *Values for Illuminating Gas and CO_2.*—Illuminating gas is often compressed to send it to outlying districts in cities, and there its pressure is reduced for distribution. The value of k here depends in each case on the value of the ratio $\frac{C_p}{C_v}$ from the

[1]Technical Thermodynamics, I, p. 121.

analysis of the particular gas compressed. The analysis and values of the gas, made by the New Orleans Gas Company, New Orleans, La., which is compressed to send it to outlying districts of New Orleans, is given in Table 26.

The value of k for this gas, 1.349, is considerably higher than the value of the gas given in Table 22, 1.332. This is due to the very much larger proportion of CO in the gas of Table 26, and illustrates well the variation of this value with gases of similar heating value, but with a different proportion of the same constituents. Lummer and Pringsheim give the value of k for CO_2 as 1.2961 from their experiments. Guldner gives the value as 1.293.

TABLE 25

ADIABATIC VALUES OF k FOR AIR OBTAINED BY VARIOUS EXPERIMENTERS

Experimenter	Value
La Place	1.403
Dulong	1.421
Wullner at 32°	1.4053
" at 212°	1.4029
Clement	1.356
Masson	1.419
Hirn	1.384
Weisbach	1.402
Cazin	1.410
Rontgen	1.405
Lummer and Pringsheim	1.4015

TABLE 26

ADIABATIC VALUES OF k FOR AN ILLUMINATING GAS

I Constituents	II Per Cent Weight	III Wt. of 1 cu. ft. lb.	IV Wt. in 1 cu. ft. of This Gas, lb.	V Ratio $\frac{kp}{kv}$	VI IV x V
CO_2	2.40	0.12267	0.00295	1.293	0.00381
C_2H_4	9.00	0.07809	0.00703	1.210	0.00851
O	0.50	0.08921	0.00045	1.418	0.000635
CO	31.00	0.07807	0.02420	1.408	0.0341
H	35.20	0.00559	0.00197	1.412	0.00278
CH_4	18.50	0.04464	0.00827	1.270	0.01050
N	3.40	0.07831	0.00266	1.408	0.00375
	Total		0.04753		0.06409

$$k = \frac{0.06409}{0.04753} = 1.349$$

APPENDIX 2

DESCRIPTION OF THE PLANT

APPENDIX 2

Description of the Plant

Apparatus Used

The engine used for the tests given in Part I is of the Corliss type, and is a part of the equipment of the Mechanical Engineering Laboratory of the University of Illinois. The plant comprises the engine and various auxiliaries which are used in connection with the engine. These auxiliaries consist of a throttle valve in the steam-pipe line, an independently fired superheater, a direct current generator with a water rheostat, a surface condenser, scales, and a tank. The instruments used consist of indicators, steam pressure gauges, thermometers, ammeters, a voltmeter, and a continuous revolution counter.

The general arrangement of the plant, with the exception of the superheater and the steam piping, was the same for both saturated and superheater steam tests. Steam was obtained from the main boiler plant of the University, located about 150 ft. away from the engine. Two pipe lines, one of 6-in. pipe for saturated steam and the other of 4-in. pipe for superheated steam, traverse the laboratory, and each is connected to the steam pipe of the engine tested.

The steam exhausts from the engine through 30 ft. of 5-in. pipe to the condenser, where the exhaust steam is condensed, and is then weighed in a large tank on a platform scales. The engine is connected by a belt to the generator, which furnishes the load. This generator is loaded upon a water rheostat located close by.

1. *The Engine.*—The engine is a small well-designed Corliss engine of a standard type built for heavy duty service. Its principal dimensions are given in Table 27. A view of the right side of the engine is shown in Fig. 31, and a view of the left side is given in Fig. 32.

The cylinder is not steam-jacketed on the ends, but is partly jacketed on the barrel by the steam chest, the latter covering about one-sixth of the barrel surface. The exhaust passages are separated from the lower part of the cylinder barrel by a dead air space formed in cylinder casting. The cylinder is shown in section in Fig. 34.

The engine is fitted with separate eccentrics for actuating the exhaust and the steam valves, thus enabling the steam valve gear to cut-off up to about 50% of the length of the stroke. The two separate eccentrics and the two wrist plates are well shown in Fig. 31.

TABLE 27

PRINCIPAL DIMENSIONS OF CORLISS ENGINE

1. Type—Horizontal single-cylinder, double eccentric, non-condensing, variable speed, heavy duty frame, Reynolds Corliss Engine.
2. Class—Belt drive for mill work.
3. Maker—Allis Chalmers Co, Milwaukee, Wisconsin
4. Rated Power of Engine—100 h. p. at 115 lb. initial pressure above atmosphere on indicator diagram, ¼ cut-off, and 120 r. p. m.
5. Cylinder dimensions:— (a) Bore (measured while hot)..........12.02 in. (b) Stroke..........24.00 in. (c) Diameter of piston rod..........2 $\frac{3}{16}$ in.
6. Clearance—in per cent of volume displaced by piston per stroke (a) Head end..........7.89 per cent (b) Crank end..........7.04 per cent
7. Speed—Controlled by fly-ball governor with variable gear ratio between main shaft and governor, giving any engine speed from 20 to 160 r. p. m Usual speed 120 r. p. m.

(*a*) *Speed Control.*—The speed of the engine is controlled by a fly-ball governor acting on the cut-off cams of the steam valve gear. Variable speed is obtained by varying the gear ratio between the main shaft and the governor. This gear ratio change is accomplished by the mechanism shown to the left of the fly-wheel in Fig. 31. The mechanism is operated as follows: The belt from the main shaft drives the concave disc to the left which is loose on the shaft; this disc drives by friction three fiber rimmed idlers which are mounted between two discs on frames supported by a stationary frame in a manner which permits the plane of the idlers with respect to the shaft to be changed; the concave disc to the right is friction-driven by the three idlers and is keyed to the shaft which is connected through bevel gears to the governor. The discs are kept in contact with the idlers by an end thrust provided by a helical spring (surrounding the shaft) which is located between the right disc and the out-board bearing; the left disc is loose on the shaft but works against a collar formed by the sleeve carrying the shaft; the spring, therefore, acting against the out-board bearing, forces the right disc against the idlers and the left disc against its collar. The hand wheel shown in the figure, by a gear and sector device not shown, changes the plane of the idlers with respect to the shaft, and therefore changes the

gear ratio between the two discs. This device permits any speed from 20 to 160 r. p. m. to be obtained. A leakage test of all valves and the piston, the engine being at rest, showed that these parts were fairly tight.

2. *The Superheater.*—The superheater is of the Foster separately-fired type, and is rated at 200° F. of superheat for a flow of 4500 lb. of steam per hour. The draft is induced by an engine-driven fan.

3. *The Condenser*—The condenser is of the Worthington surface type having 362 sq. ft. of condensing surface. Two pumps are provided; one a Worthington circulating pump drawing its water from a creek about 40 ft. away, the other, a Blake wet air pump, discharging into a tank on a platform scales. Tests at frequent intervals showed that the condenser was practically without leakage.

4. *The Generator and Water Rheostat.*—The engine was loaded on a generator and water rheostat shown in Fig. 33. This generator is of the Edison bi-polar type, and is rated at 100 kw. at 140 volts, thus giving a full load current of 715 amperes. The field was separately excited for the tests, the current being obtained from the laboratory 220-volt direct current supply.

Fig. 33 shows the arrangement of the loading part of the plant. The generator output was conducted through cables in the conduit shown to the main switch on the switch-board: here connections were made to the plates of the water rheostat. Voltmeter and ammeter connections are made on the back of the switchboard. The table shown to the right of Fig. 33 contains the voltmeter and millivoltmeter for the main load current, and the field ammeter and rheostat for controlling the voltage. The water rheostat consists of three water-tight wooden barrels, each containing two iron plates, one positive and one negative. The connections of all the barrels are in parallel. Referring to Fig. 33, and describing only one barrel, a plate connected to the negative terminal is placed on the bottom of the barrel; the positive terminal is connected through the looped wire, shown to the front, to a movable plate suspended by a rope from the long shaft hung under the frame carrying the conductors. This long shaft is turned by means of the handwheel and worm device shown. The rope holding the upper plate is tied through a hole in the shaft, so that when the shaft is turned, the rope is wound up, or vice versa, as desired. The desired load as shown by the millivoltmeter is obtained by regulating the distance between the plates and by throw-

ing salt in the water to obtain the desired resistance. An overflow from the top of each barrel is provided at the back, so that when the water boils, cold water is admitted so that the heat is carried away in the overflow water instead of by the boiling of the water. All the upper plates are lowered simultaneously by means of a long shaft. An almost absolutely constant load is maintained by this rheostat.

5. *The Instruments.*—The various instruments used were carefully calibrated.

Indicators

The indicators and the indicator rig are shown in positions in Fig. 32. Two Crosby inside spring indicators in very good condition were used for the tests. The pipe connections consist of 5 inches of ½-in. pipe for each indicator. The reducing motion is a wooden pantagraph mounted as shown. The lost motion in the indicator rig was less than .01 in.

Great care was observed in the indicator work, since the indicator diagrams themselves formed the basis of the results to be obtained. The pistons were oiled at intervals of about one hour. The springs were calibrated by steam in the indicators used, by a Cooley fluid scales tester, using a method which duplicated almost exactly the conditions under which the indicators were used during the tests.

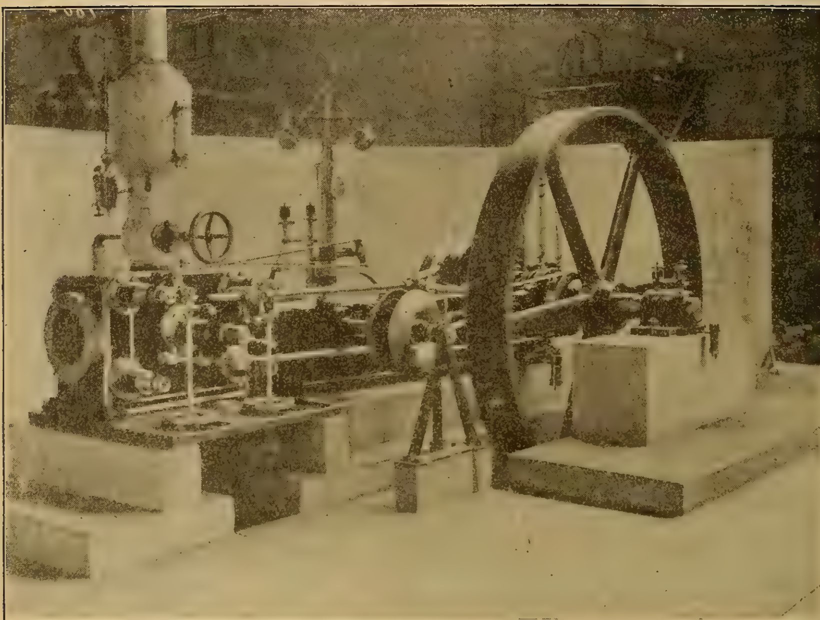

FIG. 31.

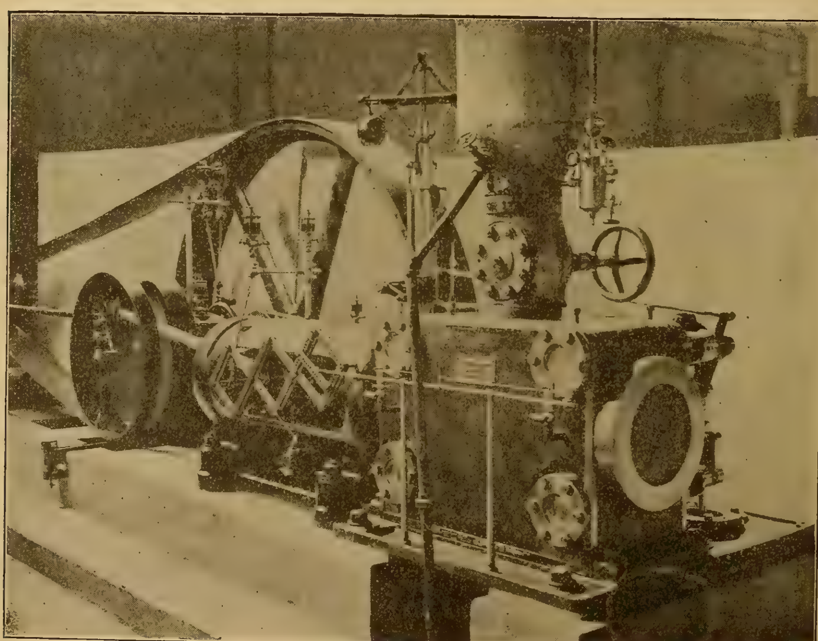

FIG.32

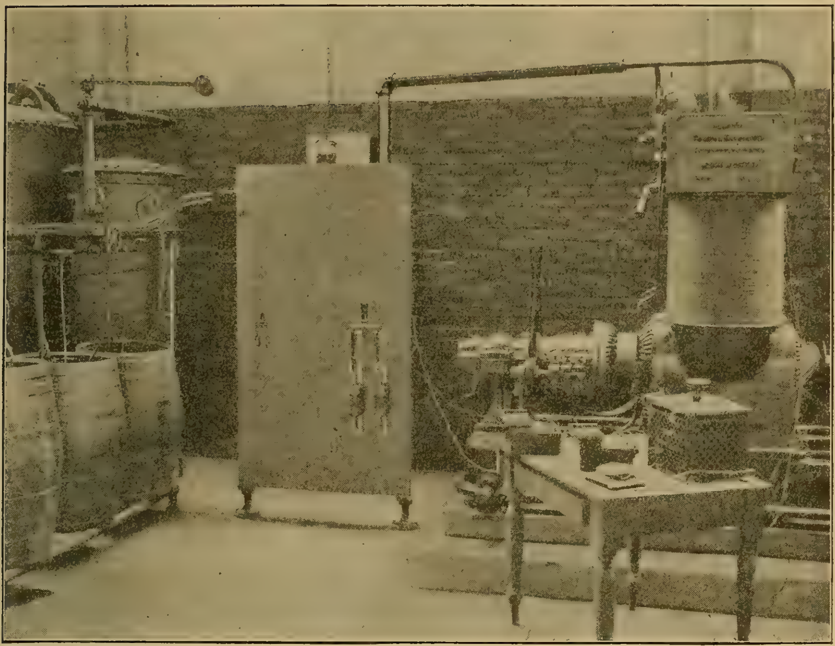

FIG. 33.

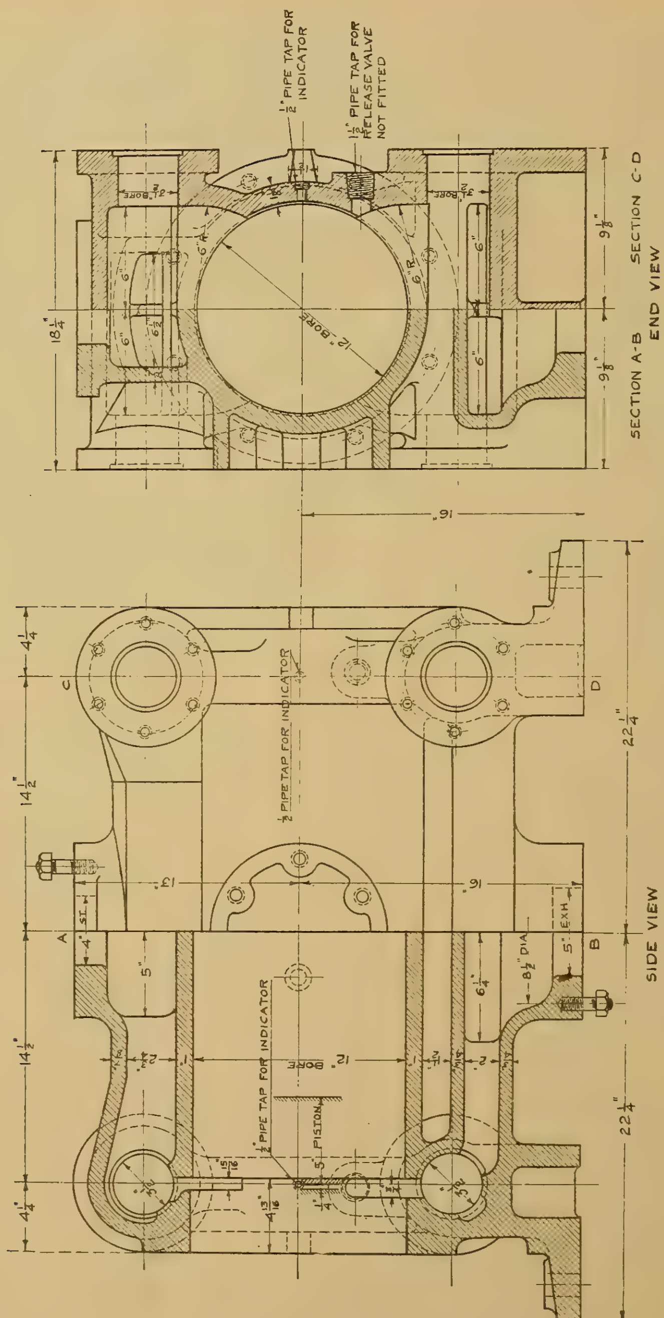

FIG. 34. SECTIONAL VIEWS OF CYLINDER OF 12-IN. X 24-IN. CORLISS ENGINE

APPENDIX 3

TEST METHODS

APPENDIX 3

Test Methods

1. *Preliminary.*—The object of the investigation given in Part 1 was to examine the values and relations of the exponent n obtained from the expansion curve of the indicator diagrams under different conditions of pressure, speed, and cut-off. The effect of changing one of these variables was studied while keeping the other two constant. The method of testing was planned therefore with a view to maintaining the conditions of pressure, speed, and cut-off as constant as was possible during any one test.

2. *Length of Tests.*—Tests may be very short when a surface condenser is used and constant conditions are maintained. Prior to running a test, the engine was operated for about one hour. It was found that after the operating conditions for one test had been maintained constant for ten minutes that the steam condensed per unit of time was almost exactly constant. After the conditions had become constant, it was found that 30 minutes of operation gave a length of test which produced trustworthy and consistent results. All observations and diagrams were taken every three minutes in response to a signal given on the even minute. Six observers were required to maintain the desired conditions of load, pressure, superheat, and to take the readings.

3. *Control of Steam Pressure and Temperature.*—The steam pressure was closely controlled by an observer, who throttled the steam in the main, before it reached the engine, to the pressure desired as shown by a gauge at the valve. Ordinarily this pressure was maintained to within three pounds of the pressure desired. It was decided to keep the steam temperature for the superheated steam tests constant at 500° F. at the superheater. The superheater was operated by an observer who was guided by the indication of a thermometer placed in the pipe carrying the steam leaving the superheater. Ordinarily, the temperature of the steam was kept within the limits of 490° to 510° F. The temperature variation at the steam chest of the engine did not exceed 2° or 3° F. during one test. The load was kept to within about 1 per cent variation from the average load during one test. A back pressure of about ½ lb. above the atmosphere was maintained at the engine by keeping the vacuum at the condenser at 1.5 in. of mercury, this vacuum being controlled by an observer who regulated an air leakage valve on the condenser.

4. *Plan of Tests.*—The testing crew became expert in control-

ling the conditions so that the variations from constant conditions were remarkably small for test work. When the conditions of one test varied for any reason more than the amounts given, this test was not used for the purposes of the investigation.

The tests were designated in the laboratory by a symbol indicating the conditions to be maintained. Thus, test 500°-100-52-120 indicated that the steam was to be superheated to 500° F. at the superheater; that 100 lb. gauge pressure was to be maintained at the engine throttle; and 52 kw. load was to be put on the generator; and that the speed was adjusted to be 120 r. p. m, measured at no load.

5. *Data for Test 52.*—All the readings to Test 52 (500° 100-52-120) are given in Table 28. A study of these readings shows the constancy of conditions that was attained.

Method of Selecting One Set of Indicator Diagrams to Represent the Average Conditions of One Test

After a test was run, the constancy of the conditions of pressure, superheat, load, the number of revolutions and the weight of the condensate for each 3-min. reading was examined. If the variations of these conditions were within the limits selected, the test was worked up in the usual manner. It was generally found that the area of each of the indicator diagrams for each end of the cylinder was within 3 per cent of the mean area. The gauge pressure for each reading at which the diagrams were taken was found in general to vary less than 3 lb. from the average. To represent the average conditions of one test, the simultaneous combination of a gauge pressure reading nearest to the average and of one set of diagrams which had an area nearest to the mean area, was sought. This combination gave one set of diagrams, taken at the average pressure, which represented the average area of all the diagrams. This mean combination condition could generally be satisfied to within ½ of 1 per cent of the average area.

The value of the average quality of the steam mixture present in the cylinder at cut-off, and the average value of n for both expansion curves were obtained from this set of diagrams (transferred to the logarithmic form) selected as representative average conditions. The unit of measurement of both quality and n was therefore the revolution.

The manner of selecting the representative diagrams is illus-

TABLE 28

Original Data (Corrected) of Test 52 (500°–100–52–120)

No. of Reading	Time of Reading	Gauge Pressure			Temperatures, Deg. F.								Area of Indicator Diagrams sq. in.		Revolution Counter		Weight of Condensate lb.		Load on Generator		Field Amperes
		In Steam Main at Throttling Valve lb. per sq. in.	Before Engine Throttle lb. per sq. in.	At Condenser inches of Mercury	Room	Steam				Water											
						In Throttling Calorimeter	After leaving Superheater	Before Engine Throttle Valve	In Steam Chest of Engine	Condensing In	Condensing Out	Condensate	Head End	Crank End	Reading	Difference per 3 min.	Gross	Difference per 3 min.	Volts	Amperes	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	2:20	99.5	97.1	1.5	66.0	380	501	453	384	42	66	73	2.87	3.17	65801		495		123	435	10.10
2	2:23	99.5	95.1	1.5	66.0	381	497	453	383	42	67	74	2.79	3.14	66109	308	601	106	124	425	10.00
3	2:26	98.5	96.1	1.5	66.0	381	499	454	384	42	67	74	2.78	3.04	66422	313	706	105	125	422	9.95
4	2:29	98.5	94.1	1.5	66.0	382	497	454	384	42	68	74	2.67	3.10	66735	313	808	102	122	415	10.00
5	2:32	99.5	96.1	1.5	66.0	383	501	454	384	43	68	74	2.69	3.01	67046	311	911	103	122	421	9.55
6	2:35	100.5	96.1	1.5	66.0	383	497	454.5	383	43	66	74	2.67	2.98	67359	313	1013	102	118	425	8.65
7	2:38	101.5	97.1	1.5	66.0	383	495	454	384	43	67	74	2.71	3.11	67669	310	1116	103	122	425	9.58
8	2:41	98.5	90.1	1.5	66.0	383	497	454	383.5	46	72	74	2.65	2.94	67983	314	1220	104	122	418	9.40
9*	2:44	99.5	94.1	1.5	66.0	381	503	454	382	52	78	75	2.70	3.05	68296	313	1325	105	123	418	9.70
10	2:47	99.5	95.1	1.5	66.0	382	493	454.5	385	54	79	77	2.72	3.09	68607	311	1428	103	123	415	10.05
11	2:50	98.5	95.1	1.5	66.0	382	497	454	386	57	80	80	2.61	2.98	68921	314	1531	103	124	405	10.00
Total	30 min.	1093.5	1046.1	16.5	726.0	4201	5477	4993.0	4222.5	506	778	823	29.86	33.61	3120	3120	1036	1036	1348	4624	106.98
Av.	3 min.	99.4	95.1	1.5	66.0	381.9	497.9	453.9	383.9	46.0	70.7	74.8	2.714	3.055		312		103.6	122.5	420.4	9.72
Per hr.															6240		2072				
Per min.																104.00					

Barometer reading 14.4 lb. per sq. in. absolute.
*Indicator diagrams selected from this reading.

trated in Table 28. The areas of the diagrams are given in columns 14 and 15. The pair taken at 2:44, the time of reading No. 9, was chosen. The steam pressure for this reading is 99.5 lb. at the throttling valve, while the average is 99.4 lb. The pressure at the engine throttle was 94.1 lb., while the average was 95.1 lb. This last pressure, 94.1 lb., was always read some time after the signal for readings, while the pressure at the throttling valve was always read at the time of the signal.

The area of the crank-end diagram of the pair selected is 2.70 sq. in., while the average is 2.714 sq. in.; the area of the head-end diagram is 3.05 sq. in., while the average is 3.055 sq. in. The crank-end diagram is 0.5 per cent lower in area than the average; the head-end diagram is 0.2 per cent lower than the average; both of them considered together are 0.35 per cent below the average. This figure, 0.35, may be taken to show the average difference, and shows that no material error is introduced by this method.

6. *Methods of Computation.*—The manner of computing the value of x_c and n is given for test 30, the representative indicator diagrams of which are given in Fig. 15, and the logarithmic diagrams in Fig. 9. The results of the computations for all tests are given in Tables 1 and 2.

The absolute pressure of cut-off was determined from the indicator diagrams at a point located by inspection, as was also the per cent of cut-off, the latter, however, not being involved in this investigation. The cut-off pressures for all the tests constituting one group were averaged and this average was used to obtain all values of x_c for one group. Test 30 belongs to group G, comprising series 6 and 14, and the average absolute pressure at cut-off was 129.0 lb.

The logarithmic diagrams of test 30, shown in Fig. 9, are the basis of the calculations for the value of x_c. The calculations which follow are given in detail in the same form as they were made for all tests.

COMPUTATION FOR THE VALUES OF x_c AND n FOR TEST 30.
(VOLUMES OBTAINED FROM FIG. 9.)

Steam present at cut-off pressure of 129.0 lb. absolute
Volume of steam present, head end ....0.482 cu. ft.
Volume of steam present, crank end............0.478 cu. ft.

Total, head and crank end ....0.960 cu. ft.

Specific volume[1] of steam at 129.0 lb. absolute = 3.478 cu. ft. per lb.

[1]Marks and Davis Steam Tables.

Weight of steam present at cut-off $\frac{0.960}{3.478} = 0.2760$ lb.

Steam regained in compression at 15.0 lb. absolute
Volume of steam present, head end............0.350 cu. ft.
Volume of steam present, crank end ....0.322 cu. ft.

Total, head and crank ends............0.672 cu. ft.
Specific volume of steam at 15.0 lb. absolute = 26.27 cu. ft. per lb.

Weight of steam retained in compression $\frac{0.672}{26.27} = 0.0256$ lb.

From Table 1, the weight of steam and water supplied = 2796 lb. per hour
Revolutions per hour..6840

Pounds of steam and water supplied per revolution $= \frac{2796}{6840} = 0.4087$ lb.

Total weight of mixture present per revolution

0.4087 lb. supplied
0.0256 lb. retained in compression

0.4343 lb. total present

$x_c = \frac{0.2760}{0.4343} = 0.635$, or 63.5% present as steam.

Value of n from point 0, Fig. 9.

Head end Length $OX = 1.575$ in.
$OY = 1.56$ in.

n, head end $= \frac{1.575}{1.56} = 1.010$

Similarly n, crank end $= \frac{1.535}{1.525} = 1.006$

Average value of n $= 1.008$

When the actual cut-off pressure was less than 129.0 lb., the line of constant weight of steam mixture on the logarithmic diagram was extended to this pressure, and the calculations made.

7. *Compression Steam.*—The point of compression was selected in the following manner from Fig. 9 for the reasons explained fully on page 49. The straight line of the compression curve on the logarithmic diagram, or the line of constant weight of steam mixture, was prolonged dotted as shown to the back pressure. In these tests the average back pressure was about 15.0 lb., and this back pressure was used to calculate all steam retained in compression. The intersection of the compression line, prolonged, with the back pressure line (15.0 lb.), extended, was taken as the volume of dry steam retained in compression. This method generally gives less steam retained than the ordinary method.

APPENDIX 4

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APPENDIX 4

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Vol. IX MAY 27, 1912 No. 29

[Entered Feb. 14, 1902, at Urbana, Ill., as second-class matter under Act of Congress of July 16, 1894]

BULLETIN NO. 59

THE EFFECTS OF COLD WEATHER UPON TRAIN RESISTANCE AND TONNAGE RATING

BY

EDWARD C. SCHMIDT

AND

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

BULLETIN No. 59 MAY, 1912

THE EFFECTS OF COLD WEATHER UPON TRAIN RESISTANCE AND TONNAGE RATING

BY EDWARD C. SCHMIDT, PROFESSOR OF RAILWAY ENGINEERING AND
F. W. MARQUIS, ASSOCIATE IN RAILWAY ENGINEERING

CONTENTS

THE EFFECTS OF COLD WEATHER UPON TRAIN RESISTANCE AND TONNAGE RATING

I. Introduction

The resistance offered by railway trains is greater in cold weather than it is under ordinary summer temperatures. Evidence of this fact will occur to all who are concerned with train operation, and its recognition has led to the practice of reducing tonnage ratings of locomotives during cold weather. This practice is almost universal among the railroads operating in the northern part of the United States and in Canada. On the few roads, running in this territory, which do not reduce ratings during cold weather, it seems probable that the ordinary summer ratings are lower than they might well be and that consequently the locomotives have a reserve tractive effort great enough to permit them to handle these same ratings throughout the winter months.

Any method of tonnage rating should recognize the three important variables which modify train resistance, viz., speed, average weight of the cars, and air temperature. The influence of speed is quite generally allowed for in establishing ratings and it is becoming more and more customary to make distinctions in rating on account of differences in car weight. The influence of the third variable, air temperature, may be as great as that of either speed or car weight, and it is proper that it should have received as general consideration in establishing winter ratings as has been accorded.

Recognizing the importance of the subject, the Railway Engineering department of the University of Illinois, two years ago, undertook tests to determine the increase in train resistance due to cold weather, and this work is still in progress. It is hoped that it may result eventually in information sufficiently specific to indicate the law according to which train resistance* and air temperature are related, and thereby to enable the reductions in rating for different air temperatures to be determined with greater certainty than is now possible; for, as will appear later, there is at present considerable diversity of practice concerning such tonnage reductions. These tests are still far from being

*Throughout the paper, train resistance means the force needed to keep the train moving at uniform speed on straight, level track and in still air. This force is expressed in pounds per ton of train weight.

completed and the data in hand do not yet warrant definite conclusions. The work has, however, gone far enough to develop some rather interesting results and it is the purpose here to present this evidence and also to present a summary of the current practice of American railroads in reducing tonnage ratings during cold weather.

The material here presented was first published in substantially the same form in the Proceedings of the Central Railway Club for January, 1912, and is reproduced by permission. The tests referred to were made possible by the courtesy of the officers of the Illinois Central Railroad.

Before presenting the experimental results, it may be helpful to examine the ways in which low air temperatures may affect tonnage rating. In establishing a rating, the purpose is to equate locomotive tractive effort and the total resistance of the train, i. e., to determine a train whose gross resistance shall equal the available tractive effort. Anything, therefore, which decreases tractive effort or which increases resistance will necessitate a reduction in rating. A drop in air temperature does both these things. Cold weather decreases tractive effort by decreasing the capacity of the locomotive boiler. This it does in two ways—first, by increasing the amount of heat lost by radiation, and second, by lowering the temperature of combustion. At low speed the reduction in boiler capacity by increased radiation probably does not amount to more than two or three per cent even in very cold weather. The decrease in combustion temperature must be so small as to be negligible in its effects on steam production. Some slight decrease in the efficiency of the performance within the cylinders probably also ensues in cold weather, but data do not exist to enable us to evaluate this effect. Cold weather further decreases tractive effort by increasing the machine friction in all the locomotive bearings. Since, however, the total machine friction is itself not generally more than eight or ten per cent, when maximum tractive effort is being developed, it is apparent that even considerable variations in this friction cannot greatly affect tractive effort. Taking all these facts into consideration, it seems likely that cold weather does not greatly reduce the tractive effort of locomotives, and that, consequently, it does not necessitate radical reductions in rating in so far as its effect upon the locomotive itself is concerned. Probably a reduction in rating of four or five per cent, even with air temperatures as low as 0°F.,

is sufficient to allow for the reduced tractive effort of the locomotive.

The influence of cold weather in increasing total train resistance is, however, greater than its influence on tractive effort. Under the conditions prevailing at ruling grades, total train resistance is made up of net resistance as above defined, together with resistances due to grade, to acceleration, and to curvature. Of these four elements of resistance only the first—the net resistance on straight level track at uniform speed—is at all affected by temperature. At the speeds at which freight trains pass ruling grades, this net resistance is composed almost entirely of those resistances which develop at the wheel tread and the resistance developed in the car journals. The former we shall call rolling resistance and the latter journal resistance. When the temperature of the air falls below the freezing point, the moisture in the road bed freezes and the whole track structure becomes less yielding. It seems reasonable to expect that under these conditions the rolling resistance will be different from what it is in summer weather. Whether it is greater or smaller does not appear, although there are some reasons for supposing it to be less on "frozen track." Whether it is greater or smaller need not concern us here, for it is altogether likely that at the speeds prevailing at ruling grades this rolling resistance is much less than the other element of resistance, viz., the journal friction. It is in journal friction, therefore, that we must seek for the explanation of the effect which cold weather is known to produce upon train resistance and consequently upon tonnage rating.

A brief review of the actions within the car journal may serve to make clearer the way in which temperature affects the journal resistance. In the journals of a car which has been standing, the oil film has been broken through and the journal and brass are probably in direct contact. The temperature of the oil and of all the bearing parts is the same as that of the air, and the lower this temperature, the more viscous is the oil. As the car starts and the journal turns, oil is brought up from the waste below and the film of oil begins to establish itself. Until this oil film is established over the whole journal, the friction is high and gives rise to the great starting resistances which prevail at this time. As the journal continues to turn, the oil and all bearing parts begin to warm up, due to the heat devel-

oped by the bearing friction. As the temperature increases, the viscosity of the oil diminishes and the resistance decreases. The temperature of the bearing continues to increase until the rate of heat production within the bearing equals the rate at which the heat is dissipated from the box and other parts, such as the axle. At this point, the bearing temperature becomes constant and the resistance reaches its minimum value and here remains. This dissipation of heat is accomplished by the air moving over the box and axle, and the rate of heat dissipation varies almost directly with the amount of the difference between the temperature of the bearing and the temperature of the surrounding air. To maintain a certain rate of dissipation of heat the *journal* temperature may be lower therefore in cold weather than in warm weather, the temperature of equilibrium attained by the journal is consequently lower in cold weather than in warm weather, and the minimum viscosity of the oil is greater. On these accounts we are prepared to find that the minimum resistance attained in cold weather is greater than in warm weather.

These statements are exemplified by the following record of journal temperatures obtained by the use of the University of Illinois dynamometer car:*

Test number	1094	2007
Average air temperature—degrees F	63	9
Approximate average test speed—m. p. h.	16	15
Maximum temperature attained by test car journal—degrees F	116	98

These tests differ chiefly in the air temperature. In the test on the colder day the temperature of equilibrium attained by the journal is 18° less than that attained on the warmer day.

In another series of tests in which the journal temperature was measured, the resulting average maximum journal temperatures attained during the tests were about 125°, 137° and 145° for constantly maintained speeds of 10, 20 and 30 miles per hour, respectively. In this case the temperature is derived from a journal of one of the cars in the test train. This car weighed 101,000 lb. and was equipped with 5 in. by 9 in. journals. The air temperature during these tests varied from 62° to 90°. These values serve to show the temperatures attained in a heavily loaded journal and also to show the influence of speed on these temperatures.

*This car is equipped with a recording thermometer, the bulb of which is inserted in a hole drilled in the body of one of the journal brasses. This instrument makes a continuous record of journal temperature. The car weighs 58,000 lb., and is equipped with 4¼ in. by 8 in. journals

II. Results of Experiments

Two years ago the Railway Engineering department of the University of Illinois completed a series of tests* to determine the influence of car weight on resistance. These tests were intended to show only the resistance prevailing in summer weather in order that they might serve as a basis for normal or "summer ratings." Tests made during cold weather have therefore been eliminated from the results. These results are here presented as Fig. 1 merely to offer a basis of comparison. In deriving the curves of Fig. 1 it was necessary to produce for each test such a curve as is shown in Fig. 2 in which the relation between speed and resistance is indicated. Fig. 2 shows the values for resistance at various speeds for test 1027 made in July, 1908, during which the air temperature varied from 64° to 80°. In such a diagram a definite relation between resistance and speed is obvious and no difficulty was experienced in drawing a curve to represent fairly this relation. In these respects Fig. 2 is quite characteristic of the entire series of 32 tests which led to the conclusions embodied in Fig. 1. It should be borne in mind that these tests were made in warm weather.

As the tests progressed, however, and cold weather was encountered, the plotted values of resistance and speed exhibited no such obvious relation. Fig. 3 shows the resistances at various speeds obtained from test 1041, the first test made in cold weather. This test was made on October 31, 1908 and the air temperature varied from 30° at the beginning to 42° at the end of the test. If there is a definite relation between resistance and speed for this test, Fig. 3 certainly does not disclose it and it would require considerable hardihood to try to draw a curve for the points there shown. The attempt to discover a reason for the discordance among resistance values disclosed in Fig. 3 by a comparison of the conditions prevailing in test 1041 with the conditions of the preceding tests, made it clear that this test differed chiefly in being run during cold weather. The explanation was sought in this fact.

In Fig. 3 the resistance values, for speeds in the neighborhood of 15 miles per hour, vary from 8.9 to 12.6 lb. per ton, and similar variations occur at other speeds. If cold weather causes these variations, it does so through its influence on journal temperature. It was conceived, therefore, that the variations were

*Freight Train Resistance: Bulletin 43 of the Engineering Experiment Station of the University of Illinois.

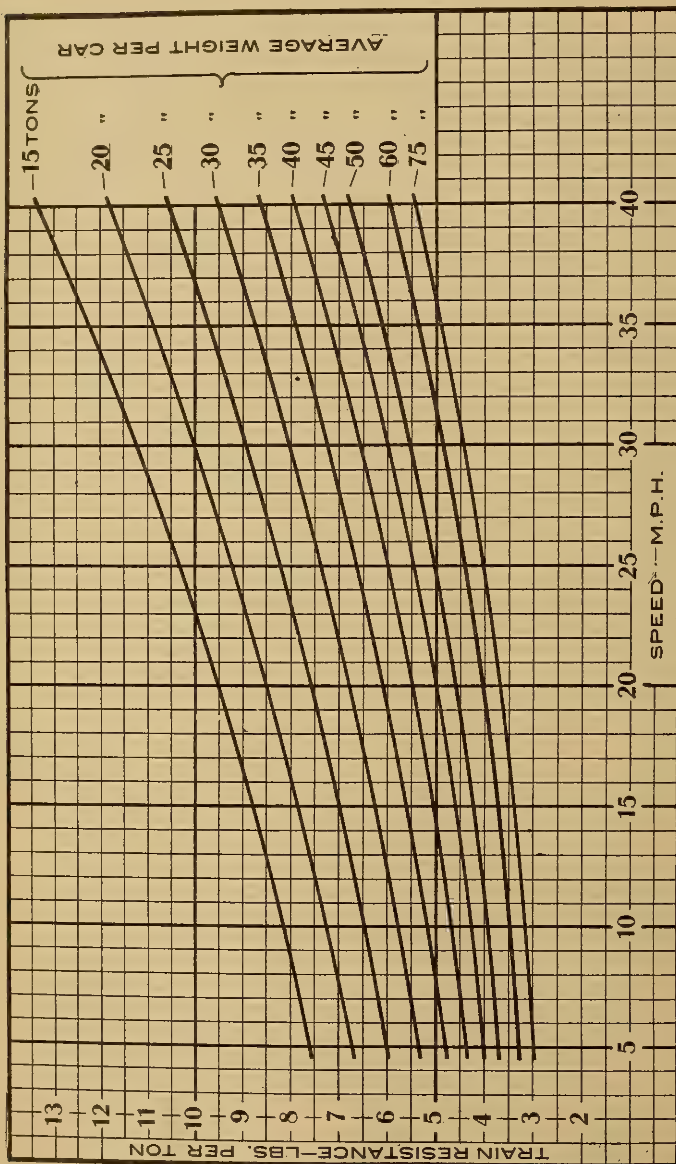

FIG. 1. SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR TRAINS COMPOSED OF CARS OF VARIOUS AVERAGE WEIGHTS—APPLICABLE IN SUMMER TEMPERATURES ONLY

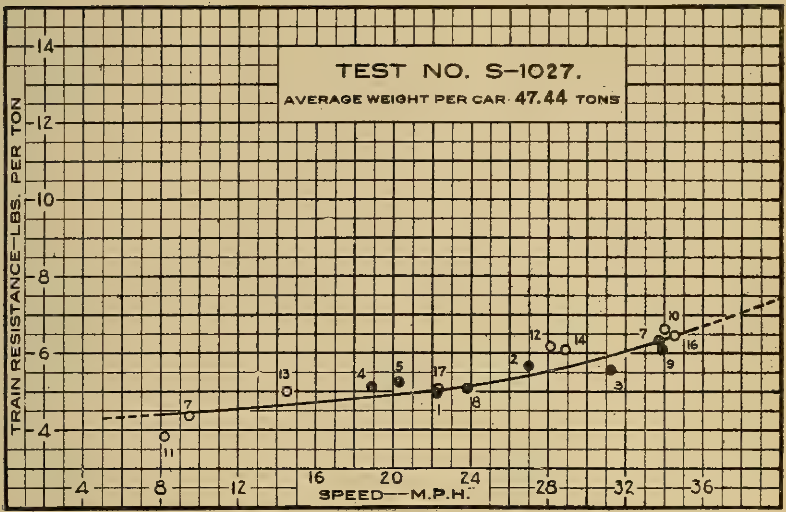

FIG. 2. SHOWING THE RELATION BETWEEN RESISTANCE AND SPEED FOR TEST 1027—AIR TEMPERATURE VARIED FROM 64° AT THE BEGINNING TO 80° AT THE END OF THE TEST

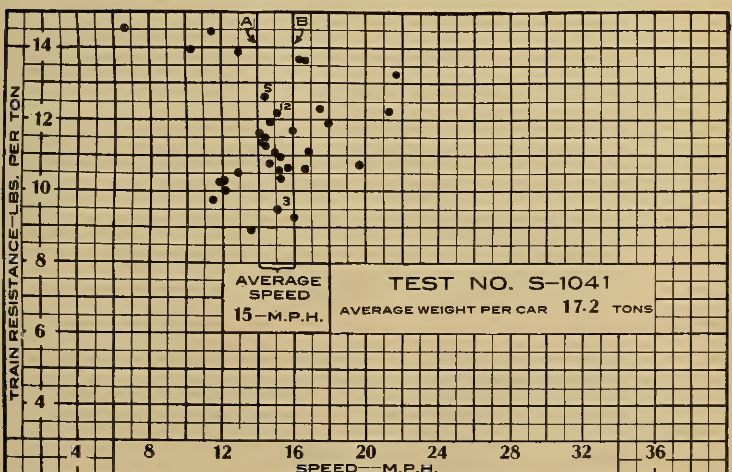

FIG. 3. SHOWING THE RELATION BETWEEN RESISTANCE AND SPEED FOR TEST 1041—AIR TEMPERATURE VARIED FROM 30° AT THE BEGINNING TO 42° AT THE END OF THE TEST

due to differences in journal temperature, and that these differences were, in their turn, due to the fact that most of the points* in Fig. 3 applied to the period during which the journals were warming up. In other words, it was assumed that cold weather had unusually delayed in this test the time at which the temperature of equilibrium of the journal became established.

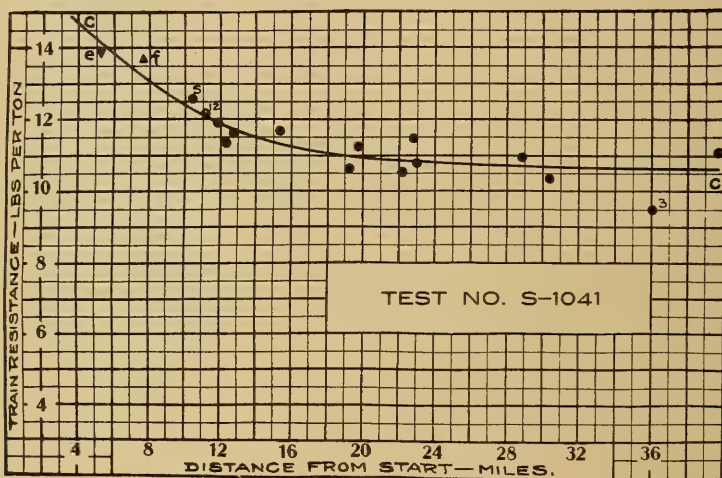

FIG. 4. SHOWING THE DECREASE IN RESISTANCE AS THE TRAIN OF TEST 1041 PROGRESSES. THE RESISTANCES APPLY TO A SPEED OF ABOUT 15 MILES PER HOUR

If these assumptions are correct, it might be expected that a diagram showing the resistance values and the corresponding journal temperatures would disclose their intimate relation. No record of journal temperature was available at this time and it was consequently impossible to produce such a diagram. If, however, the journal temperature was varying, it must have been increasing as the train moved further and further from the starting point, and it was concluded that if, for each point on the road at which resistance had been determined, its value were plotted with respect to the distance of that point from the beginning of the run, such a plot would reveal a regular variation of resistance with distance due to the influence of distance upon

*Each point in this and succeeding diagrams represents the resistance value applying to a particular position of the train upon the road. It may define the momentary resistance as the train passes a particular point, or it may define the average resistance during the time the train passes a short track section.

journal temperature. It was hoped that such a diagram would offer an explanation of the discordance among the values shown in Fig. 3. This assumption was tested in the following manner:

On Fig. 3 the two lines A and B were drawn embracing all points whose speed values lie between 14 and 16 miles per hour. All points lying within the belt defined by lines A and B pertain, therefore, to speeds which are not far from 15 miles per hour. The resistance values of the points lying within this zone were next plotted as Fig. 4, in which vertical distances again denote resistance, and horizontal distances represent miles run from the starting point. The points which in zone AB of Fig. 3 have the highest resistance values are found in Fig. 4 near the beginning of the run, whereas those having the lowest resistance values fall at the end of the run, and in general the points* in

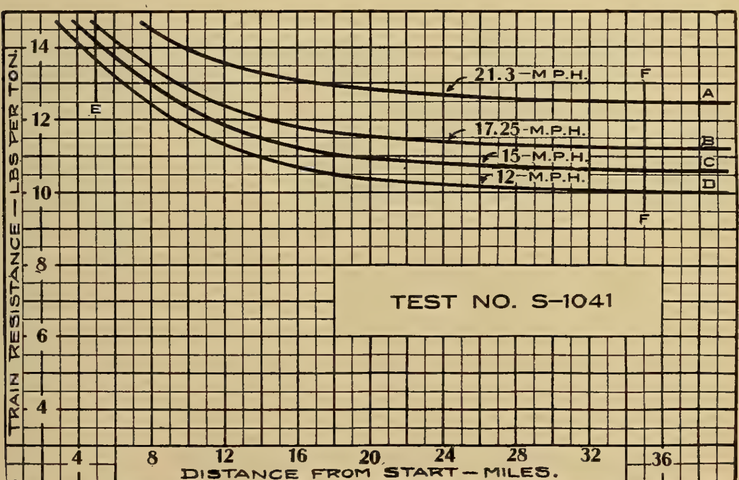

FIG. 5. SHOWING THE DECREASE IN RESISTANCE AS THE TRAIN OF TEST 1041 PROGRESSES—THE CURVES APPLY TO FOUR DIFFERENT SPEEDS

Fig 4 so arranged themselves that their resistance values constantly decrease as the distance increases. A few of the points in Fig. 4 are numbered. These numbers in Fig. 3 denote the corresponding points. Bearing in mind the facts that all values apply to the same train and to approximately the same speed, it

*Points e and f in Fig. 4 do not lie within zone AB; but, since they correspond to speeds not far from 15 miles per hour, (13.0 and 16.7 m.p.h. respectively) they are there plotted in order to fill out the exhibit for the first 10 miles.

is apparent that neither variations in car weight nor variations in speed can account for the regular decrease in resistance shown in Fig. 4. This decrease must therefore be due to the only remaining variable which exerts any important influence upon resistance, viz., journal temperature. It is therefore assumed that in Fig. 4 the regular decrease in resistance as the train progresses is due to the fact that the journal temperatures are constantly increasing and that oil viscosity and journal resistance are therefore diminishing. This assumption underlies the further discussion of this and of the following figures. The test conditions* make it difficult to directly measure the temperature of the journals in the train and the results are therefore presented in terms of distance run by the test train. The journal temperatures are intimately related to this distance unless the test is run under widely varying speeds.

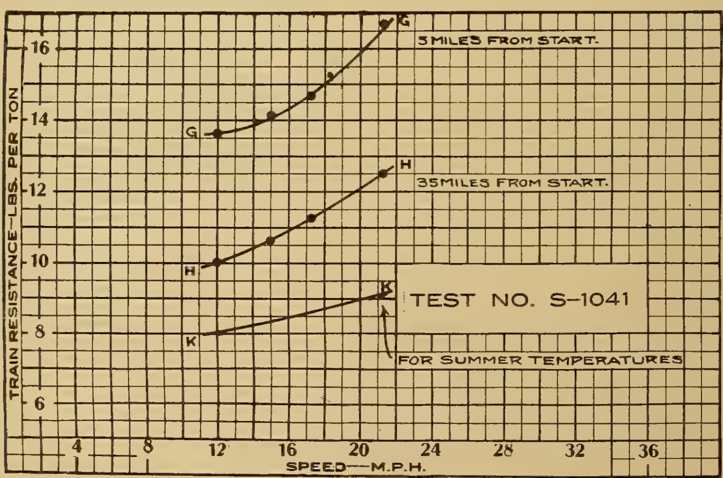

FIG. 6. SHOWING THE RELATION BETWEEN RESISTANCE AND SPEED FOR TEST 1041 AT 5 MILES AND AT 35 MILES FROM THE BEGINNING OF THE RUN

The curve CC drawn in Fig. 4 represents approximately the rate at which resistance changes with distance run, and it applies to a speed of about 15 miles per hour. It is apparent from this curve that the resistance at the beginning of the run is about 14 lb. per ton at this speed and that it constantly decreases until the train has progressed about 35 miles, at which point the

*The trains tested are freight trains of the Illinois Central Railroad, accepted as they come in the regular service. It has not proved feasible, thus far, to directly measure the temperature of any of the journals of the cars composing these trains.

resistance reaches its minimum value of about 10.5 lb. per ton and the journal temperature reaches its maximum. Trains of like character when run at similar speeds in warm weather reach their minimum resistance within the first eight or ten miles of their run, and their minimum resistance is less than that attained in the train whose performance is exhibited in Fig. 4.

Fig. 5 also shows for this same test (1041) the decrease in resistance as the train progresses. The curve C there shown is reproduced from Fig. 4, and the additional curves A, B and D there drawn were derived by a process similar to that just explained in the discussion of Fig. 4. The curves A, B, C and D denote the approximate resistance for speeds of 21.3, 17.3, 15 and 12 miles per hour, respectively. All four curves show that the minimum resistance is reached at about 35 miles from the beginning of the run. For widely different speeds this distance would be different. Fig. 5 offers a means whereby we may determine the relation between resistance and speed at different points in the run. This relation is found in the following manner. On Fig. 5 the lines EE and FF are drawn, intersecting all four curves at points corresponding respectively to 5 and 35 miles from the start. The points at which the line FF cuts curves A, B, C and D determine four resistance values which correspond respectively to speeds 21.3, 17.3, 15 and 12 miles per hour. These corresponding values of resistance and speed constitute the co-ordinates of the four points shown on the curve H of Fig. 6 and serve to define this curve. The values corresponding to the intersections of the line EE with the four curves of Fig. 5 serve similarly to define the curve G of Fig. 6. In Fig. 6 vertical distances represent resistance and horizontal distances represent speed, the curves G, and H represent, therefore, the variations of resistance with speed for test 1041, and present train resistance curves in their usual form. The curve G shows the resistance at about 5 miles from the beginning of the run, when the journals are still cold. The curve H shows the resistance at 35 miles from the beginning when the journals have attained their maximum temperature. Fig. 6 presents the same information as is embodied in Fig. 5; but in a more familiar form. It should be recalled that Fig. 6 applies to a test made when the air temperature varied from 30° to 42° and when the wind was light. The train was composed of cars weighing, on the average, 17.2 tons.

In order to compare the resistance shown by curves G and H

with the resistance prevailing in warm weather, the curve K is drawn in Fig. 6. This curve is derived from Fig. 1 and shows the approximate resistance in summer weather of a train composed of cars weighing 17.2 tons. Curve K is therefore comparable with either G or H. Curve H represents resistances which are approximately 25 to 30 per cent greater than those represented by curve K and we may conclude that for the train of test 1041 even the minimum resistance attained after the train had run 35 miles is about 25 per cent greater than the resistance of a similar train in warm weather. Whether the change from summer temperatures to a temperature of 30° will always result in an increase of 25 per cent in the resistance does not appear, and the data in hand do not as yet warrant generalizations of this sort. Attention is again called to the fact that the term resistance as here used means resistance on level track, and consequently a difference of 25 per cent in resistance does not necessarily require a difference of 25 per cent in tonnage rating. This statement is developed beyond.

Fig. 3 to 6 constitute what is perhaps unneeded evidence of the effect of low air temperature upon train resistance and upon tonnage rating. They show a way in which quantitative expression may be given to this effect. These four figures also serve to show the methods employed in the study of this problem which is now in progress at the University of Illinois. When this work has progressed far enough to cover all ordinary ranges of air temperature and all ordinary car weights, it may result in information which will enable tonnage reductions to be determined more systematically than seems now to be possible. It may be of interest to present a few additional diagrams touching one or two other phases of the subject.

Fig. 7 and 8 are similar to Fig. 3 and 4 and they lead to similar conclusions. Fig. 7 and 8 apply to test 1045 during which the air temperature varied from 22° to 26°, and for which the train was composed of cars averaging 49.2 tons in weight. In Fig. 7 the resistance values are plotted with respect to speed and the same difficulty of discovering any relation between resistance and speed presents itself as was presented in Fig. 3. When, however, these resistance values are plotted with respect to distance from the beginning of the run, as they are in Fig. 8, they arrange themselves in a more orderly way. All the points of Fig. 7 are plotted in Fig. 8. Points 8, 9 and

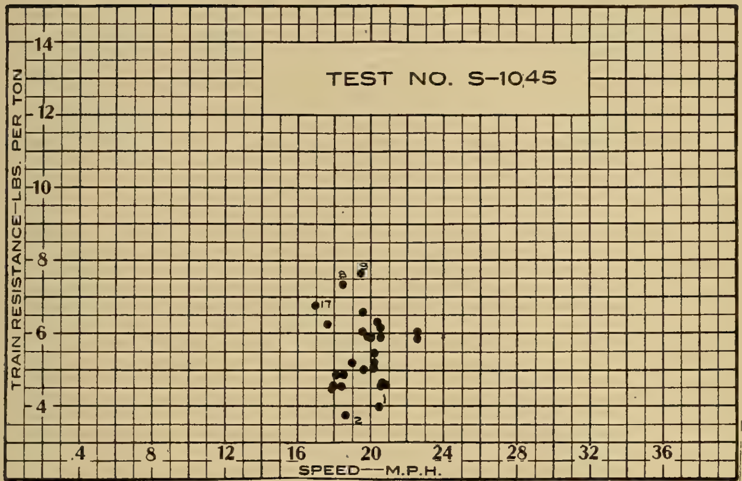

FIG. 7. SHOWING THE RELATION BETWEEN RESISTANCE AND SPEED FOR TEST 1045—AIR TEMPERATURE VARIED FROM 22° AT THE BEGINNING TO 26° AT THE END OF THE TEST

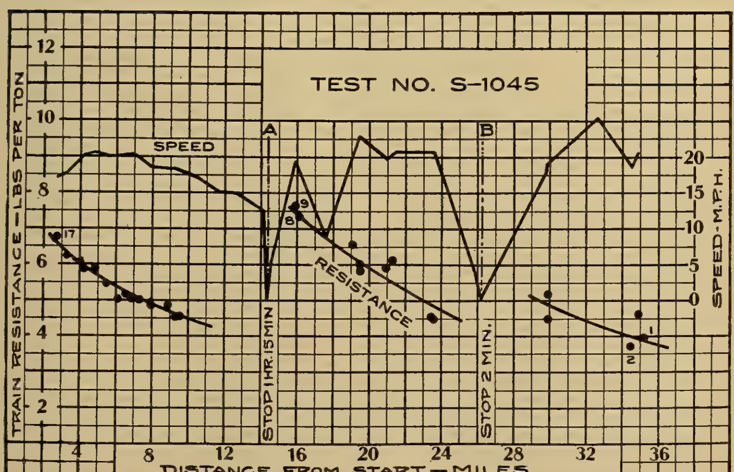

FIG. 8. SHOWING THE DECREASE IN RESISTANCE AS THE TRAIN OF TEST 1045 PROGRESSES, AND THE INFLUENCE OF STOPS UPON RESISTANCE

17, which in Fig. 7 correspond to the highest values of resistance, fall in Fig. 8 either at the beginning of the run or immediately beyond the stop at A; whereas points 1 and 2 which have the lowest resistance values fall at the end of the run. In Fig. 8 the speed during the run is indicated by the line in the upper part of the diagram. It will be noticed that the train was brought up to a speed of about 20 miles per hour within the first three miles of the run and that the speed was thereafter maintained at approximately 20 miles, except in the immediate neighborhood of the two stops which are indicated at A and B. The speed for all but four of the points for which resistance is calculated lies between 18 and 21 miles per hour, and the speeds for these four points lie close to this range. Since the speed is so nearly uniform for all these points, its influence in modifying resistance is practically eliminated, and such changes in resistance as are indicated on the diagram are chiefly due to changes in journal temperature. At point A about 14 miles from the start the train was stopped for one hour and fifteen minutes, the air temperature at that time being 23°. Again at B, 12 miles farther along, a stop of two minutes' duration was made. It is interesting to note the effect of these stops upon the resistance. The resistance in the beginning is in the neighborhood of seven pounds per ton. It steadily decreases as the train progresses, until at the point A, where the train was first detained, it had fallen to about four or four and one-half pounds. Upon leaving A the train's speed was immediately raised to its general value of 20 miles per hour and the resistance is found to have risen again to about the same value which it had at the original starting point. From there on, the resistance again decreases steadily until the point B is reached, after which there is a slight increase in resistance due, probably, not so much to the two-minute stop as to the cooling of the journals during the considerably longer period in which the speed at this point was low, while the train was approaching and leaving B. The diagram serves well to show how important the effect of such a stop as that at A may be. If the ruling grade in this case had occurred just beyond A, it is entirely likely that the increased train resistance would have stalled the train at that point. There is no evidence in this diagram that the minimum resistance of this train at this speed is reached during the test. Indeed, if the resistance curves there drawn are accepted as correct, it seems clear that

the stops have delayed the establishment of this minimum resistance beyond the test limits. Comparison with resistance in warm weather is therefore unwarranted.

During test 1084, the results of which are presented in Fig. 9, the air temperature varied from 1° below zero at the beginning to 5° above zero at the end of the test. These temperatures are lower than any others prevailing during the tests here discussed, and Fig. 9 is introduced primarily on that account. It exhibits the same facts as may be derived from Fig. 4 and 8 and needs but little additional comment. The speed during this test was increased from 8 miles per hour near the start to 20 miles per hour at the point A, beyond which it was maintained almost uniform at 20 miles per hour. The resistance values derived for the first 10 miles of the run are not numerous enough to offer much information. Between A and B, however, the points are more numerous and indicate clearly the usual decrease in resistance as the train progresses. The resistance, which at A is 20 pounds per ton, has decreased at B—24 miles from the start—to about 16 pounds per ton, and probably it would have continued to decrease had the test been continued beyond this point. The normal resistance in summer weather for a train of this car weight (16.5 tons), as derived from Fig. 1, is 9.5 pounds per ton. The train of this test has, therefore, a resistance 68 per cent in excess

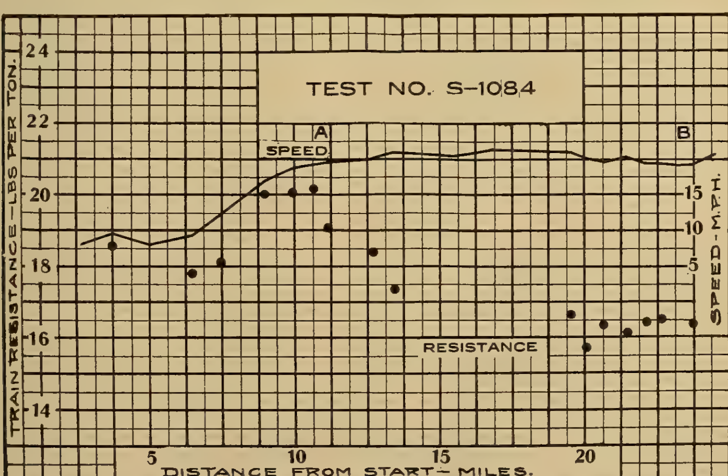

FIG. 9. SHOWING THE DECREASE IN RESISTANCE AS THE TRAIN OF TEST 1084 PROGRESSES—AIR TEMPERATURE VARIED FROM 1° BELOW TO 5° ABOVE ZERO

of the normal, even after running 24 miles. Part of this excess is doubtless due to the fact that a strong wind prevailed during the test; most of it, however, is probably due to low temperature.

Fig. 10 and 11 apply to test 1086, during which there were very light winds and the air temperature varied between 28° and 30°. The test train was composed of cars whose weights averaged 59.5 tons. Fig. 10 shows the resistance values plotted with respect to speed, and it differs from similar preceding diagrams only in that the speeds vary throughout a greater range. This figure will be used, as have the others, to show the influence of journal temperature; but before doing so it may be of interest to show how plausibly this exhibit might be so construed as to lead to wrong conclusions.

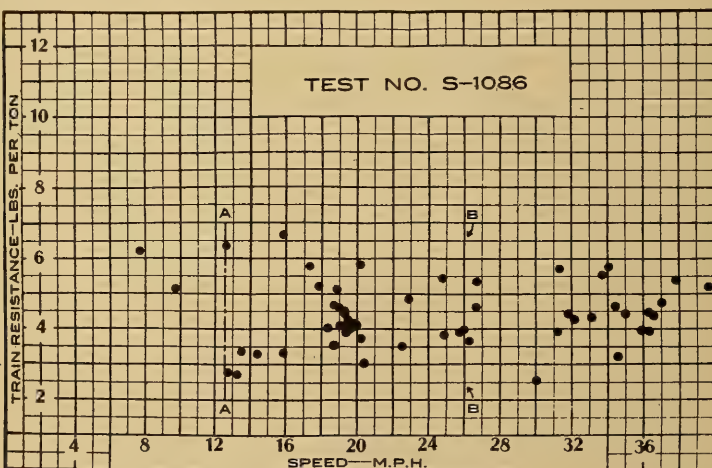

FIG. 10. SHOWING RESISTANCE AND SPEED FOR TEST 1086—AIR TEMPERATURE VARIED FROM 28° TO 30°

The diagram exhibits considerable variations in the resistance values, even at like speeds. Let it be assumed, however, that speed is the only important influence at work in causing this variation. Presupposing that no record of wind velocity is available it might seem justifiable to ascribe much of this variation to the variations in wind resistance, and also to occasional changes in such elements of resistance as flange friction. Making such allowances, the discordance among points in the diagram might appear no greater than should be expected. Such considerations might easily lead to the belief that the diagram does actually represent the true relation between average train resist-

ance and speed, and it would therefore appear justifiable to represent this relation by a line drawn among the points of Fig. 10. An attempt to thus express the assumed relation would probably result in a horizontal straight line lying at a height corresponding to about 4.5 pounds per ton. Such a process would consequently lead to the conclusion that for this train the resistance is the same for all speeds up to about 40 miles per hour; that is, that train resistance is independent of speed. It is obvious, however, from what has preceded, that in causing the variations in resistance shown in Fig. 10, the journal temperature plays at least as important a part as the speed. Fig. 11 will make it clear that there is no warrant for the above conclusion in this case.

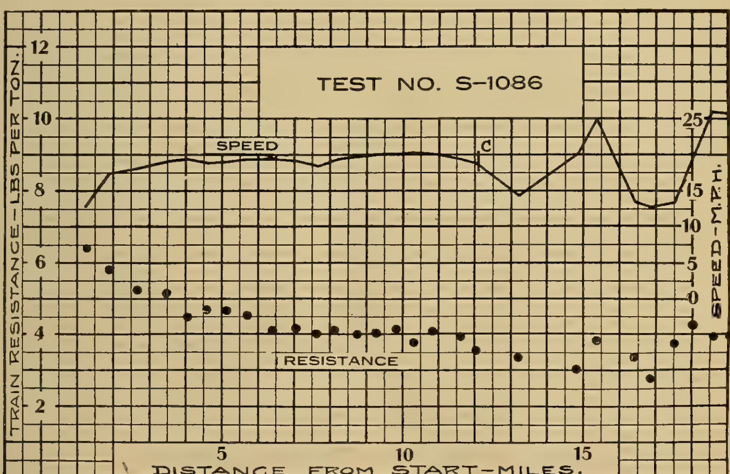

FIG. 11. SHOWING THE VARIATION OF RESISTANCE WITH DISTANCE AND WITH SPEED FOR TEST 1086

Fig. 11 comprises the points which in Fig. 10 lie between the lines A and B, corresponding to speed limits of 12.5 and 26 miles per hour. B is chosen at this point merely to reduce the length of Fig. 11. The resistance value for each point lying within this zone is plotted in Fig. 11 with respect to the distance of this point from the start. The upper line in Fig. 11 again represents the speed, which was quite uniform and near 20 miles per hour for the first 12 miles of the run—up to the point C. Beyond C the speed varied considerably. As in the tests previously discussed, the resistance decreases with great regularity during the first 12

miles, until at C the journals have apparently attained their maximum temperature for a speed of 20 miles per hour, and the resistance has reached its minimum value for this speed. Beyond C, therefore, the influence of journal temperature upon resistance largely disappears and the resistance thereafter responds, in its variation, quite definitely to changes in speed.

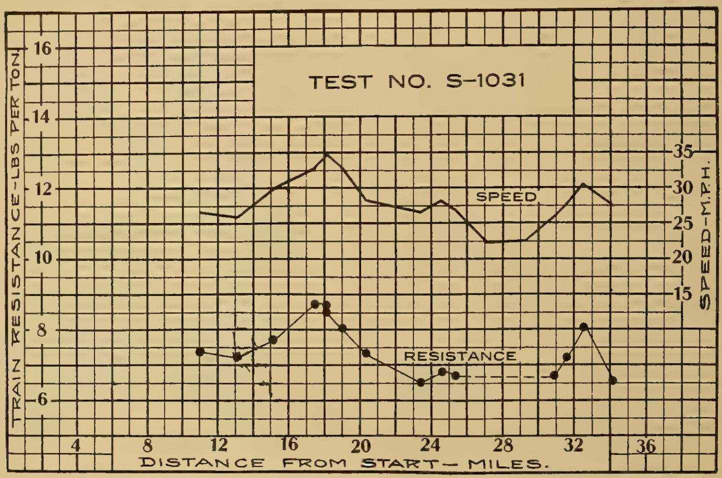

FIG. 12. SHOWING THE INTIMATE RELATION OF RESISTANCE AND SPEED AFTER THE JOURNALS HAVE BECOME WARM

Reference was made above to the definite response made by resistance to changes in speed, after the journals have become thoroughly warmed up. Fig. 12 is introduced to further illustrate this. It applies to test 1031, which was made in warm weather and which was selected on this account. During test 1031 the air temperature varied between 70° at the start and 82° at the end of the test. Within the first eleven miles the journals had assumed their maximum temperature and the record is presented only for that portion of the run lying beyond this point. As before, the upper line in Fig. 12 represents speed and the lower line represents resistance. The diagram reveals the intimate relation which exists between resistance and speed when the journals are warm. Every change in speed is closely followed by a corresponding change in resistance.

III. The Effect of Grade on Tonnage Reductions

It was stated above that an increase in net train resistance, of say 30 per cent, due to low temperature, does not necessarily require a like reduction in train tonnage. This is due to the facts that net train resistance, which here denotes merely the resistance on level track, is not the only resistance which absorbs the tractive effort, and that the other resistances are unaffected by temperature.

The process of rating locomotives consists essentially in specifying a train whose gross resistance shall equal the available tractive effort. Since ratings are made to meet the conditions which exist at the ruling grades, this gross resistance must always consist of net resistance, as above defined, and of grade resistance.* Of these two elements the grade resistance is almost always the greater. Obviously neither air temperature nor any other external condition can affect the grade resistance, which is modified only by difference in grade. Since the larger element of gross resistance remains unaffected, the reductions in rating in cold weather need not be as great as the variations which cold weather causes in the smaller element of gross resistance; that is, in the net train resistance. Neither are these tonnage reductions the same for different grades. An example may serve to make this clearer.

Let us assume that it be required to find the summer and winter ratings for a certain class of locomotives on two divisions of a road, which we here designate as division A and division B. The ruling grade on A is one-half per cent, and that on B is one per cent. The resistance due to grade alone is 20 pounds per ton of train weight for each per cent of grade, and the grade resistance on division A is therefore 10 pounds per ton while on division B it is 20 pounds per ton. Now assume also that the net train resistance for the desired speed is 4.5 pounds per ton in summer and that in winter it is 33⅓ per cent greater, namely, 6.0 pounds per ton. We assume further that in summer the available tractive effort on grade A for the class of engines under consideration is 32,000 lb. and on grade B 30,500 lb. If the effect of cold weather upon the engine itself be assumed such as to cause a reduction of

*Acceleration and curve resistance may also be components of this gross resistance. They are, however, ignored here, since their consideration is not necessary to the argument, although their presence may modify its conclusion.

five per cent in tractive effort, we find the available tractive effort in winter to be 30,400 lb. on division A and 28,970 lb. on division B. We have now available enough information to calculate the tonnage ratings. On division A, for example, the gross resistance in summer is 10+4.5=14.5 lb. per ton, the tractive effort is 32,000 lb. and the tonnage is consequently 32,000÷14.5=2,207 tons. The winter tonnage on division A is 30,400÷(10+6.0)=1,900 tons. The proper winter tonnage on division A is found therefore to be (2207—1,900)÷2207=14 per cent less than the summer tonnage. Similarly for division B the tonnage reduction for winter weather is found to be 10 per cent. The results of these calculations are summarized in the following table:

		Division A	Division B
Ruling Grade—per cent		½	1
Tractive Effort in Summer,	pounds	32,000	30,500
Tractive Effort in Winter,	pounds	30,400	28,970
Grade Resistance,	pounds per ton	10	20
Net Resistance in Summer,	" " "	4.5	4.5
Net Resistance in Winter,	" " "	6.0	6.0
Gross Resistance in Summer,	" " "	14.5	24.5
Gross Resistance in Winter,	" " "	16.0	26.0
Tonnage in Summer,	tons	2,207	1,245
Tonnage in Winter,	"	1,900	1,115
Tonnage Reduction,	per cent	14	10

It is apparent from these calculations that an increase in net resistance of 33⅓ per cent necessitates a reduction in rating on division A of only 14 per cent and on division B this reduction need be only 10 per cent. Not only are the tonnage reductions in both cases considerably less than the difference in net resistance, but the reductions are different on the different grades. The greater grade requires the smaller tonnage reduction. If the ruling grades on a particular road do not differ greatly on the different divisions, it would be an unnecessary refinement of practice to discriminate between divisions in establishing tonnage reductions for winter weather. If, on the other hand, a road runs in both level and mountainous country, it is not only logical but economical to make such distinctions. The information at hand concerning current practice indicates that these facts have received little consideration, or at any rate, no application in the establishment of certain existing tonnage rating systems. On other roads, however, the facts are duly recognized and embodied in their rating practice.

There are a few roads operating almost exclusively in mountain territory which find it unnecessary to make reductions in rating for low temperatures. The ruling grades on these

TONNAGE RATING—SUMMARY OF CURRENT PRACTICE

Railroad	Weather and Temperature Range for Normal Rating	Tonnage Reductions for Various Weather and Temperature Conditions—Per cents of Normal Ratings								Additional Reduction for Unusual Weather Conditions
		Temperature Range	Tonnage Reduction Per cent	Temperature Range	Tonnage Reduction Per cent	Temperature Range	Tonnage Reduction Per cent	Temperature Range	Tonnage Reduction Per cent	
1	2	3	4	5	6	7	8	9	10	11
N. Y. Central and Hudson River	Above 50°	More or Less Discretionary. For 20° Drop it would be about 10%								Governed by Special Orders
Chicago Great Western	Above 45° light winds	45° to 25°	8	25° to 0°	16	0° and below	25			Heavy winds—8% otherwise governed by Special Orders
Chicago and Eastern Illinois	Above 45°	45° to 32°	10	32° to 15°	20	15° to 0°	30	0° to 10° below	35	Not Stated
Chesapeake and Ohio	Above 45°	45° to 35°	5	35° to 25°	10	25° to 15°	15	15° to 5°	20	
		5° to 0°	25	0° and 5° below 0	30					Governed by Special Orders
Boston and Albany	Above 40°	Summer to winter rating	50 to 150 tons	Excessive cold	Governed by special orders					Governed by Special Orders
Pennsylvania	Above 40°	40° to 20°	5 to 7%	20° to 10°	15	Below 10°	30			Governed by Special Orders
Northern Pacific. Lines East of Paradise Montana	Above 40°	40° to 20°	10	20° to 0°	20	Below 0°	35			Governed by Special Orders
Central of New Jersey	Above 40°	40° to 20°	10	20° to 0°	20					Governed by Special Orders
Pittsburg and Lake Erie	Above 40°	40° to 20°	210 tons	20° to 10°	420 tons	Zero weather	640 to 860 tons			Governed by Special Orders
Baltimore and Ohio	Above 35°	35° to 20°	B	20° to 0°	C	0° and below	D	B, C and D adjustments vary from 0 to 30%		Governed by Special Orders
Minneapolis, St. Paul and Sault St. Marie	Above 35°	20° to 10°	100 tons	10° to 0°	220 tons	0° to 10° below	300 tons	10° below and lower	400 tons	Governed by Special Orders
Erie	Above 33°	33° to 23°	5	23° to 13°	10	13° to 3°	15	Below 3°	20	Near Freezing. 0 to 5% Red
Missouri Pacific	Above 32°	32° to 0°	15							Governed by Special Orders
Missouri Kansas and Texas	Above 32°	Below 32°	10							Governed by Special Orders
Wabash	Above 32°	Governed by Special Orders								Governed by Special Orders
Delaware Lackawanna and Western	Above 32°	Below 32°	10	Extreme Conditions	15					
Louisville and Nashville	Above 32°	32° to 0°	10 to 20							Snow 4" and above 10 to 20 Per cent. None for wind
Chicago Burlington and Quincy	Above 30° and little wind	30° to 0° no wind	10 to 15	Below 0° no wind	15% to 20%	30° to 0° strong Head or Side wind	15 to 20	Below 0° heavy wind	20 to 30	Strong Head or Side wind 10 to 15% Same for frosty rail
Chicago Milwaukee and St. Paul	Above 30°	30° to 20	5	20° to 10°	10	10° Above 0° to 10° Below	20	10° Below to 20° below	30	Governed by Special Orders
Chicago, St Paul, Minneapolis and Omaha	Above 30°	30° to 10°	10	10° Above to 10° Below 0°	15	10° Below and lower	25			Governed by Special Orders
Denver and Rio Grande Utah Lines	Above 30°	30° to 0°	15	Below 0°	25					Governed by Special Orders
Norfolk and Western	Above 30°	At 30°	50 to 100 tons	30° to 20°	100 to 150 tons	20° to 10°	200 to 250 tons	10° to 0°	300 to 350 tons	Governed by Special Orders
Lehigh Valley	Above 25°	About 5% Reduction for Each 10° Drop in Temperature.								Governed by Special Orders
Duluth and Iron Range	Above 25°	25° to 0°	10	0° to 10° Below	20	More than 10° Below	25			Governed by Special Orders
Great Northern	Above 25°	25° to 5° Frosty or Wet	10	5° Above 0° to 10° Below	20	10° Below and Colder	25			Governed by Special Orders
Chicago Milwaukee and Puget Sound	Above 25°	25° to 10°	25 to 50	Reductions Vary on Different Parts of Line						Governed by Special Orders
Chicago and North Western	Above 25°	25° to 0°	10 to 25							Governed by Special Orders
Mobile and Ohio	Above 15°	15° to 0°	5 to 10	0° or Below	10 to 15	These Reductions Depend Upon Length of Train				About 15 Per cent During Heavy Snow Storm
Canadian Pacific	Above 15°	15° to 10°	5	10° to 5°	6	5° to 0°	6 to 7	0° to 5° Below	6 to 10	Bad Rail 5 Per cent
		5° to 10° Below 0°	8 to 12	10° to 15° Below 0°	10 to 14	15° to 20° Below 0°	12 to 16	20° to 25° Below 0°	14 to 17	
		25° to 30° Below 0°	10 to 20	More than 30° below 0° with Storm—25 Per cent						
Boston and Maine	Above 15°	15° to 0°	7 to 15	0° to 15° Below 0°	12 to 18	15° Below to 30° Below	17 to 22			Bad Rail 7 to 15 Per cent
Illinois Central	Above 10° light wind No snow	Governed by Special Orders								Governed by Special Orders
Buffalo, Rochester and Pittsburg	Above 5°	Governed by Special Orders								Governed by Special Orders
Intercolonial of Canada	Above 0°	Below 0°	100 tons							Governed by Special Orders
Grand Trunk Pacific	Above 0°	0° to 5° Below 0°	5	5° Below to 10° below 0°	10	10° Below to 15° below 0°	15	15° Below and under	20	Governed by Special Orders
Central Vermont	Above 0°	Reduction of 150 Tons for Each 8°								Governed by Special Orders
Rock Island	Above 0°	Below 0° no wind	20	Below 0° Heavy Wind	25 to 30					Stormy Weather and Bad Rail—10 Per cent
Denver and Rio Grande, Colorado Lines	Above 0°	0° to 10° or 15° Below	10	10° or 15° to 30° Below	20	At Times only Engine is Run to keep Line Open				At Times 50 Per cent Governed by Special Orders
Colorado and Southern	Above 0°	0° to 25° Below	10 to 25							Governed by Special Orders
Big Four	Above 0°	Matter Left to Division Officials and Governed by Special Orders								Governed by Special Orders
Toledo, St Louis and Western	Above 0° No Storms	20° and Below with Storm	10 to 30	A 10 Per cent Reduction Account Severe Cold Might Be Increased to 30 Per cent if Accompanied By Heavy Snowfall and High Wind.						Governed by Special Orders
Grand Trunk	Matter Left to Operating Official in Charge									
New York, New Haven and Hartford	Matter Left to Division Officials									
Pere Marquette	Matter Left to Division Officials Largely. Bad Storms and Very Cold Weather 20 to 30 Per cent									
New York Central Lines	Matter Left to Division Officials									
Union Pacific	Matter Left to Division Officials									
Michigan Central	Matter Left to Division Officials A, B, C, and D Ratings, A=100%, B=90%, C=80%, D=70%									Governed by Special Orders
Northern Pacific, west of Paradise Montana	Matter Left to Division Officials									
Atchison Topeka and Santa Fe. (Eastern Lines)	Matter Left to Division Officials									Governed by Special Orders
Atchison Topeka and Santa Fe. (Coast Lines)	No Reductions Except in Albuquerque Division Where a Reduction of 100 Tons is Made in Winter									Governed by Special Orders
Southern Pacific*	No Reductions									Governed by Special Orders
Oregon Short Line	No Reductions									
Kansas City Southern	No Reductions									Severe Storm and Cold Reduce 5, 10 or 15 Per cent by Special Orders
El Paso and Southwestern	No Reductions									
St. Louis and San Francisco	Matter Left To Sub-Division Officials									
Southern	No Reductions									Governed by Special Orders
Seaboard Air Line	No Reductions									Governed by Special Orders
Atlantic Coast Line	No Reductions									
Virginian	No Reductions									

roads are, of course, heavy. The foregoing example illustrates how the effect of heavy grades may disguise and almost nullify great variations in net resistance, and it offers, therefore, some justification for the practice which ignores distinctions between summer and winter ratings in such territory.

These facts serve also to show the necessity for care in adopting on one road the practice which has proved satisfactory on another. Unless the ruling grades are nearly alike, the system of tonnage reductions which has proved itself satisfactory on one road ought not to be transplanted to another without due consideration of these facts, even though the weather conditions are identical.

IV. A Summary of Current Practice

In connection with this investigation, a considerable amount of information has been collected from the railroads of the country concerning their tonnage rating practice. The attempt has been made to summarize this information and present it in the table which is here included, in the expectation that it would be useful to have such information assembled in somewhat compact form. It is believed that the table fairly represents, in most cases, the practice of the various roads as stated by their own officers. It has, however, been difficult occasionally to force into the form and limits of the table all the information available, and in a few cases the tabular statement scarcely represents all the facts. It is difficult, for example, to present in tabular form the limitations placed by some roads upon the application of their general practice. It is hardly possible to indicate in the table the degree of authority given on these roads to trainmasters and dispatchers, under which they may vary from the usual practice.

The roads are arranged in the table in the order of the air temperature limits which determine the normal rating and which appear in the first column. This arrangement was adopted because it makes easier the direct comparison of figures appearing in the later columns. At the same time, it brings together roads which operate in very different territory and under very different weather conditions and these facts should be borne in mind in making comparisons. Examination of the table makes it evident at once that there is great diversity of practice. Not only are different tonnage reductions made for similar temperatures,

but the range in temperature which is considered to warrant tonnage reduction varies from a few degrees to 30 or 40 degrees. Some roads have only one rating schedule in addition to their normal schedule; others operating under weather conditions not radically different have as many as ten additional schedules. Most of the differences in practice are, however, not surprising when it is considered that the roads included, represent practically the entire United States and Canada and represent, therefore, the greatest variety in weather conditions and topography. Most of the differences in practice disclosed by the table are quite sufficiently explained by the differences in the weather conditions prevailing in the territory served.

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BULLETIN NO. 60

THE COKING OF COAL AT LOW TEMPERATURES

(WITH A PRELIMINARY STUDY OF THE BY-PRODUCTS)

BY

S. W. PARR

AND

H. L. OLIN

UNIVERSITY OF ILLINOIS

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

BULLETIN NO. 60 JUNE 1912

THE COKING OF COAL AT LOW TEMPERATURES

(WITH A PRELIMINARY STUDY OF THE BY-PRODUCTS)

BY S. W. PARR, PROFESSOR OF APPLIED CHEMISTRY, AND H. L. OLIN, RESEARCH FELLOW, DEPARTMENT OF CHEMISTRY

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THE COKING OF COAL AT LOW TEMPERATURES

I. Introduction

1. *Purpose of the Investigation.*—The investigations discussed in this bulletin had two general purposes in view: (1) to discover some fundamental facts pertaining to the properties and characteristics of bituminous coals; (2) to determine the feasibility of modifying the composition of raw coal in order that a different type of fuel might be produced, or possibly an alteration accomplished of the entire fuel content into forms better suited to present-day requirements.

2. *Scope of Previous Investigations.*—In earlier experiments[1] (1907-1908), the information developed was mainly of the type indicated under the first division; for example, the experiments early indicated the important role played by small amounts of oxygen in the gases surrounding the heated masses of coal. The ease with which carbonaceous matter absorbed or united with oxygen was so striking that it seemed desirable to follow the matter into detail regarding the temperatures at which oxidation takes place, and its effect upon the material in hand. As a result, the whole matter of coal oxidation at low temperatures was opened up as one of extreme importance. One fundamental fact brought out in the study[2] was the absorbent power of freshly-mined coal for oxygen, and the part oxygen played in producing certain changes in the coal and promoting the initial form of deterioration in storage. Again[3], the prime element in all the phenomena was seen to be that of oxidation. It will thus be seen that these preliminary studies on low temperature distillation, while mainly bringing into view what might be termed the scientific or fundamental properties of the material, at the same time determined facts which have had much to do with developing the practical application of

[1]The present investigations are a continuation of the work carried on in 1907-8 and presented as a preliminary report under the title of "The Modification of Illinois Coal by Low Temperaature Distillation", Bulletin No. 24, University of Illinois, Engineering Experiment Station, by S. W. Parr and C. K. Francis. 1908.

[2]"The Occluded Gases in Coal", Bulletin No. 32, University of Illinois, Engineering Experi ment Station, by S. W. Parr and Perry Barker.

[3]"The Weathering of Coal", Bulletin No. 38, University of Illinois, Engineering Experiment Station, by S. W. Parr and W. F. Wheeler; also "The Spontaneous Combustion of Coal", Bulletin No. 46, University of Illinois, Engineering Experiment Station, by S. W. Parr and F. W. Kressmann.

the information in its relation to storage and spontaneous combustion.

In the second phase of the earlier study, i. e., its industrial side as related to the development of a special type of fuel, it seemed to be established that below a certain temperature, say 700° F., the heavy hydrocarbons, those chiefly responsible for the formation of smoke, could be driven off, yielding a gas of high illuminating power, a tar with high percentage of volatile oil, and a solid which, while it could be burned without smoke, was friable and not well adapted to ordinary use as a fuel.

3. *Outline of Present Investigation.*—In the present studies, the friable or non-coking tendency of the earlier product has been found to depend directly upon the amount of oxidation that has occurred both in the preliminary exposure at ordinary temperature and in the process of heating to moderately high temperatures.

The fact that a coke of good texture could be produced when a careful exclusion of oxygen had been effected, has given special interest to the present experiments. In addition, important facts have developed in connection with the study of the various by-products. These by-products have also been more or less modified in their characteristics by the exclusion of oxygen.

Briefly outlined, the present studies have developed three lines of industrial interest.

First: The possibility of developing a smokeless fuel of good texture and admirably suited to domestic as well as to general industrial use where absence of smoke is essential. The accompanying by-products promise to be of special value. These consist of (a) Ammonia, though smaller in quantity than the yield obtained at higher temperatures; (b) Illuminating gas of high candle-power and high heat value; and (c) Tar, which is composed almost entirely of oils, with a minimum amount of pitch and free carbon. Some of the oils produced are of peculiar structure and may have more than passing interest, two of the fractions, for example, being readily oxidizable. The iodine absorption numbers of the lighter fraction are found to be as high as 165.

Second: They suggest a possible method for the manufacture of producer gas which would be free from present difficulties attending the use of bituminous coal, and would convert a

much higher per cent of the fuel into the gaseous form. In view of recent developments in the matter of combustion, efficiencies are possible[1] where gaseous fuel is available which are almost revolutionary in character.

Third: There are opened up interesting possibilities in the production of coke, briquettes or other forms of fuel in a dense and stable form to meet certain requirements of shipping, storing, foundry, and other industrial uses. Certain facts developed in these studies will be found to throw some light on the problem of coking, which is at present but little understood[2].

It is not intended here to enter into a discussion of these three main topics; they will be taken up again after the details of the experiments have been set forth. The results of the experiments may then with better understanding be made to enter into the conclusions reached.

II. Experimental Work

4. *Apparatus.*—The apparatus employed is illustrated in Fig. 1. From the high pressure main at *A*, steam was admitted to *BB*, a ⅝-in. pipe 11 ft. long, fitted with two return elbows. The steam was then heated by a 26-burner combustion furnace, *CC*. The retort D, 18 in. by 8 in., containing the coal, was fitted with a head *J* held in place with set-screws and sealed with asbestos. From the retort, the distillates were conducted by a pipe to a condenser *E* connected in turn with a large wash bottle *F*. Here the oils and tars were collected while the gases passed on to the gasometer *G*. A Hoskins nickel-nichrome thermocouple, inserted through a stuffing box *S* and joined to a millivoltmeter *K* measured the temperature of the retort contents. A battery of burners placed directly under the retort provided a means for securing additional heat, which was retained by means of an asbestos-lined oven which entirely surrounded both the retort and the furnace.

5. *Use of Superheated Steam.*—Superheated steam was used in this series of experiments as a medium for carrying the

[1]Surface Combustion. Proc. Am. Gas Inst., 1911. By Prof. W. A. Bone. In this article Prof. Bone gives data showing an efficiency in the generation of steam by use of the principle of surface combustion of 94.2 per cent. It should be noted, however, that this efficiency is based upon the net heating value of gas.

[2]"The question as to what really is the factor that produces the coking tendency characteristic of some coals has been a matter of some speculation among manufacturers and users of coke for two hundred years and we are no nearer to its solution now than were the investigators of two centuries ago."—Iron Age, 1907. F. C. Keighley.

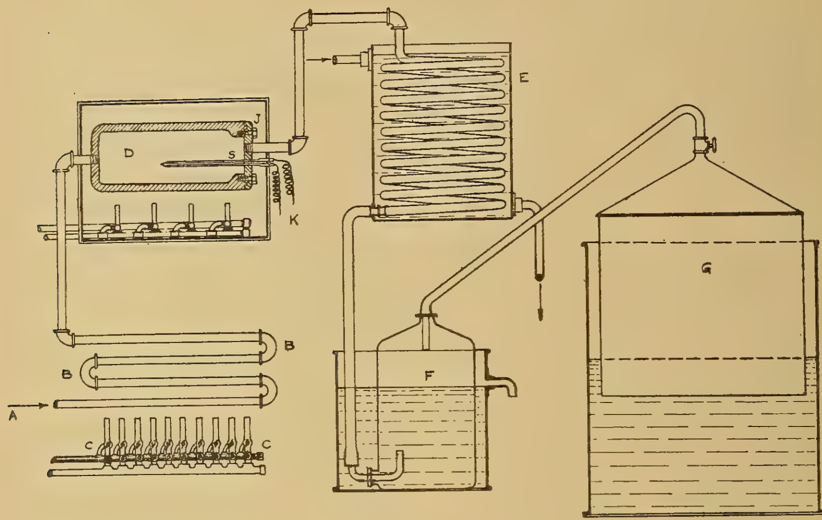

FIG. 1

heat into the coal mass, in order to distribute the heat evenly throughout the coal and thus obviate the necessity for revolving the container. In the earlier experiments (1907-1908), the carbonization was carried on in a cylinder heated externally and mounted on hollow trunions in order to make possible the turning of the receptacle, while at the same time the hollow bearings permitted the admission of various inert gases at one end and the discharge of the distillates at the other. With that device, the frequent turning over of the coal seemed to be unfavorable to the formation of coke having a homogeneous texture. Moreover, the mechanical features were not easily installed. There was positive evidence also of the activity of small quantities of oxygen, which entered by leakage or as an impurity in the circulating gas employed, thereby acting as a disturbing element. There seemed sufficient reasons, therefore, for employing a fixed retort and using superheated steam as the medium for conveying the heat and also for securing a suitable atmosphere for the distillation. As will be seen later under the discussion of the coking of coal, the use of steam in this manner has other advantages which, while not fully appreciated at first, are directly in line with the fundamental conditions upon which depends the property of coke formation.

6. *Coal Used.*—Table 1 gives the data concerning the coals used. It should be noted that since these studies were made for the

purpose of testing the coking powers of the different coals and not to determine their relative commercial values, many of the samples selected were cleaner than the general run-of-mine. The low ash and sulphur percentages result from the exclusion of pyrites.

TABLE 1

COMPOSITION OF COAL

Mines Counties—Illinois	Moisture	Ash	Volatile Matter	Fixed Carbon	Sulphur	B. t. u.
Vermilion	8.80	8.72	43.05	39.43	2.88	12673
Franklin	6.84	7.38	37.96	47.82	1.33	12770
Saline	3.93	5.80	37.86	52.41	1.54	13593
Macon	8.70	12.12	39.30	40.88	2.30	11417
Perry	7.19	10.05	35.42	47.34	.80	12153
Williamson	5.30	8.55	36 50	49.65	2.77	12640

7. *Operation.*—A quantity of coal sufficient for one run only from 2500 to 3000 grams, was crushed at one time. In the first experiments, the pieces ranged from ¼ in. to buckwheat size, the dust being removed by a sieve. At first the coal was put directly into the retort, but it was found that the circulation of the steam was retarded, delaying the heating of the mass. To remedy this, a cylindrical sheet-iron container, 6 in. in diameter, perforated with small holes, was made to hold the charge. This shell (see Fig. 2) being smaller than the retort and having a sur-

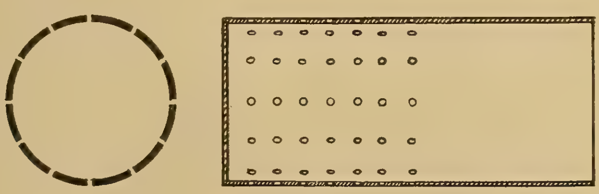

FIG. 2

rounding space of about 1 in., allowed a free distribution of heat. It was used throughout the remaining runs of the series.

Steam was admitted from the main and allowed to blow through the system until the air was entirely displaced. The combustion furnace was next started and then the burners under the retort. The coal was not stirred after heating had begun.

Table 2 exhibits the average working conditions. By improving the facilities for applying external heat to the retort, the time of the later runs was reduced to an average of about five hours.

TABLE 2

TEST CONDITIONS: FIRST SERIES

Run No.	3	4	5	6	7
Weight of coal[1], grams	4800	5351	2195	3498	3398
Weight of residue, grams	4030	4112	1895	2810	2895
Max. temp. (degrees C.)	475°	515°	450°	410°	430°
Ratio of coke	84%	76.8%	86.3%	80.3%	85.2%

[1]Nineteen runs were made in the first series, using Williamson Co. coal for the first 10 tests. In the other tests, the coal came from the following counties in the order given, Vermilion, Williamson, Franklin, Saline, Macon, Vermilion, Vermilion, Williamson, Vermilion.

8. *Distribution of Products.*—Table 3 illustrates the distribution of products.

TABLE 3

EXPERIMENT[2] NO. 11

Coal used[3]	Electric Mine, Danville, Ill.
Temperature (average)	450°
Time of distillation	5 hr.
Volatile matter in original coal not including moisture	43.00
Volatile matter in coke residue	27.95
Volatile matter in coke residue referred to original coal	22.01
Loss in weight of original coal, volatile matter only, not including moisture	20.28
Total volatile matter derived as above, not including moisture	42.29
Total material, removed by distillation including moisture	29.10

[2]Selected as a typical example.

[3]For methods of calculating percentages of coal constituents in this and succeeding tables see Bul. No. 16, p. 209, Ill. Geol. Sur.

TABLE 4

YIELD OF PRODUCTS FOR DIFFERENT PERIODS OF HEATING

Time of Heating	3 hr.	6 hr.
Coal	3000 grams	4000 grams
Coke	2327 grams	2902 grams
Per cent coke	77.50%	72.50%
Weight of tar	238.5 grams	316.0 grams
Per cent tar	7.93%	7.90%
Weight of total water	208.5 grams	348.4 grams
Per cent free moisture	3.38% or 6.93%	3.00% or 8.71%
" " water constitution	3.55%	5.71%
Volume of gas at 760 mm. and 0°	87 liters	134.7 liters
Calculated to cu. ft. per lb. of coal	.46 cu. ft.	.54 cu. ft.

From the preceding tables a fair indication is given of the ratio of distribution of the main products of decomposition. A study of these three products, gas, tar and coke, has been made, sufficient to determine their general characteristics and value.

III. Gases

9. *Analysis of the Gas.*—The methods of Hempel were used in making all gas analyses. For absorbing the illuminants, bromine water checked with the results from fuming sulphuric acid and was free from the disagreeable properties of the latter. The paraffin hydrocarbons were determined by the use of the explosion pipette. Hydrogen was determined separately with palladium sponge, a variation from the ordinary industrial method necessary when higher paraffins are present in the gas. Calorific values were determined with the Parr gas calorimeter.

It is impossible to determine absolutely the paraffin content of a gas by any methods now in general use, when more than two of the homologues of methane are present. However, by measuring the contraction of the gases and the amount of CO_2 produced in burning them in the explosion pipette, the total volume of the hydrocarbons having the general formula $C_n H_{2n+2}$ may be determined together with the average value for n. On the assumption that the higher homologues are all ethane, the percentages of methane and ethane may then be computed[1].

Several analyses of the gases obtained early in the work were made, but on account of air leakage in the gasometer, the results obtained were misleading. Table 5 shows the average of results obtained under satisfactory conditions, from gas evolved at an average temperature of 400°.

TABLE 5

Gas From Danville Electric Mine Coal

H_2S	CO_2	Illuminants	CO	H_2	C_2H_6	CH_4	N_2	B. t. u.
3.2	5.7	8.3	5.2	5.0	14.4	51.4	5.7	1032

The computed heat value of this gas was 1024 B. t. u. and agrees closely with that determined directly. Heat values of the different gases as given by Abady[2] were used as the basis of calculation.[3]

From the agreement between the observed heat value as shown by the gas calorimeter and the calculated value as derived

[1]Abady, Gas Analyst's Manual, p. 356, 1902.

[2]Gas Analyst's Manual, p. 521, 1902.

[3]According to J. H. Coste (Chemical Engineer, February, 1911) it has been found from Julius Thomsen's figures that the average calorific value of the unsaturated hydrocarbons is equivalent to that of propylene, C_3H_6.

from the constituents, indirect evidence is obtained as to the correctness of the assumption concerning the composition of hydrocarbons assumed to be present in the higher forms.

10. *Heat Value.*—It will be noted that this gas is relatively of high heating value, 1024 B. t. u. per cu. ft. Compared with ordinary city gas at 600 B. t. u. per cu. ft., this gas has a heat value about 70 per cent greater, i. e., 1 cu. ft. at 1024 B. t. u. would be equal to nearly 1.7 cu. ft. at 600 B. t. u.

11. *Sulphuretted Hydrogen.*—The gas is practically free from naphthalene but has a considerable content of H_2S. The latter feature is unexpected, since the temperature of decomposition of FeS_2 is 1000° C. and above. Doubtless, therefore, the sulphuretted hydrogen present is in the main due to the breaking down of the organic sulphur. It seems to be entirely in the form of H_2S and, therefore, easily removable by the usual methods of purification. Some of the coke residue from a coal having originally 4 per cent of sulphur was examined to see if any of the iron pyrites, FeS_2, had been broken down by the temperature employed to ferrous sulphide, FeS. Five grams were treated with a large excess of dilute hydrochloric acid. The mass was thoroughly washed and the percentage of sulphur remaining determined. Test No. 1 gave 3.55%; No. 2, under identical conditions, 3.70%. A quantity of the same residue kept well moistened was then exposed to air and sunlight for a period of twelve days in order to oxidize any FeS present to a sulphate. After washing, the sulphur content was 3.74%, indicating that FeS in the original sample was absent.[1] It is evident, therefore, that the pyritic iron had been little affected by the temperatures of the retort.

12. *Ammonia.*—Any by-product process for the carbonization of coal would, of course, take account of the nitrogen liberated in the form of NH_3. At the temperature employed in these experiments, it would not be expected that any considerable part of the nitrogen organically present would be decomposed. The following values are shown in a distillation[2] varying in temperature from 375°-400°C. In this work the entire distillate from a run of 3000 grams was retained and the total ammonia of the liquor determined. It was found to contain ammonia as NH_3 sufficient to

[1]On the subject of the decomposition of pyrite, Peters, in *Principles of Copper Smelting*, p. 268, quotes Sticht as saying "At dull red heat FeS_2 loses 3/7 of its sulphur and becomes Fe_7S_8. At 1200°, it becomes for the first time FeS".

[2]Experiments by Mr. E. C. Hull, Fellow in Chemistry, University of Illinois, Engineering Experiment Station, March, 1909.

represent a yield of 0.8 lb. per ton of coal, somewhat less than ¼ of the yield from high temperature distillation. It is not certain that the value of this product would pay for its recovery.

13. *Decomposition of Oxygen Compounds.*—The oxygen compounds upon decomposing form water. They are, therefore, often referred to as the water of constitution. They are properly considered under this division, though not forming permanent gases. It is a question of great interest whether any decomposing action in connection with the temperatures employed has taken place. If such decomposition has occurred, it has by so much enriched the fuel value of the remaining coke for the reason that these compounds are inert and noncombustible and, when present, by so much increase in effect the ash factor so far as combustion is concerned. The fact of their decomposition is shown by the increase of water content in the distillate over and above that which would normally occur from a condensation of the hygroscopic moisture alone. While this fact was not available in the case of distillation with superheated steam, the point was well established in the previous experiments[1], as also by experiments conducted by Mr. E. C. Hull, not heretofore published, in which careful measurement was kept of the amount of water distillate recovered from the coal used.[2] Thus, from the work of the latter we have the following:

TABLE 6

	3000 Grams Coal Distilled for 3 hr. Temp. 300° to 400°	4000 Grams Coal Distilled for 6 hr. Temp. 300° to 400°
Weight of water in distillate	208.5	348.4
Weight of free moisture in original coal	102.0	120 0
Excess water from decomposition of oxygen compound in coal	106.5	228.4
Per cent of water from decomposition of oxygen compound	3.55	5.71

14. *Summary of Data Concerning the Gaseous Product.*—Distillation of Illinois coals at temperatures averaging 450° C. and not exceeding 500° C. produces a gas having a heating value exceeding 1000 B. t. u. per cu. ft. The yield approximates ½ cu. ft. per lb. of coal which, at the heat value present, would represent a yield of 1.00 cu. ft. per lb. of a gas with a heat value of 500 B. t. u. per cu. ft. The ammonia yield is low, being approximately 3 lb. of ammonium sulphate per ton of coal. Decom-

[1]Bulletin No. 24, University of Illinois, Engineering Experiment Station, Parr and Francis.

[2]See also Porter and Ovitz. Bulletin No. 1, U. S. Bureau of Mines, p. 26–28.

position at this temperature extends to the oxygen compounds, which are in the main carried off and appear in the condensate instead of in the gaseous product. This feature will be referred to again under the discussion of the composition and properties of the coke residue.

IV. Tar

15. *Composition.*—As already noted, the amount of tar recovered from the distillations approximates ½ of the yield of volatile matter and in the sample noted where a direct weighing was made (Table 6), this material represents very nearly 8% by weight of the original coal. An exhaustive study of this material would be an elaborate topic for research in itself. We can, therefore, give only the general characteristics of the material as found by fractional distillation as follows:

TABLE 7

Fractions From Low Temperature Tar

Amount of tar (exclusive of water carried over)	375 grams	
Light oil (20°-100°)	39.1 "	10.5%
Fraction (b) (100°-200°)	109.1 "	29.1
" (c) (200°-240°)	111.8 "	29.8
" (d) (240°-275°)	20.6 "	5.5
Coke residue	80.0 "	21.3

From the results as given in Table 7, it will be seen that 75% of the material classed as tar is in reality oils of different specific gravities and thus of much greater value than the pitch proper. This latter product, moreover, is much smaller in amount than is produced with high temperature distillation. In the latter case over one-half of the tar is pitch, with a considerable content of free carbon suspended in the material. The low temperature product is approximately one-fifth a pitch residue with some suspended carbon present, seemingly depending on the extent to which the temperature of the coal mass has been carried above 400°.

16. *Properties of Oils.*—The further examination of the oils distilled from the tar has developed the interesting fact that these oils are readily oxidizable. As a measure of this property the iodine absorption number was determined with results as given in Table 8. It is realized, of course, that the iodine absorption

must include or represent other activities than simple oxidation especially in a complex mixture where members of the aromatic series are present.

TABLE 8

IODINE ABSORPTION OF OIL DISTILLATE

Fraction *b*, **100°–200° (29%)**	**Iodine No. 165**
Fraction *c*, **200°–240° (29.8%)**	**115–125**

Further study of the oils recovered is necessary in order to determine their specific values. Their ready oxidizability opens up a very interesting and suggestive field. For example, this feature is a marked characteristic of drying oils, turpentines, etc., used in paint mixtures. The question arises as to whether these oils will have drying qualities, i. e., will they not simply evaporate, leaving no residue, or will they oxidize in such a manner as to produce a film-covering, which will serve as a paint vehicle. Or, in a mixture with a drying oil such as linseed or similar oil, will they promote the peculiar properties of such oils which make them of value for paint mixtures? While only a few general points in this connection have been developed, they indicate characteristics of great interest and, possibly, value. It seems fair to conclude that in some measure at least the iodine absorption numbers are an indication of the avidity of the oils for oxygen. This is shown by the rapid discoloration of the oil when exposed to the air and to the fact that the lighter fraction will yield a dry film on glass at a 45° angle when exposed for 24 hr. under the usual standard requirements for such test. The second fraction has also drying properties, but the process is much slower. Or, rather, a fractionation appears to take place in which the drying oil forms a hard gelatinous film while the non-drying portion segregates into minute globules which are more or less enveloped by the films of oxidized oil. At least, it may be said of the oils which make up the element of the tar, they are available directly as fuel or for enriching or carburetting water gas. For example, if the process were continued to include the manufacture of water gas from the coke residue, the oil of the tar would doubtless enter into the reaction in the same manner as the crude petroleum now used, and thus would furnish the needed enrichment without the clogging effect which results when the attempt is made to use the raw coal directly in the manufacture of water gas.

V. Coke

17. *Yield of Coke.*—The yield of coke, under average conditions, as already noted in previous tables, is approximately 75% to 80%. This factor will, of course, vary greatly with the amount of ash originally in the coal and on the temperature at which the distillation has been carried on. These items of variation are shown in the following table where material of widely varying composition was used.

TABLE 9

Composition of Coke Residues

	Experiment No. 11 Vermilion Co.	Experiment No. 13 Franklin Co.	Experiment No. 14 Saline Co.
Moisture	.34	.40	.28
Ash	11.15	9.28	6.97
Volatile matter	27.61	26.60	23.50
Fixed Carbon	59.90	63.72	69.23
Sulphur	2.58	1.21	1.20
B. t. u.	12892	13446	13746

TABLE 10

Showing the Yield of Coke From Various Coals Referred to Original Coal—Dry Basis

	Experiment No. 11	Experiment No. 13	Experiment No. 14
Ash	9.56	7.92	6.04
Volatile matter expelled	25.48	18.00	19.12
Residual coke	78.10	84.72	84.86

18. *Reactions Involved.*—In the transformation illustrated by the change from the composition as given for the raw coal in Table 1 and the residual coke as shown by the table above, No. 9, certain facts may be deduced as follows:

First: there has been, seemingly, a decomposition of the volatile matter in a manner which would increase slightly the factor for fixed carbon. For example, if the fixed carbon be calculated as indicated in Table 11 to a percentage of ash corresponding to that of the raw coal, comparisons will be obtained as follows:

TABLE 11

Comparisons of Fixed Carbon in Original Coals and Residues, Dry Basis

	Experiment No. 11 Vermilion Co.	Experiment No. 13 Franklin Co.	Experiment No. 14 Saline Co.
Fixed carbon in original coal	43.24	51.30	54.56
Fixed carbon in coke residue referred to original ash	46.80	54.10	59.70

19. *Oxygen Removed.*—As has already been stated, the decompositions occurring at temperatures in the neighborhood of 400° C. include the liberation of oxygen, or, as it is frequently designated, the water of constitution. Since this ingredient of the raw coal is non-combustible[1], it has the same function as so much ash. Its removal, therefore, serves to make of the resulting material a richer or more concentrated fuel. This feature is still further promoted by the removal of the hygroscopic or free moisture which usually exceeds in amount the water of composition. This point may be illustrated by the accompanying table wherein the heat values per pound of the original coal are compared with the heat values per pound of the residual coke. There is also given an estimate of the amount of non-combustible material removed in the form of water in the process of decomposition.

TABLE 12

Samples	B. t. u. per lb. As Received	B. t. u. After Treatment per lb.	Gain Thermal Units	Gain per cent	Estimated Loss of Total Non-combustible Free and Combined Moisture
Williamson Co....................	12695	13150	455	3.60	10.30
Saline Co.........................	13583	13746	163	1.63	8.93
Vermilion Co.....................	12673	12892	219	1.72	13.30

20. *Properties, Porosity, Hardness, etc.*—The coke material obtained by this process varies in character somewhat with the kind of coal used, and also the amount of pressure employed during the carbonization. The Williamson Co. coal, for example, gives a coke of much finer texture and less porosity than the coal from Vermilion Co. With a view to determining the reason for this greater porosity or to finding the conditions that would modify it, the attempt was made to carry on a test with the coal sample under pressure. To this end the following apparatus was used:

21. *Apparatus.*—A, Fig. 3, is an iron cylinder, 8 in. by 4 in., fitted with screw caps B and B[1], which received the coal. The movable piston C to which is attached a long rod D, is pressed against the charge by tightening the nuts EE. The cylinder is perforated with small holes to allow the escape of gases. This contrivance was fitted into the retort originally used and heat was applied as before.

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[1]Bulletin No. 3, Illinois Geological Survey, p. 32-33.

Exhibit 1 shows the results obtained when pressure is applied slowly during the entire heating period. The outer portions passing through the temporary state of fusion soon harden and form a wall which resists external pressure. The inner core, therefore, is extremely porous. When sufficient pressure is applied, the outer part fractures and, as in this case, the residue comes out broken up into small pieces. The coke shown in the figure is from coal from Perry Co. The specific gravity of the outer portions of the mass is .733 against .652 when coked without pressure.

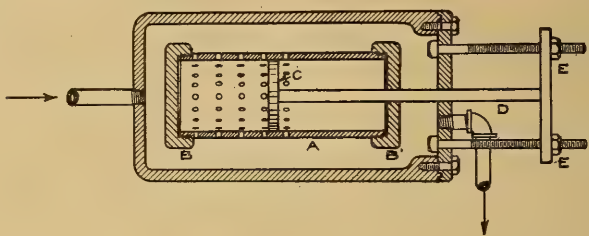

FIG. 3

It was evident, therefore, that in order to get a firm block, pressure must be constant. In the next run, the charge was rammed into the cylinder and the piston was screwed up tightly but not moved after heating had begun. The resulting column cohered well and showed the same increase in specific gravity as the one mentioned above.

22. *Illustrations of Various Products.*—An interesting feature of the product is the complete fusion of the mass, where proper conditions exist, i. e., the individual particles of coal of buck-wheat or pea size have completely lost their identity, the resulting homogeneous mass showing no lines of demarcation from the original pieces of coal. The texture, however, in some cases is finer or closer than in others. These points are well illustrated in photographs of typical masses as reproduced in exhibits 2 and 3, for coals from southern Illinois. Exhibit 4 represents a somewhat coarser texture. It was made from Vermilion Co. coal. For the composition of these samples, reference is made to Table 9, p. 14. Exhibit 2 from Saline Co. coal showed a crushing strength of 750 lb. per sq. in[1]; exhibit 3 from a Franklin

[1]John Fulton, (Coke, p. 331.) gives 1200 lb. per sq. in. as the ultimate crushing strength of standard Connellsville coke; by-product coke is, in general, considerably stronger.

The crushing strength is important in reference to the load or burden the coke can with stand in the furnace without crushing.

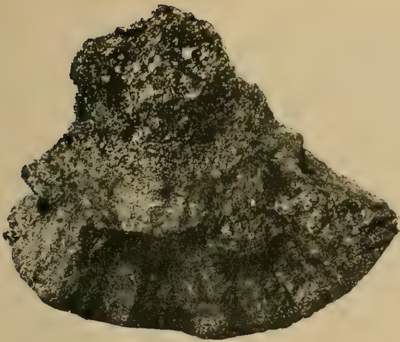
Exhibit 1


Exhibit 2


Exhibit 3

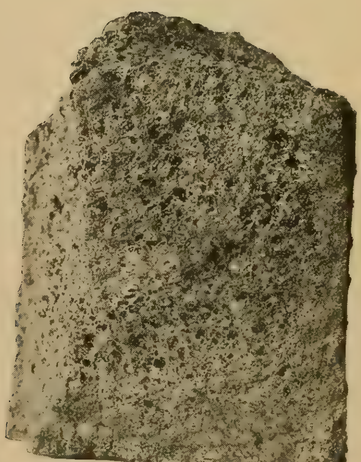
Exhibit 4

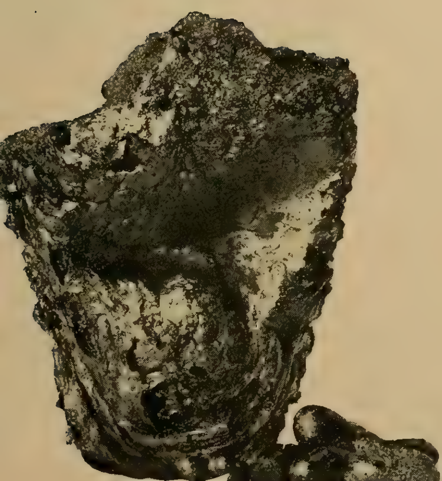
Exhibit 5

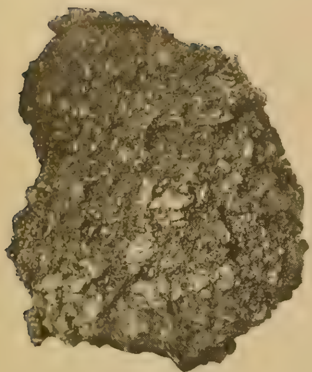

Exhibit 6

Exhibit 7

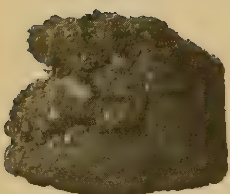

Exhibit 8

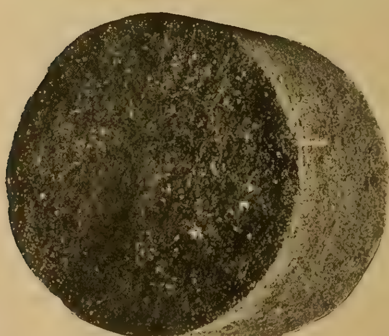

Exhibit 9

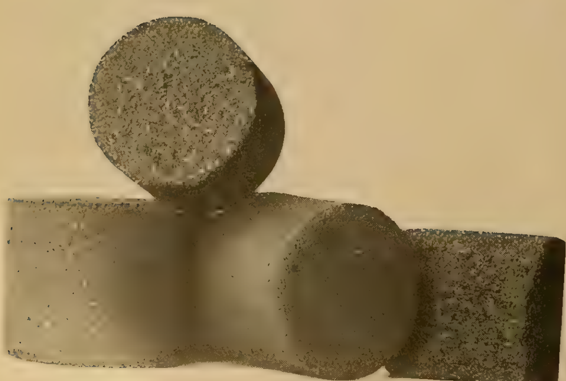

Exhibit 10

Co. sample crushed at 900 lb. On account of its coarse cellular structure, exhibit 4 showed little rigidity, and broke down at a pressure of 300 lb.

23. *Resume Relating to the Coke Product.*—It is evident upon examination of the coke product obtained, as above described, that we have here a fuel of firm texture, not readily broken down by handling and producible in the most convenient sizes for handling and for efficiency in combustion. It is, moreover, in a more concentrated form, in that for the most part, the free moisture and the water of constitution have both been removed. Thus in freshly mined coal there would be eliminated from 15 to 20 per cent of inactive material. Again, the heavy hydrocarbons have been removed. These are the constituents most directly responsible for the formation of smoke in the combustion of untreated coal. It is to be noted further that because this coke has been subjected to a temperature just approaching a red heat, it will not begin to evolve volatile matter, when thrown upon the fire, before it again comes up to or passes that temperature.

The effect of this point is twofold: first, there is obviated the cooling effect which must be necessary in the vaporization of moisture in the raw coal which also lowers the temperature just when a high temperature is needed for burning the heavy hydrocarbons; and second, the remaining gases to be evolved consist almost wholly of ethane or marsh gas (CH_4) and hydrogen, both of which are readily combustible. The hydrogen, of course, burns with a non-luminous flame and is incapable of making smoke. The marsh gas (CH_4), though it has carbon in its composition, adds but little luminosity to the flame and is almost incapable of producing smoke in the process of combustion.

It may be well to analyze briefly the processes of combustion as they occur in an ordinary hand-fired furnace. The first result of throwing a mass of coal upon a fire is to lower the temperature during the time of volatilization of the moisture in the coal. Theoretically, the temperature of the mass during this process would remain at or slightly below 100° C.

Other factors tending to lower the temperature would be the specific heat[1] of the coal and the heat necessary to effect the decomposition, since it is probable that the decomposition reactions are endothermic up to approximately 300° C[2].

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[1]Bulletin No. 46, University of Illinois Engineering Experiment Station, Parr and Kressman, p. 34.

[2]Bulletin No. 24, University of Illinois, Engineering Experiment Station. Parr and Francis, p. 46—47.

It is to be noted that during this depression of the general temperature there is being distilled from the coal such volatile substances as are liberated at these lower temperatures. This point can best be illustrated by means of the accompanying diagram. In Fig. 4, the region between the lines A and B may be assumed to include those volatile constituents that are driven off at a temperature below 400° C. This area includes the free moisture of the coal, the combined moisture or water of constitution, or as some prefer, the oxygen compounds of the coal, shown on the chart as inert volatile; and, in addition, some of the pure hydrocarbons which constitute a portion of the true volatile combustible matter. It is, moreover, the nature of this latter or volatile combustible material with which we are just now concerned in this discussion of the processes of combustion. It is to be noted first that this volatile matter contains the

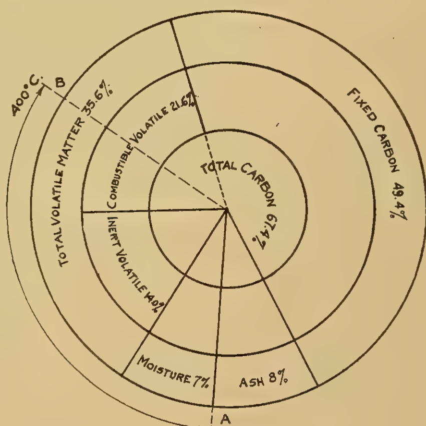

FIG. 4

bulk of the heavy hydrocarbons. By this is meant that they belong to the higher series of any of the homologous compounds present which in general are characterized by a higher percentage of carbon. For example, if the series is that of marsh gas or methane CH_4, i. e., $C_n H_{2n+2}$, then the next higher order of this series would be ethane or C_2H_6, and the next, propane or C_3H_8. The carbon percentages, respectively, being 75, 80, 81.8, etc. Again a very considerable part of the volatile matter delivered at this temperature belongs to the methylene series $C_n H_{2n}$, and the first known member of the series is ethylene or olefiant gas, C_2H_4, with a carbon percentage of 92.92. Moreover,

this last compound may be made to break down under higher heat into members of other series, as acetylene, C_4H_4, benzene, C_6H_6, and naphthalene, $C_{10}H_8$. Other members of the ethylene or paraffin series are found which ally the resulting complex mixture quite closely to the very complicated compounds with which we are familiar in petroleum.

The point to be noted in this phase of the discussion is the fact that these compounds discharged at this relatively low temperature, and having these high percentages of carbon, are the most difficult of complete combustion without the formation of smoke.

It is not necessary here to discuss the mechanics of combustion of hydrocarbons. As a result of the researches of H. B. Dixon[1] and of Professor Bone[2] the selective theory of oxygen for hydrogen or the dogma of "preferential combustion of hydrogen" has been obliged to give place to the theory of the intermediate formation of "oxygenated" or "hydroxylated" molecules. In any event, or whatever the theory finally developed by Professor Bone in his most important researches on combustion, the fact remains that these heavier hydrocarbons are the most difficult of all with which to effect complete combustion, and that even under favorable circumstances the tendency in their combustion is to form condensation products in which free carbon largely predominates. The faulty reaction is thereby made visible to the eye as smoke. A good illustration of this fact is found in acetylene gas, which requires a special burner with special provision for an extra oxygen supply in order to produce a smokeless flame.

Smokeless combustion of raw coal is secured, therefore, by observing the principles indicated above; i. e., there must be uniform and gradual accession of fresh coal and a combustion chamber maintained at a sufficiently high temperature, and the same extending over a sufficient space to permit of ultimate mixing and contact of the oxygen with the combustible gases. Other conditions such as accelerating the reaction by introducing the principle of surface combustion, as developed by Professor Bone, may at some time be added to the mechanical and physical conditions now in vogue. But while these provisions are readily adapted to large steam generating units, they are impossible of application to the larger members of combus-

[1]Phil. Trans. 1893, 159; Trans. Chem. Soc. 61. 873 (1892);
[2]Chem. News, 102, 309 (1911).

tion processes such as are common to the small plant, house heaters, and possibly to locomotives. It is these latter cases especially that demand a modified fuel which can be burned without the formation of smoke.

It will thus be seen that in the low temperature distillation of coal, processes have been put into operation which have taken out the heavy smoke-producing ingredients, and have also removed the moisture, both free and combined, which are chiefly responsible for the depression of temperatures under ordinary conditions. There is left, moreover, as volatile matter, practically these volatile substances only; methane CH_4 and hydrogen, which most easily of all the gaseous products from coal, maintain a smokeless combustion.

VI. The Formation of Coke

The experiments as thus far conducted seem to throw some light upon the matter of coke formation. In this discussion of the theories involved, it may be helpful to formulate certain hypothetical conditions which have had more or less confirmation in these studies, as follows:

First: For the formation of coke there must be present certain bodies which have a rather definite melting point.

Second: The temperature at which decomposition takes place must be above the melting point.

Third: Where the compounds that satisfy the first and second conditions are unsaturated, it is possible by subjecting them to oxidation to so lower the temperature of decomposition as to alter the second condition prescribed, in which case coking will not occur.

Discussion of Conditions.—The first condition prescribed above may be well illustrated by the behavior of sucrose or cane sugar. This substance has a rather low melting point, say, 160° C. This melting point, however, is just below the temperature of decomposition. Where this point is reached, gaseous products in the form of steam, etc., are delivered, leaving behind, if the high temperature is continued, a mass of coke. On the other hand, if starch is heated in a similar manner, it does not melt but its first action is that of decomposition. When this is carried to completion, there remains not a strongly coherent mass but easily disintegrated particles of carbon. Pure cellulose behaves in a still more striking manner, showing no fusion properties whatever as may be demonstrated by distilling in a closed

tube some cotton fiber, or other form of cellulose such as filter paper beaten to a pulp and dried. Wood, however, if not disintegrated, as in the form of sawdust, has enough resinous material closely associated with the fibrous structure to bind the carbon filaments resulting from the decomposition of the cellulose by reason of the fact that these gums, or resins, have a melting point below that of their decomposition temperatures, and thus form a binding film of carbon throughout the mass, producing a sort of coking effect which we find in charcoal.

In the case of Illinois coals, we find the first prerequisite formulated above, as present in a marked degree. As an illustration of the fact of a low melting point, reference is made to exhibit 5, which is a photograph of a mass of such material, which exuded from a sample of Vermilion Co. coal, subjected to the usual treatment as described on p. 7. The lump shown is a part of a mass that flowed out of the container, forming a bubble-like puddle. It would seem, therefore, that this type of coal from the north Danville field (Electric mine) has the first essential for coke formation in a marked degree.

As illustrating the conditions which exist where oxidation had been allowed to take place, an example is given in exhibit 6. This was made from a weathered sample of coal from Niantic. It had little if any tendency to fuse; the individual particles of coal still retain their form and the mass may be easily crumbled between the fingers. It should be noted that this result is not due to any inherent quality possessed by the original coal; a Danville mine sample, for instance, weathered to a like degree, gives the same results.

Test No. 9.—Another verification of this point, though in a more marked manner, was the result of test No. 9. The coal used was the fine material which had collected from the preparation of the previous tests, all of which had given excellent samples of coke in their freshly prepared condition. A quantity of coal passing through a 10-mesh sieve had accumulated through a period of about six weeks and had been stored in an ordinary coal hod in the grinding room. After being heated for eleven hours under conditions identical with those of the preceding runs, it showed no signs of fusion and was entirely without coking properties.

It is evident from these tests that the very great avidity of fresh coal for oxygen is evidence of the presence of those compounds which satisfy the first of the hypothetical conditions,

p. 17, and the subjection of the coal to oxidation destroys the fusion property of the fresh coal and produces a condition corresponding to that described under the third proposition, in which the coking property is lost.

Other studies on the nature of the coking process were carried out as follows: The apparent plasticity exhibited by the coal during certain stages of the treatment suggested the idea of compressing it into a briquette at the time when it would most easily yield to pressure and when it would presumably cohere without requiring an artificial binder. Accordingly, a cupel press with a pressure of 500 lb. was provided and the retort was charged with Danville mine coal. At the time of maximum evolution of gas, the heat was suddenly shut off and the retort quickly opened. It was found at this point that the outer and hotter portion of the mass was hard and unyielding. A soft inner core was discovered, however, and portions of this were put into the press. The resulting briquette is shown in exhibit 8. The escaping gases have swollen it considerably. Determinations of the amount of volatile matter possessed by the coal when in the plastic condition showed that this constituent had been reduced very little,—from 38 % to 30 %. In short, the state of fusion seems to exist in early stages of distillation but disappears before the process has proceeded far.

In one of the earlier tests with Danville mine coal using the apparatus described in Fig. 3, the extreme fusibility of this type was again demonstrated. As the piston was slowly forced in, pencils of bituminous matter were squeezed out through the holes of the cylinder. Exhibit 5 includes some of these nodules. The fact that there was a selective separation of bitumen is proved by a comparison of the ash values, the residue as a whole having 13 %, the nodules, 8 % of ash.

The readiness with which the cementing material ran to waste seemed to indicate that the coal contained a superfluous amount of it—more than was necessary for binding itself together. The correctness of this view was shown by a series of runs in which crushed gas house coke and anthracite were heated with varying amounts of bituminous coal.

Exhibit 7 shows the hard firm product resulting from the mixture of equal parts of Majestic bituminous coal and gas house coke, both crushed to 20 mesh. Fairly good results were obtained in the next run with 75 % of the coke and only 25 % of Danville Electric coal. In like manner, powdered an-

thracite and bituminous coal in ratios varying from 1 : 1 to 3 : 1 were firmly cemented together. Pitchy material no longer exuded from the retort, being absorbed, seemingly, by the added substance.

One of the essential factors in this scheme for briquetting loose infusible material with bituminous coal is the use of the press for keeping the two substances in close contact. On account of the difficulty of applying such a contrivance in industrial work, attempts were made to attain the desired end by using temporary binders, i. e., substances which might hold the particles together closely until the permanent coal binder could relieve them.

Mixtures of Danville mine coal and Danville mine coke residue No. 17 in the proportion 3 : 1 were thoroughly moistened with water and pressed (1) in the cupel machine and (2) in a testing machine up to 1000 ℔. per sq. in. Neither of the briquettes survived the subsequent heating, being disintegrated, seemingly, by the escaping steam. The same effect, though to a much less degree, was noted when coal tar was employed. The resulting briquette retained its shape, but was rather soft and friable. Crude molasses, of all the binding materials tride, proved to be the best for this purpose. Different percentages of the molasses, ranging from 5 to 15, were tested out at different times. Below ten per cent the strength of the briquette was much diminished. Exhibit 9 is a 3:1 mixture of Danville mine coke residue and fresh Danville mine coal, both ground to 20 mesh, first bound with 11 % of molasses and then pressed in the cupel machine. The cake was next heated in the retort under the atmospheric conditions of all the preceding runs. This briquette 2 in. high and 2 in. in diameter, has a crushing strength of 550 ℔. per sq. in. Exhibit 10 shows anthracite briquettes made in the same way. They have a specific gravity of 1.02 and crush at 650 ℔. per sq. in.

These tests seem to show that the fusible substance of Illinois coals is the true binding material in the coking process; that it is present in such abundance as to produce a coke of too open and spongy a character as a result of the evolution of the large amount of gaseous products which result from its decomposition. In this respect, it is paralleled by the behavior of sugar in the process of coking, which yields as a result of the large volume of escaping gases a very porous mass of sugar, coke or carbon. However, if the raw coal is mixed with a considerable

amount of material which has already gone through the coking process, or which has at least given off the larger part of its gases, and then has been reduced to a fine division like breeze, the cementing material of the fresh coal is able to disseminate throughout the mass, and the gases may also escape without blowing it into a spongy mass, with the result that a coke of good texture is formed. Exactly in a similar way, if molasses or other sucrose or glucose material be substituted for the fresh coal, we shall have again the formation of a dense coke capable of retaining its shape under conditions of firing much better than where a plastic binder is used. In both cases a strongly cohering mass is produced which meets the requirements of handling, storage, and combustion with the greatest efficiency and the least formation of smoke. A small admixture of raw coal may thus be made to serve the purpose of a binder for material otherwise wasted as coke breeze at a cost which would enable it to compete with the pitch binders now in use. It also suggests a process of fractional coking, or coking in two stages. The first result at the lower temperature furnishes a product which, when ground to a moderate degree of fineness and mixed with a small portion of fresh raw coal, would furnish the essential conditions for producing a coke of dense nature with a binder so distributed as to give the material a strength quite comparable with that produced by coals of the regular coking variety. Moreover, an advantage would be evident in such material, especially for use in household appliances, in that it would be more lively in combustion and less difficult of manipulation in the matter of maintaining a fire than coke made by the usual methods.

One point further is to be noted in this connection. It was said at the beginning of the discussion that superheated steam was employed for the purpose of conveying heat into the material so that it would not be necessary to revolve the apparatus in order to secure an even distribution of heat. It is seen from the above detail of the essential conditions to be observed in the coking of coals, at least of this class, that an atmosphere free from oxygen is of prime importance. Indeed following the indicated requirement, the coal should be fresh, or as recently mined as possible, and in any event retained in larger sizes than in a broken down or a fine state of division, in order that the least possible opportunity be given for the absorption of oxygen. Furthermore, by first admitting steam or bringing the coal into an atmosphere of

superheated steam, the effect is to drive out such oxygen as has been occluded or absorbed by the coal and as yet not chemically combined. This is also brought about at temperatures and other conditions least conducive to a reaction between oxygen and the coal substance. Moreover, from former experiments,[1] it has been shown that no reaction at these temperatures takes place between the steam itself and the coal. These principles have an important bearing on certain recent tendencies to concentrate gas production and coke manufacture in large units and distribute the gaseous products at high pressure. From the above, it would seem that the nearer such units were located to the mine or pit-mouth, the better. If it is found, as seems probable, that the coke residue is a suitable material for further continuation of the gas-making process for the manufacture of producer gas, then the above advantages and essential conditions would be magnified.

A discussion[2] by Prof. Lewes, relating to English coals, bears such a striking resemblance to the facts developed in our own work on Illinois coals that the references have especial interest in this connection. Lewes develops his theories on the basis of the existence in coals of four types of degradation products which have all come from two original forms of vegetation; viz: cellulose or lignose, and resinous bodies such as the spores of the lycopodia. The first form of vegetation, i. e., the cellulose, has produced the coal compounds of the humic and ulmic types, while the resinous bodies have produced the other three, viz: (1): resinous bodies with but little alteration; (2) isometric or other slight modifications in form rather than of composition; and (3) decomposition products from resins produced by the action of heat and pressure and consisting of a long series of both saturated and unsaturated compounds, hydrocarbons of the hydro-aromatic series, and saturated hydrocarbons, like hexane, pentane, etc.

"All these degradation products of the original vegetation are to be found in the bituminous coals, the residual body and humus forming the basis, which is luted together by the hydrocarbons and resins, and the characteristics of the various kinds of coal are dependent upon the proportions in which the four groups of the conglomerate are present. * * * * The resin bodies and hydrocarbons which form the cementing portion in the coal melt between 300° and 320°C, and if a coarsely powdered sample of the coal becomes pasty or semi-fluid at this temperature,

[1] Parr and Francis, p. 45, Bulletin No. 24, Engineering Experiment Station.

[2] A recent contribution to the theories of coke formation is made in a lecture by Vivian B. Lewes, Professor of Chemistry, Royal Naval College, Greenwich, England, published in Progressive Age, Dec. 15, 1911, page 1030.

it is a strong inference that the coal will coke on carbonization, a fact noted by Anderson, and which I have found very useful in practice as a rough test. About these temperatures also the resin bodies and hydrocarbons begin to decompose.

The resin bodies at low temperature yield saturated hydrocarbons, unsaturated, chiefly hexahydrides or naphthenes, together with some oxygenated compounds, while the hydrocarbons yield paraffins and liquid products, all these primary constituents undergoing further decompositions at slightly higher temperatures. The liquids so produced begin to distill out as tar vapors and hydrocarbon gases, and leave behind with the residuum pitch, which at 500°C forms a mass already well coked together if the residuum from the humus is not too large in quantity; the coke formed at this temperature is, however, soft, but if the heat be now raised to 1000°C, the pitch residue undergoes further decomposition, yielding gas and leaving carbon, which binds the mass into a hard coke."

He discusses further the action of oxygen upon certain of the initial constituents, referring to the investigations of Boudouard.

"Boudouard has shown that when coal is weathered humus bodies are produced and the coking power lessened or destroyed. In seven samples of various coals the humus constituents were increased by the oxidation, which seems to show that the action of the absorbed oxygen is to attack the resin compounds, and as we know that carbon dioxide and moisture are the chief products of the earlier stages of heating of masses of coal, it seems probable that the result is a conversion of resinic into humus bodies with evolution of these gases, and it is this change which leads to the serious deterioration in the gas and tar made coal which has been too long in store, while the fact that a cannel coal like Boghead or a shale do not weather is partly due to their dense structure and also is an indication that the resin bodies of which they are chiefly composed are of a different type, a fact borne out by their resistance to certain coal solvents which freely attack the ordinary resin matter."

A continuation of studies along this line is being made. Mention has been made concerning the adaptability of the coke thus produced to use in suction gas producers for furnishing fuel to gas engines. Its freedom from tar, oils, and the heavier products of distillation, which clog and render impossible the use of raw bituminous coals of this type, would seem to offer a solution of these fundamental difficulties. Further studies along this line are also being made.

VII. Summary and Conclusions

1. Coals of the Illinois type can be coked at a temperature approximately 400° or 450° C.

2. The gaseous products consist chiefly of illuminants of high candle-power, and represent, together with the condensible

material under (3) following, the chief elements involved in formation of smoke in the ordinary combustion of raw coal. The nitrogen of the coal is liberated as NH_3, at these temperatures, in amounts representing approximately 20% of the total nitrogen present.

3. The condensible distillate consists largely of oils with the minimum amount of tar and free carbon. The oils represent positive values for fuel, for carburetting water gas, or for other specific uses on account of their chemical characteristics as unsaturated compounds.

4. The coke residue has special characteristics which seem to make it of value as a concentrated fuel, capable of combustion without the formation of smoke, suitable for storing without the possibility of spontaneous combustion, and presumably adapted to the manufacture of gas for use in the suction gas producer.

5. Certain facts seem to have been developed concerning the principles involved in the formation of coke which may open the way to the production of a kind of coke of such texture and strength as to make it acceptable for uses that are not now possible with coke made from similar coal, but formed under ordinary conditions, such as are found in the ordinary gas-house retort practice, or that of the by-product coke-oven.

Other considerations[1] are pertinent in this connection, such as losses and pollution of the atmosphere which accompany the production of smoke[2].

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[1]As illustrating the present-day appreciation of matters connected with fuel economy and activity of thought concerning remedial measures, a quotation is here made from the presidential address of Sir William Ramsey before the British Association for the Advancement of Science at York, Eng., July, 1911. (Science, Vol. 34, p. 302, Sept., 1911.)

"The domestic fire problem is also one which claims our instant attention. It is best grappled with from the point of view of smoke. Although the actual thermal loss of energy in the form of smoke is small, still the presence of smoke is a sign of waste of fuel and careless stoking. In works, mechanical stokers, which insure regularity in firing and complete combustion of fuel are more and more widely replacing hand-firing. But we are still utterly wasteful in our consumption of fuel in domestic fires. These considerations would point to the conversion at the pit-mouth of the energy, using as intermediary, turbines or preferably, gas-engines; and distributing the electrical energy to where it is wanted. The use of gas engines may, if desired, be accompanied by the production of half-distilled coal, a fuel which burns nearly without smoke, and one which is suitable for domestic fires.

[2]It is not necessary to multiply arguments for the prevention of smoke. However, a recent article in the Journal of the Society of Chemical Industry, December 15, 1911, by Prof. J. B. Cohen and A. G. Ruston, contains some very striking facts developed in their study of the smoke problem. A few extracts may be given as follows:

"The average per cent of soot passing up the chimney, in 12 analyses including eight of Yorkshire coals, two of Durham coals, and two of Wigan coals, amounted to 6.5 per cent on the carbon burnt. This quantity 6.5 per cent seems a very high figure, representing an annual loss of nearly two million tons on the estimated domestic consumption of 32 million tons. The average deposit of soot over the whole of Leeds will therefore correspond to at least 220 tons per square mile per annum. The tar contained in the soot adheres so tenaciously to everything that it is not easily removed by the rain. The leaves of trees and evergreens in particular get coated with this black deposit. Unfortunately, it does more than blacken the vegetation; it covers the whole leaf over with a kind of varnish, and fills up the pores or stomata, thus checking the natural process of transpiration and assimilation. It is in fact no uncommon thing to find in the case of leaves of conifers grown in Leeds that 80 per cent of the stomata are choked up with tar."

Further studies have in mind the carrying out of the processes as indicated with apparatus involving the continuous feature, subjecting the mass at the point of greatest fusibility to the pressure of the oncoming material and producing the coke in amount sufficient for testing its properties in the gas producer and for combustion in other ways which would test its properties as a smokeless fuel.

APPENDIX

APPENDIX

I. INTRODUCTION.

HISTORICAL[1]

STUDIES IN THE LOW TEMPERATURE DISTILLATION OF COAL.

Researches in the low temperature distillation of bituminous coals have been carried on at the University of Illinois since 1902[2]. In a series of preliminary experiments, on heating coal to temperatures ranging from 250° to 500° for periods of less than an hour, it was found that the percentage of fixed carbon was increased by more than 25% and that there was a corresponding decrease in volatile matter to a point where the formation of smoke was prevented altogether.

In order to eliminate as far as possible those variables which would result from oxidation, Parr and Francis in continuing this work heated Illinois coals in non-oxidizing atmospheres. Choosing nitrogen first as the most suitable medium for this purpose, a careful study was made of the quantity and composition of the gases and heavy residues produced at different temperatures below 400° C. With a view to securing an absolutely inert atmosphere, after finding that the ordinary commercial nitrogen was contaminated with oxygen, the air in the retort was displaced by steam.

The coals heated in these media underwent changes which rendered them smokeless in ordinary combustion. However, on account of the rotary motion given the retort in order to equalize the temperatures, the coke product came out in a loose granular state much like that of the original coal.

In the course of some of the experiments, while using oxygen as the atmospheric medium, rises in the thermometer readings were observed at unexpectedly low temperatures, seemingly independent of the amount of external heat supplied. This suggested the idea of a second series of tests entirely separate from the first, to determine the temperatures at which oxidation begins. The method of procedure was to allow the retort to cool slowly

[1]In assembling the literature relating to the carbonization of coal, it has been planned to bring together first, all of the references to low temperature distillation including the studies of the by-products, followed by references to the theories concerning coke formation and the carbonization of coal in general.

[2]Ill. Geol. Surv., Bull. No. 4, by S. W. Parr, p. 97, 1906.

until a drop of say 50° had been recorded. Oxygen was then admitted and the resulting temperature was noted. A rise was considered proof of oxidation. This was repeated until a point was reached where no rise in temperature occurred on the readmission of oxygen. In this way it was found that pulverized bituminous coals in pure oxygen began to oxidize at about 125° and that they ignited at about 160°. With diluted oxygen the temperatures were somewhat higher.

While making some of their tests with atmospheres of steam it was observed that near the temperature of 315° the mercury made an abrupt rise incommensurate with the amount of external heat added. After allowing the coal to cool to 300° and then again heating up to 315° the same phenomenon was observed. No appearance of carbon dioxide accompanied this sudden rise. A tentative explanation is that it was due to the exothermic character of decompositions occurring at that stage.

In considering the subject of oxidation temperatures,[1] it was found that freshly-mined coal immediately begins to exude hydrocarbons and to absorb oxygen and that it retains its avidity for oxygen for an indefinite length of time. The exact result of this absorption was not fully determined, but it seems probable that under favorable temperature conditions it would tend to hasten combustion.

Constam and Schläpfer[2] publishing "Studies in the Gasifying of the Principal Types of Coal" report that the percentage of oxides of carbon included in the gases given off in distilling coal varies with the oxygen content of the coal itself.

R. T. Chamberlain[3], studying the causes of mine dust explosions, found that fresh coal absorbs a large quantity of oxygen but that even under a vacuum it gives off very little. He determined further that coal bottled in air for several weeks yields some carbon dioxide but an amount equivalent to only a small part of the oxygen absorbed. This he thought, might be due to the presence of unsaturated compounds in the coal, which form addition products with oxygen.

Mahler and Charion[4] found that when dry air was passed over pulverized coal at temperatures below 100°, measurable quantities of water, carbon dioxide and carbon monoxide were

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[1]Bulletin No. 32, By Parr and Barker, Engineering Experiment Station, University of Illinois. (1910).

[2]Jour. Gasbel. 49, 741. 774. (1906).

[3]Bulletin No. 383. U. S. Geol. Sur. (1909).

[4]Compt. rend. 150, 1521, 1604. (1910).

given off. Between 125° and 200° the liberation of water was so greatly accelerated as to indicate the splitting off of water of constitution. Above 150° the water contained considerable quantities of acetic acid, from 20% to 40% of the total condensate, and showed, in addition, traces of acetones, aldehydes, and methyl alcohol. The upper limit of temperature in their studies was 200°.

Porter and Ovitz[1] made an extended study of the volatile matter of coal with a view to determining the influence of the gas composition factor on the efficiency in the use of coal in various industrial processes with special reference to gas producer, coke oven and gas retort operation.

Their investigations show that the composition of the volatile matter of a coal depends largely upon the character of the coal itself. The gases from the younger coals of the West compared with those from the coals of the Appalachian region have high percentages of carbon dioxide and carbon monoxide. Because of the readiness with which these gases are given off even at comparatively low temperatures (300°-500°), the writers conclude that these western coals contain compounds having a direct carbon linkage such as the complex alcohols, aldehydes and acids. They show, further, that contrary to the theory of Dulong, who assumed that in combustion all the oxygen of a coal combined with hydrogen, in the case of certain low grade highly oxygenated coals nearly two-thirds of the oxygen appears in the volatile products in union with carbon, and that this fact accounts for the discrepancy between the determined heat value and that calculated by Dulong's method.

Higher hydrocarbons such as ethane are produced in greatest abundance from the eastern coals and they, consequently, yield more smoke in combustion. In general, however, the gas evolved from any coal subjected to moderate heat only, is rich in the higher paraffins such as ethane and propane. In the case of Connellsville coal, at furnace temperatures of 500° and 600° these higher hydrocarbons constitute about 50% of the total paraffin content. At about 800° the percentage reaches a maximum, when it rapidly falls on account of decomposition by heat.

They conclude that the nature of the volatile products distilled from coal in the early stages of heating varies in accordance with the smoke producing tendencies of that coal. They

[1]The Volatile Matter of Coal. Bull. 1, Bureau of Mines. 1910.

include among the smoke-producing constituents, tar, benzene, ethylene, and the higher homologues of methane.

E. Boernstein[1], subjecting eight Westphalian coals to a maximum temperature of 450°, reports that the gaseous products of distillation did not exhibit differences corresponding with those shown by the coals themselves. Compared with ordinary coal gas, they were characterized by a higher content of heavy hydrocarbons (5% to 14%) and of methane and its homologues (55% to 76%), and a lower content of hydrogen (5% to 16%). The tars had a specific gravity between .95 and .98, began to distill at about 70° to 80°, and were found to contain no aniline, thiophene, naphthalene, or anthracene. He states that the solid paraffin content ranged from .3% to 2% (m. pt. 55° to 60°).

Inasmuch as in modern gas retort operation portions of the coal do not reach their maximum temperature for one or two hours, the subject of low temperature distillation is of real importance to the gas industry. In a paper read before the Michigan Gas Association, White, Park and Dunkley[2], report the results of their studies of the primary reactions involved in heating American coals to 500°.

Gas evolution commences only above 300° and that given off in the 300° to 350° interval contains from 25% to 40% of ethane. Above the latter point the yield of ethane diminishes and very little is produced between 450° to 500°. The illuminants decrease with increasing temperature starting with 8% at 300° and going down to zero at 500°. Methane starting with small amounts reaches its maximum in the 400° to 450° interval. They call attention to the similarity of the gases produced at low temperature to natural gas and suggest that the latter was also produced at low temperature. They give the following results of analyses:

TABLE 13

AVERAGE YIELD AND COMPOSITION OF GAS FROM COAL HEATED FOR SIX TO EIGHT HOURS AT TEMPERATURES OF 300°-500°

Coal Volume in cu. ft. per lb. of	Pittsburgh, Penna.	Bay City, Mich.	Zeigler Ill.
Coal	1.42	1.15	0.63
CO_2	2.9	16.2	13.1
Illum	2.2	4.1	1.6
CO	6.2	5.0	5.8
H_2	26.3	16.4	13.9
CH_4	47.0	37.8	38.0
C_2H	13 2	11.8	19.5
N_2	2.7	9.1	7.8
Calculated B. t. u.	902	778	871

[1] Jour. Soc. Chem. Ind. 25-213. (1906).
[2] Am. Gas Light Jour. 89-621. (1906).

The apparent similarity between the gases evolved from coal at low temperatures and natural gas, gives interest to the work of Cady and McFarland[1] on the composition of the natural gases of Kansas. They proved the presence of paraffins heavier than methane and ethane, by condensing higher boiling hydrocarbons along with the methane in a bulb surrounded with liquid air. Some of these remained liquid up to ordinary temperatures and had an odor similar to that of light boiling petroleum distillates. The quantity of this residue varied in the different gases.

Professor V. B. Lewes[2] in discussing the relative merits of high and low temperatures for gas distillation, gives parallel tables showing the net cost of 1000 cu. ft. of gas produced by each of the two processes.

TABLE 14

COST OF 1000 CU. FT. OF GAS

	(1) High (900°)		(2) Low (400°)	
		pence		pence
Coal		13.30		26.50
Operating expenses		6.74		5.50
		20.04		32.00
LESS VALUE OF RESIDUALS PRODUCED				
Coke	.82 cwt.	6.11	2.4 cwt.	17.64
Tar	.9 gal.	1.30	4.6 gal.	6.90
NH_4 products		2.11		2.80
		9.52		27.34

NET COST OF GAS

10.52 .... 4.66
B. t. u. of gas 592 .... 750.

He points out that although the coke residues are figured at the same price, coke (2) is really more valuable since it contains 15% of volatile matter which increases its calorific value. He states also that the low temperature tar distillates contain valuable fractions of a character different from those obtained from ordinary gas tar, one of which is especially suitable for use in motors as a fuel.

Burgess and Wheeler[3] working on the problem of the prevention of mine dust explosions, and recognizing the relationship

[1]Jour. of Am. Chem. Soc. 29, 1523. (1907).
[2]Engineering. 85–410. (1908).
[3]Jour. Chem. Soc. 97–1917. (1910).

that exists between the character of the volatile matter escaping from a heated coal, and its degree of inflammability, studied the composition of the gases evolved at different temperatures.

They found that with all coals whether bituminous, semi-bituminous, or anthracite, there was a well-defined decomposition point at a temperature between 700° and 800° which corresponds to a marked increase in the quantity of hydrogen evolved. This increase they attribute to the thermal decomposition of one or more of the higher homologues of methane yielding hydrogen and carbon. Ethane, propane, butane, and, probably, higher members of the paraffin series, form a large percentage of the gases given off at temperatures below 450°; above 700° they no longer appear.

They believe that the smoke producing elements consist almost entirely of the higher paraffins and differ from Porter and Ovitz in excluding ethylene and the related unsaturated gases from this class. This view is based upon experiments made showing that ethylene decomposing at 600°, deposited very little carbon.

A typical analysis of the gases obtained is given below.

TABLE 15

GAS FROM COAL FROM ABERTILLERY, SOUTH WALES (BITUMINOUS)

Coal (C)	Temp.	Illum.	CO_2	CO	H_2	CH_4	C_2H_6
	500°	5.8	3.9	4.7	8.0	64.5	11.0
	600°	4.9	3.2	6.4	25.0	47.2	12.4
	700°	2.8	3.4	7.4	34.7	46.2	4.2
	800°	2.8	2.5	9.8	50.8	28.6	4.7
	1100°	4.2	1.4	13.0	60.7	18.8	1.8

In a second paper[2] they discuss the results obtained by subjecting coals to a series of fractional distillations in a vacuum and determining the compositions of the gases evolved within well defined limits of temperature. They succeeded by prolonged exhaustion at a low temperature, in removing entirely the paraffin-yielding constituents and leaving behind a compound which decomposed at a comparatively high temperature, yielding only hydrogen. They conclude, therefore, that coal is composed largely of two types of compounds, the one unstable, giving no hydrogen, the other more stable yielding hydrogen only.

[1]Jour. Soc. Chem. Ind. 5, 2. (1886).

[2]Jour. Chem. Soc. April, 1911, p. 649.

G. E. Davis[1], discussing the tars formed under different conditions, says that at low temperatures are produced mainly such hydrocarbons as belong to the paraffin series having the general formula $C_n H_{2n+2}$, along with the olefines $C_n H_{2n}$. The lower members of these series are liquid, and, furnished in the pure state, are illuminating and lubricating oils; the higher ones are solid and form commercial paraffin. They are always accompanied by phenols. Liquid products prevail and among the watery substances acetic acid predominates.

If, on the other hand, the coal has been decomposed at a very high temperature, the molecules are grouped quite differently. While olefines and acetylenes occur more or less the paraffins disappear almost entirely with the resultant deposition of carbon.

Some of this carbon set free is deposited in the retort in a compact graphitoidal form; some occurs in a state of extremely fine division in the tar and forms a constituent of the pitch or coke remaining behind. At the same time the action of heat effects molecular condensations by which process compounds of a higher molecular weight are formed, such as naphthalene, anthracene and phenanthrene.

Behrens[2] found that the tar obtained in the distillation of coal in the ordinary fire-clay gas-retorts (operated at high temperatures) was much richer in benzene, toluene, naphthalene, etc., than the tar made in Pauwel's coke ovens (operated at low temperatures) from the same kind of coal.

Lunge[3] thinks that at low temperatures most of the nitrogen of the tar is in the form of aniline and fatty amines (ethylamine, propylamine, amylamine); at high temperatures in the form of pyridine bases, picoline, lutidine, viridine, etc. He admits that the statement needs verification by more detailed investigations. In general, at high temperatures the tendency to complete dissociation becomes far more pronounced; the products approach more and more to free carbon on the one hand and free hydrogen on the other.

Watson Smith[4] states that naphthalene increases with rise of temperature. This is true also of anthracene, which is then found in the creosote oil coming over before the anthracene oil proper. Carbolic acid is also an important constituent.

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[1]Jour. Soc. Chem. Ind. 5, 2. (1886).
[2]Dingler's Polyt. Journal 208, 362.
[3]Coal Tar and Ammonia. p. 26, (1900).
[4]Jour. Soc. Chem. Ind. 8, 950, (1890).

II. Studies in General on the Carbonization of Coal

F. C. Keighley[1] argues that since the chemical constituents of coals from any horizon are not necessarily indicative of their coking properties, it is reasonable to assume that an important factor determining the coking quality must be one of a physical character and not altogether chemical.

It is known, he says, that the finest coking coals not only are of the bituminous class, but their structure is such that upon fracture they exhibit a fingery or prismatic form and separate vertically, while the more difficult coking coals and the ones of a bituminous character that cannot be coked at all, are of a laminated structure and upon fracture break into cubical form and have a tendency to separate horizontally instead of vertically. This he thinks would indicate that the coking property depends very largely upon the arrangement of the small particles of coal composing the seam. If these lie in the seam with their longer axes horizontal to the bedding of the seam they are unfavorable to the coking process. On the other hand, if they are perpendicular to the strike of the seam, i.e., at right angles with its bedding, the coking tendency is much more pronounced. He suggests that the superiority of Connellsville coke may be due to the structure given it in the process of formation by the peculiar geological movements of the region in which it is found.

M. A. Pishel[2] suggests a simple practical test for coking coal. Pulverize the coal to 100 mesh in an agate mortar. Pour out the dust and observe its condition. If it adheres strongly to the mortar, it will probably make good coke, he says. If there is little adhesion, coking properties are absent. In his experimental work he tested more than 150 different specimens. Of the four Illinois coals tried, none stuck to the mortar while most of the Eastern coals adhered. He offers no theory to account for this phenomenon.

Groves and Thorp[3] classify coals with respect to their coking properties as sand coals, those devoid of coking powers; sinter coals, those possessing it to a relatively slight degree; coking coals, those which produce a good quality of coke, and anthracite.

[1] Iron Age. 80-364. Aug. 1907. Mines and Minerals Oct. 1907.
[2] Econ. Geol. June-July 1908. p. 265-270.
[3] Chem. Tech. Vol. 1, p. 122 (ed. 1889)

They give the following analytical table made up from the work of Richardson, Regnault and others:

TABLE 16

	(Percentage)		
	C	H	O
Sand coal	77	5	18
Sinter coal	83	5	12
Coking coal	87	5	8
Anthracite	95	3	2

TABLE 17

Anthracite	80C + 88H + 0
Blanzy sinter	80C + 128H + 60
Lancashire cannel sinter	80C + 128H + 30
Mons coking	80C + 24CH + 50
Grand Croix-highly coking	80C + 112H + 30

It will be observed from Table 16 that the amount of hydrogen in the first three varieties is identical, while the oxygen diminishes as the coking property is developed. The Grand Croix coal (Table 17) has only half the amount of hydrogen contained in the coking coal from Mons. Anthracite, consisting almost entirely of carbon, may be considered a kind of natural coke.

They state in conclusion, however, that Stein of Dresden has shown that coking and non-coking coals may have the same ultimate composition and that simple analyses, therefore, cannot determine absolutely the coking property of the coal. They suggest that the real source of coking lies in a resinoid body or bodies identical in composition with the coke itself.

White and others[1] mention the work of Ste. Clare Deville, consulting chemist of the Paris Gas Company, who, on the basis of results of nearly 2000 tests, divided coals into groups according to the relations of their percentages of oxygen to hydrogen. He found that all coking coals contain a percentage of oxygen approximately twice that of hydrogen.

They reasoned that possibly the artificial application of heat which gives as its first products water and other compounds rich in oxygen, would lower the relatively high oxygen of the non-coking coals and possibly bring them into the coking class. They found, however, that coals which were originally non-coking were not improved in this respect even though the oxygen-

[1]Am. Gas Light Jour. 89-621. (1906)

hydrogen ratio was brought down to 2 to 1. The coking coals tested sintered together during the heating and if the resultant mass was heated to redness it retained its shape and gave a good coke. If, however, it was powdered before being heated, it remained a powder.

Dr. Haberman[1], in studying the spontaneous heating of coals, noted the fact that long storage tends to destroy both gasifying and coking properties. He found that those coals that oxidized the most and gave the greatest rise in temperature absorbed the largest quantities of bromine.

Professor Fischer[2] of Göttingen, working on the same problem, mentions the loss of coking suffered by oxidized coals. He too suggests the bromine absorption test for determining the chemical activity of the fuel.

Parr and Lindgren[3] doing work on the weathering of coal at the University of Illinois observed that in volatile matter determinations, samples exposed for several months gave powdery residues instead of coke as in the case when fresh coals were used.

David White[4], in his bulletin "The Effect of Oxygen in Coal," after discussing the negative calorific value of the oxygen and the transition between various grades of coal due to progressive devolatilization brought about more or less directly by dynamic forces, takes up a study of the relative proportion of oxygen, hydrogen, and carbon, in coking coals with special reference to a theory framed to explain the coking quality.

He mentions the work of Regnault[5] and Bertrand,[6] who found that the high percentage of volatile matter and the high illuminating value of certain bogheads and oil shales are due to the presence of immense numbers of supposed gelatinous algæ which, in these coals, seem to have exercised a selective attraction for certain bituminous compounds. Likewise, the conditions of accumulation and deposition attending the origin of many coals were doubtless favorable for the mingling of algæ and different animal remains with the debris of higher plant types.

Mr. White thinks it is more than probable that the substances of these lower organisms contributed as ingredients to

[1]Schillings Jour. fur Gasbel. 49-419, (1906)
[2]The Gas World. April 13, 1901.
[3]Unpublished reports of supplementary studies to Bulletin No. 17, University of Illinois, Engineering Experiment Station. (1911)
[4]Bull. U. S. G. S. 382 (1909)
[5]Regnault, B. Les micro organismes des combustibles fossiles, St. Etienne, 1903.
[6]Bull. Soc. d'hist. Nat. Antun, Vol. 9, 1897, p. 193.

the mass of coal-forming material, and that they, therefore, exerted some influence on the character and quality of the final residues. He considers the higher percentage of bituminous matter in the older and more altered condition of the fuel, due to concentration as the result of devolatilization of the coal by dynamochemical processes, the larger part of the concentration being the result of loss of oxygen, this loss being disproportionately great as compared with that of hydrogen. Thus, the progressive deoxygenation of the organic matter accomplishes bituminization.

Now, he continues, the qualities of fusibility and swelling concurrent with bituminization which appear to characterize fuels known to contain quantities of gelatinous micro-algæ, are also necessary to the coking quality in coals, and he thinks it permissible, therefore, to inquire whether the coking property may not be due to some unascertained proportion of gelatinous algal matter entering into the original mass from which the coal was formed and imparting to it this fusibility and tendency to swell.

While the presence of micro-algal ingredients has been noted in peats and even in some brown coals, yet it is very evident that their detection by microscopical means in the highly metamorphosed coking coals, is so difficult as to be practically impossible. The evidence of chemical analysis must therefore, be called into service. The coals, he says, whose large volatile combustible matter contains relatively the highest hydrogen and the lowest oxygen, thus approaching nearest the bitumen analyses, are those in which the organic remains described as micro-algæ are most predominant and best preserved. If then, in the high volatile coals high bituminization and gelatinous algal ingredients go together and the presence of the latter causes the coal to fuse and swell, we may conclude that high volatile coals that show sufficiently high bituminization will coke by the ordinary process. The degree of bituminization is indicated by the relative excess of hydrogen as compared with the diminished oxygen in dry coal and is expressed by the ratio H:O.

Data covering the tests of over 300 coals from different localities furnished by the U. S. Geological Survey are given. It was found that those coals having a H:O ratio of 59 or more, coke by the ordinary commercial process. Nearly all below 59 and above 55 so far as tested, make a coke. Those below 55 us-

ually give a poor and dark product. The best cokes obtained by the ordinary process were made from coals having a ratio of 60 or over. It was noted however that with coals with a fixed carbon value of over 79 per cent the rule breaks down.

He remarks that his hypothesis appears to harmonize with the tendency of coking coals to cohere when reduced to fine powder, discussed by M. A. Pishel.

O. Boudouard[1] took up the study of coals with the specific purpose of determining the causes of coking and selected for experiments samples of (1) English anthracite, (2) Courrieres (¼ bituminous), (3) Belgian forge coal, (4) Forge coal of unknown origin which has lain in the laboratory several years, (5) Bruay (¾ bituminous), (6) Coal of unknown origin, (7) Lignite.

The following table gives the results of the approximate analyses:

TABLE 18

COMPOSITION OF COALS BEFORE TREATMENT

	1	2	3	4	5	6	7
Fixed carbon..........	88.6	89.5	70.5	79.1	39.3	51.4	37.3
Ash.	2.5	1.6	4.6	2.6	3.1	2.3	4.2
Volatile matter.....	8.8	8.8	24.8	18.1	37.6	46.2	58.4
Character of coke	powdery	powdery	hard	hard	hard	slightly caked	powdery
Hardness[2]	0	0	3	3	3	0	0

These coals were successively subjected for periods of 105 hr. each to the action of air at 15° and 100°. After the first treatment little change in the coal and in the appearance and character of the coke was noted except that No. 6 and 7 showed traces of humic acid. In contrast with this, after being heated at 100°, none had retained their coking powers and all but (1) and (2) contained humic acid. A marked increase in weight due to oxygen absorption was observed, amounting in some cases to nearly 5 per cent.

He further treated 25 grams of each of the coals studied with 150 grams of concentrated nitric acid for a period of 2½ months. Analyses of the residues gave the following results:

[1]Bull de Ca. Sec. Chim. 5 (series 4) 365-390 (1909).

[2]The relative hardness of the coke is indicated by the figures 3, 2, 1, 0,—3 denoting a hard compact coke, 0, a powdery residue.

TABLE 19

COMPOSITION OF COALS AFTER TREATMENT WITH NITRIC ACID

	1	2	3	4	5	6	7
Per cent change in weight . . .	+ 15.6	+ 26.0	+ 6.4	+ 20.4	+ 17.2	− 14.0	− 36.8
Fixed carbon.	68.1	54.7	56.5	51.5	49.6	43.2	39.4
Ash.....	1.8	.41	1.5	6.1	1.6	.72	.61
Vol. matter . .	30.1	44.8	41.9	42.2	48.7	56.0	59.9
Appearance of coke	powdered	powdered	powdered	powdered	traces of agglomerate	traces of agglomerate	powdered
Humic Acid . .	0	0	15 per cent	8 per cent	50 per cent	40 per cent	27 per cent

Organic solvents such as ligroin, pyridine, benzene, carbon disulphide, carbon tetrachloride and the like, modified in no appreciable way the quality of the coke produced. Concentrated sulphuric acid destroyed the coking power; concentrated hydrochloric acid had no effect.

In none of these coals did humic acid exist before treatment and since its presence was always constant in the same oxidized coals which had in the process lost their coking powers, working on the theory that the carbohydrates were responsible for the origin of the acid, he found that starch or sugar treated with bromine water, for instance, yielded humic acid much like that obtained from coal.

It is probable, he thinks, that the hydrocarbonaceous substances giving rise to this acid do not exist in a single form but in a state of great condensation, and polymerization is a result of the decomposition of the living matter, the principal characteristics of this series of processes being the disintegration of the plant tissues and the accumulation of carbon at the expense of hydrogen and oxygen.

In his comparative studies of natural and oxidized coals, he noted that the production of a very small quantity of humic acid (less than 1 per cent) marked the disappearance of the coking qualities of the original sample.

In this connection the theories advanced by Professor Lewes, already referred to, on page 25, are of interest, harmonizing as they do with Boudouard's work and presenting some of the most modern lines of thought on this subject.

Dennstedt and Bunz[1] hold with Boudouard that humic acids

[1]Zeitsch. f. ang. Chem. 21, 1825. (1908).

are the ultimate oxidation products of coals and the most inflammable coals are those that produce the largest quantities of the acid.

The exact nature and composition of the so-called humic acids, however, seem to be unknown. Boudouard[1] quotes the results of several experimenters who produced the substance by treating sugar with acids. The empirical formulas (no structural formulas are attempted,) range from $C_{24}H_{18}O_9$ (Stein) to $C_{40}H_{24}O_{12}$ (Mulder). He himself proposes $C_{18}H_{14}O_9$ as the composition of humic products he obtained by extracting oxidized coal with potassium hydroxide.

W. C. Anderson in studying the varying coking tendencies of a number of Scotch coals, concluded that cementation is caused by the decomposition of two classes of substances; (1) resinous materials soluble in caustic potash, which break down on rapid ignition; and (2) non-saponifiable substances, some of which were volatile at 300°, others being stable at this temperature.

III. Summary of Opinions

A very brief review of the literature covering the decomposition that takes place at low temperatures in the distillation of coal, is sufficient to prove to the student that the problem in all its phases is distinctly modern. A glance at the bibliography will show that few, if any, references date back more than ten years and that most of the publications on the subject have appeared within the last two or three. Indeed, Burgess and Wheeler[2] writing in 1910, remark that "previous work has been very scanty". Furthermore, almost without exception, those investigators who have already made reports announce that their first articles are more or less incomplete and that they expect to continue along the same lines of study.

While the development of the subject is evidently still in its infancy, yet results from different sources are in many cases entirely consistent. Of particular interest in that it bears a close relationship to the problem of smoke prevention, is the fact, mentioned by nearly all authorities, that the heavy smoke-producing benzines and paraffins of high carbon content are given off at

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[1]Bull. de la Soc. Chim. 5 (series 4) 378. (1909).
Jour. Soc. Chem. Ind., 17-1013. Nov., 1898.
[2]Jour. Chem. Soc. 97—1917 (1910)

low temperatures and are practically eliminated at 500°. Attempts to separate and estimate the higher homologues of methane contained in early distillates, however, have not been entirely successful on account of a lack of adequate methods of gas analysis. Cady and McFarland[1], using liquid air, got perhaps the best results but even their scheme leaves much to be desired. Writers reporting the paraffin content of the gases studied therefore have been obliged to estimate the heavier members as "ethane", or, using the formula $C_n H_{2n+2}$, to give average values of n.

It is generally agreed further, that as temperatures rise above 500°, methane and hydrogen are the principal gas constituents, being decomposition products of not only the coal itself but of some of the gases given off at the lower temperatures. Below 400°, hydrogen is present in very small amounts. It seems fairly well established, therefore, that the density and, consequently, the calorific value of a gas varies inversely with the temperature at which it is evolved and that a very moderate heating of the coal is sufficient to remove enough of the smoke producing elements to make the combustion of the residue clean and economical.

With a very small amount of work done in determining the character of the low temperature tar distillates, a fruitful field is left for future investigation. Paraffin oils, valuable for lubricating and power generating, seem to predominate, while the equally important aromatic derivatives, as anthracene, are present to a less extent than in the high temperature runs.

The investigations of Parr and Francis prove that coal, modified by the application of moderate heat gains valuable properties and that it retains a high calorific value. In the use of certain types of coal, however, such as those of the central west, the problem of putting the residues into marketable condition demands a solution before the process can be made an economic success.

Much has been written in attempts to explain the causes of coking, or at least to define the conditions that govern it. From the work of Parr, Chamberlain, Boudouard, and others, who have studied the reactions taking place at low temperatures, it has been proved that oxygen absorption goes on rapidly when fresh coal is exposed to the atmosphere. It has been shown

[1]Jour. Am. Chem. Soc. 29—1523 (1907)

further that this absorption weakens or destroys altogether any coking properties that the original coal may have. In other words a high oxygen-hydrogen ratio marks the absence of fusibility and cementation.

The structures of the organic compounds of the coal which furnish the cementing material for coke and which are apparently attacked by oxygen, have not been determined and seem to vary somewhat in different types of coals. However they yield, on oxidation, humic acids of varying composition which decompose into powdery residues. Because of the complex nature of these substances and the difficulty experienced in isolating and identifying them, the matter of coking is still an open problem and the explanations advanced are largely hypothetical.

**Bulletin No. 1.* Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

**Circular No. 1.* High-Speed Tool Steels, by L. P. Breckenridge. 1905 *None available.*

**Bulletin No 2.* Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks 1905 *None available.*

**Circular No. 2.* Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

**Circular No. 3.* Fuel Tests with Illinois Coal. (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907,) by L.P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

**Bulletin No. 3.* The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

**Bulletin No. 4* Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

**Bulletin No. 5.* Resistance of Tubes to Collapse, by Albert P. Carman. 1906. *Fifteen cents.*

**Bulletin No. 6.* Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *Thirty-five cents.*

**Bulletin No. 7.* Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr and Henry B. Dirks. 1906. *Thirty-five cents.*

**Bulletin No. 8* Tests of Concrete: I. Shear; II. Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9 An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Reprint available Sept. 1, 1912. *Fifty cents.*

**Bulletin No. 10.* Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

**Bulletin No. 11.* The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

**Bulletin No. 12.* Tests of Reinforced Concrete T-beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

**Bulletin No. 13.* An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *Fifty cents.*

**Bulletin No. 14* Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot 1907. *None available.*

**Bulletin No. 15.* How to Burn Illinois Coal without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

**Bulletin No. 16.* A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

**Bulletin No 17* The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *Twenty cents.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

**Bulletin No. 19.* Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *Twenty-five cents.*

**Bulletin No. 20.* Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908 *None available.*

Bulletin No. 21 Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908 *Fifteen cents.*

**Bulletin No. 22.* Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *Thirty-five cents.*

Bulletin No. 23. Voids, Settlement and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

Bulletin No 24. The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*

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Bulletin No. 30. On the Rate of Formation of Carbon Monoxide in Gas Producers, by J. K. Clement, L. H. Adams, and C. N. Haskins. 1909. *Twenty-five cents.*

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* Out of print; price attached.

N. B.—A limited supply of bulletins, the titles of which are not starred, is available for gratuitous distribution.

Bulletin No. 31. Fuel Tests with House-heating Boilers, by J. M. Snodgrass. 1909. *Fifty-five cents*

Bulletin No. 32. The Occluded Gases in Coal, by S. W. Parr and Perry Barker. 1909. *Fifteen cents.*

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